Unification Without Extra Dimensions

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February 21, 2023

1 Introduction

Recently there has been a development of theories related to invariant Planck Voltages [3, 4, 5]. This is part of a larger trend in extending special relativity by postulating additional invariants of nature, another prominent example being doubly special relativity [1, 2, 6]. In a previous paper, we showed that postulating an invariant voltage at the Planck Scale can lead to unification by automatically producing the Poisson equation for gravitation from the Poisson equation for electrostatics. In this paper, we further develop the theory from a Lagrangian standpoint. We derive a modified electrostatic Lagrangian for a single dimension, and we demonstrate that the resulting field equations produce a hyperbolic tangent function for the potential. This function has horizontal asymptotes at the positive and negative Planck Voltages. It is shown that the new leading corrective term in the Taylor series for the new electrostatic potential is in fact the classical gravitational potential. We then discuss how this Lagrangian could be derived from a version of calculus in which the derivatives are formulated to be consistent with a non-additive field space. It is then discussed how the modified derivatives may allow this theory to be generalized beyond electrostatics to produce a unified theory of electricity, magnetism, and gravitation. We then discuss corresponding modifications for the Dirac equation and its implications for quantum gravity. The basic theory advanced in this paper is a unique form of unified field theory that does not assume the existence of extra dimensions, in contrast to nearly all leading theories on this subject to date.

2 Lagrangian Formulation

Having demonstrated that it is possible to derive Poisson's equation for gravitation from Poisson's equation for electrostatics, the next logical step is to develop the theory from a Lagrangian standpoint. As a simple example, we begin with the Lagrangian for a free electrostatic field in a single dimension denoted as x.

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 \tag{1}$$

The simplest way to develop a Lagrangian for this theory would be to modify the usual Maxwell Lagrangian by substituting the "voltage-boosted" potential formula directly into that Lagrangian. Inserting the "voltage-boosted" formula for the potential ϕ gives

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\frac{\partial}{\partial_x}\left(\frac{\phi}{\sqrt{1-\frac{\phi^2}{V_p^2}}}\right)\right)^2 \tag{2}$$

In the above transformation, it has been assumed that $U_1 = 0$ for simplicity. The idea is to transform from a "voltage frame" where energy is zero at the origin to another "voltage frame" where it is non-zero. After some mathematics, the Lagrangian is shown to be the following:

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\left(\frac{\partial\phi}{\partial x}\right)^2\left(\frac{1}{1-\frac{\phi^2}{V_p^2}}\right) + \left(\frac{\partial\phi}{\partial x}\right)^2\frac{\phi^4}{V_p^4}\frac{1}{\left(1-\frac{\phi^2}{V_p^2}\right)^3} + \left(\frac{\partial\phi}{\partial x}\right)^2\frac{\phi^2}{V_p^2}\frac{1}{\left(1-\frac{\phi^2}{V_p^2}\right)^2}\right) \tag{3}$$

Applying the Taylor series to the factors of $1/(1-\frac{\phi^2}{V_p^2})$ yields

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(1+\frac{2\phi^2}{V_p^2}+\frac{3\phi^4}{V_p^4}+\mathcal{O}(5)\right)\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 \tag{4}$$

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 + \frac{\epsilon_0}{2}\frac{2\phi^2}{V_p^2}\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 + \mathcal{O}(3) \tag{5}$$

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 + \epsilon_0\frac{\phi^2}{V_p^2}\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 + \mathcal{O}(3) \tag{6}$$

This strongly suggests that the above Lagrangian is actually an approximation to the following simpler formula. We now take the formula below to be the actual Lagrangian, with additional theoretical and mathematical justification being provided in future sections.

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\frac{1}{1-\frac{\phi^2}{V_p^2}}\right)^2\left(\frac{\partial(\phi)}{\partial(x)}\right)^2 \tag{7}$$

3 Euler-Lagrange Equations

The Euler-Lagrange equations can then be applied to the new Lagrangian to obtain the field equations. The field equations are found to be the following:

$$\frac{\partial^2(\phi)}{\partial x^2} * \left(1 - \frac{\phi^2}{V_p^2}\right) = -\frac{2}{V_p^2} * \phi \left(\frac{\partial(\phi)}{\partial x}\right)^2 \tag{8}$$

These equations admit an exact closed form solution which is as follows:

$$\phi(x) = -V_p * \tanh\left(\frac{C_1}{V_p} x + \frac{C_2 C_1}{V_p}\right) \tag{9}$$

Now the free electrostatic potential no longer has a linear form, but rather the form of a hyperbolic tangent. Deriving a hyperbolic tangent function is a pleasing result, as it allows the scalar potential to be non-additive and is reminiscent of velocity addition in special relativity. The function can be approximated as

$$\phi(x) \approx -C_1 * x - C_2 * C_1 \tag{10}$$

Note that since the space is 1-dimensional, the electric field is constant and does not decay with the x-coordinate. Clearly C_1 is to be identified with the electric field in the x-direction, while $C_1 * C_2$ is the negative of the potential at the origin. This allows us to re-write the full formula as

$$\phi(x) = -V_p * \tanh\left(\frac{E_x}{V_p} x - \frac{\phi_0}{V_p}\right) \tag{11}$$

Note that the hyperbolic tangent function has horizontal asymptotes at $+V_p$ and $-V_p$, the positive and negative Planck voltages. In a 1-dimensional space, the classic Maxwell electrostatic field Lagrangian causes the electric field to be constant so that the potentials will eventually reach positive and negative infinity far enough from the zero point of the potential . In contrast, the Lagrangian derived in this paper will make that voltage gradually level off so that it never quite reaches a negative or positive Planck Voltage. Note that The Taylor polynomial expansion for this expression around $x = 0$ is as follows:

$$\phi(x) \approx -V_p \tanh(-\frac{\phi_0}{V_p}) - E_x * sech^2(-\frac{\phi_0}{V_p}) * x + \frac{E_x^2}{V_p} \tanh(-\frac{\phi_0}{V_p}) * x^2 \tag{12}$$

Further approximating that $\phi << V_p$ gives the following formula.

$$\phi(x) \approx \phi_0 - E_x * x - \frac{(E_x)^2}{(V_p)^2} * \phi_0 * x^2 \tag{13}$$

We also have the following formula for potential energy:

$$PE(x) \approx q\phi_0 - qE_x * x - q\frac{(E_x)^2}{(V_p)^2} * \phi_0 * x^2 \tag{14}$$

We now seek to understand this new term in the electrostatic field equation and to demonstrate that it is, in reality, a term for the gravitational potential energy. We note that in 1-dimensional space, the electric energy density can be written as follows:

$$\rho_{energy} = \rho\phi = \epsilon_0 (E_x)^2 = \epsilon_0 * (\frac{Q}{\epsilon_0})^2 = \epsilon_0 * \frac{Q^2}{(\epsilon_0)^2} \tag{15}$$

Note that per our initial and final conditions, all of this energy was derived from a gauge transformation rather than from self-interaction. Because of this, we have omitted the usual factor of one-half from the above formula, where the one-half is typically inserted to prevent an erroneous double summation of interacting particle energies. The total energy bound between a given $-x$ and $+x$ is then

$$E_{enc} = 2x * \epsilon_0 * \frac{Q^2}{(\epsilon_0)^2} = 2x * \frac{Q^2}{\epsilon_0} \tag{16}$$

The strength of the gravitational field in a single dimension is then as follows:

$$g_x(x) = -\frac{GE_{enc}}{c^4} = \frac{4\pi GQ^2}{\epsilon_0 c^4} * 2x \tag{17}$$

For a test charge q, this force corresponds to the following potential energy

$$\phi_g(x) = \frac{4\pi GQ^2 q}{\epsilon_0 c^4} x^2 = q\frac{(E_x)^2}{V_p^2} * x^2 \tag{18}$$

Now we can rewrite the electrical potential energy formula as follows:

$$PE(x) \approx q\phi_0 - qE_x * x - q\phi_0\phi_g \tag{19}$$

$$PE(x) \approx q\phi_0 - qE_x * x - \Delta E_{test}\phi_g \tag{20}$$

The equation above emphasizes that the shifting potential changes the energy of the test particle, which then interacts with the existing gravitational field of the electromagnetic field energy. We can also write this as follows, to highlight the analogy with magnetism.

$$PE(x) \approx q\phi_{electric} - q\phi_g * \Delta V \tag{21}$$

In summary, here we found that the classical gravitational potential is one of the leading terms among a series that "corrects" the Maxwell electrostatic Lagrangian in order to enforce a maximum voltage. The higher order terms in the correction will be explored in future papers. They are clearly not connected to classical gravitation. It is not clear yet whether these new terms may be related to general relativistic-corrections to classical gravitation or whether they constitute a new prediction.

4 Next Steps

By inspection of the Lagrangian derived in this paper, it is clear that it could also be derived by directly modifying the definition of the derivative such that

$$\frac{d_m}{dx} = \left(\frac{1}{1 - \frac{\phi^2}{V_p^2}}\right) * \frac{d}{dx} \tag{22}$$

$$\mathcal{L} = \frac{\epsilon_0}{2}\left(\frac{\partial_m(\phi)}{\partial_m(x)}\right)^2 \tag{23}$$

Such a derivative is no longer invariant under additions of a constant k. But it can be shown to have the following property instead:

$$\frac{d_m}{dx}\left(\frac{\phi(x) + k}{1 + \frac{k*\phi(x)}{V_p^2}}\right) = \frac{d_m}{dx}(\phi(x)) \tag{24}$$

In other words, the modified derivative function is invariant under translations in a "non-additive" potential space. The "composition of translations" law in such a space would have to be be as follows.

$$\phi' = \frac{\phi + \Delta\phi}{1 + \frac{\phi\Delta\phi}{V_p^2}} \tag{25}$$

But the above formula is exactly the new composition of voltages law. This reveals an alternate path to deriving our Lagrangian that is more readily generalizable. We simply begin with a non-additive space, modify the derivative function for that new space, and modify the Lagrangian by substituting the modified derivative function. The modified derivative formula can be derived from the following limit. The limit is believed to generally be equivalent to the formula listed above for most simple functions.

$$\frac{d_m}{dx} = \lim_{\Delta x \to 0} \frac{\phi(x + \Delta x) - \phi(x)}{\Delta x * (1 - \frac{\phi(x)\phi(x+\Delta x)}{V_p^2})} \tag{26}$$

In future papers, this theory will be generalized to multiple physical dimensions of space. In addition, another goal will be to generalize this concept so that we can make similar changes to the full Maxwell electromagnetic field Lagrangian. We will begin with the standard Maxwell Lagrangian and will modify the definition of the derivative in the Faraday tensor formula using a generalized form of a non-additive derivative. This modified derivative will be constructed to be consistent with the elecromagnetic 4-potential being non-additive. The modified derivative will be suitable for a multi-dimensional complex space that is non-additive, and may perhaps be related to the Anti De-Sitter space covariant derivative. Future papers will also focus on the quantum gravity implications, namely the modification of the Dirac Lagrangian required for compatibility with this new theory. It is anticipated that such a Dirac Lagrangian must be compatible with theories of non-additive energy and momentum such as doubly special relativity. From doubly-special relativity, examples of such Dirac Lagrangians are already known [5].

5 Acknowledgments

Thank you to my friend Richard Kassman for reviewing this theory, providing feedback, and encouraging me to continue to develop it further.

6 References

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