Nonequilibrium Dynamics and General Relativity

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Abstract

Complex Ginzburg-Landau equation (CGLE) is a universal model of nonequilibrium dynamical systems. Focusing on the primordial stages of cosmological evolution, this work points out that the connection between CGLE and the Navier-Stokes (NS) equation bridges the gap between fluid flows and the mathematics of General Relativity (GR).

Key words: Complex Ginzburg-Landau equation, Navier-Stokes equation, gauge-gravity duality, dimensional reduction, continuous dimensions.

1. Introduction

CGLE is considered a paradigm of non-equilibrium statistical physics and dynamic critical phenomena. It encodes many key properties of collective phenomena with space-time dependence, and it models the generic onset of chaos, turbulence, and spatiotemporal patterns in extended systems [1-3]. We recently argued that applying CGLE to the chaotic dynamics of interacting fields yields unforeseen solutions to the challenges raised by high-energy theory [4-5]. The goal of this work is to expand our findings to the possible link between CGLE and the high temperature / long wavelength limit of GR.

Let's begin with the observation that there are (at least) four distinct routes leading from nonequilibrium dynamics to GR:

1. The emergence of a nonvanishing K-entropy in the unstable sector of gravitational dynamics, the N-body problem ($N > 2$) of cosmology in near or non-equilibrium conditions [6-8].

2. The emergence of a spacetime equipped with continuous dimensionality above the Fermi scale follows from several premises, one of them being the onset of Hamiltonian chaos and fractional dynamics [9-10]. Along the same

lines, it can be argued that fractional dynamics in flat spacetime is formally equivalent to classical dynamics on curved manifolds [11].

3. The geometry of Hamiltonian systems is dual to geometry on curved manifolds [12].

4. Thermodynamics of Black Holes lends support for the multifractal interpretation of horizon dynamics [13].

We believe that, besides 1) - 4), a scenario worthy of investigation is the *fluid-gravity correspondence* inspired by the gauge-gravity duality of string theory [14]. A drawback of this duality is that it operates with a negative cosmological constant, clearly at odds with current astrophysical observations. It was found in [15] that, applying the gauge-gravity conjecture to a 1+1 spacetime endowed with continuous dimensions leads to a positive cosmological constant. Besides leading to a positive cosmological constant, setting the fluid-gravity duality in 1+1 dimensions brings up two attractive features, namely, a) a low dimensional metric is compatible with the framework of *dimensional reduction* (DR) applied to the primordial stages of Universe evolution [16], b) the *duality* of hydrodynamics and high-temperature/long-wavelength gravitational dynamics in 1 + 1 dimensions necessarily turns into an *identity*, as continuous dimensions automatically overlap within an infinite range of positive values.

The paper is divided into four sections. Section 2 lists the main couple of assumptions underlying the approach, while section 3 and 4 delve into the route connecting CGLE, relativistic NS equation and General Relativity, following the straightforward diagram shown below:

CGL Equation $\Rightarrow$ NS Equation $\Rightarrow$ General Relativity

2. Assumptions

A1) The DR conjecture asserts that the number of spacetime dimensions monotonically drops with the boost in the observation scale. In a nutshell, the DR expectation is that spacetime becomes two-dimensional near the Big-Bang singularity. This conjecture is backed up by several cosmological scenarios, including the BKL ansatz and the Kasnerian regime of metric fluctuations in the primordial Universe [17-18].

A2) When applied to the fluid-gravity correspondence, DR yields a positive cosmological constant in 1+1 dimensions [15]. It is conceivable that the cosmological constant stays unchanged as the Universe expands and cools off, on account of inherent *memory effects* attributed to nonequilibrium dynamics.

3. From CGLE to the NS equation

The standard form of CGLE is given by,

$$\partial_t z = az + (1+ic_1)\Delta z - (1-ic_3) z|z|^{2\sigma} \tag{1}$$

in which z is a complex-valued field, the parameters a and σ are positive and the coefficients c_1 and c_3 are real [1-2]. The *nonlinear Schrödinger equation* (NSE) is a particular embodiment of the CGLE in the limit $a \to 0$, namely [19]

$$-i\partial_t z = c_1 \Delta z + c_3 z|z|^{2\sigma} \tag{2a}$$

In what follows we set $\sigma = 1$. In natural units ($\hbar = 1$), the quantum-mechanical version of (2a) reads,

$$i\frac{\partial}{\partial t} z(x,t) = [-\frac{1}{2m}\nabla^2 + V(x,t)] z(x,t) \tag{2b}$$

where $V(x,t)$ is the potential function. The *Madelung transformation* enables one to turn (2b) into the quantum Euler equation for compressible potential flows [20]. To this end, taking the complex-valued field in the canonical form,

$$z(x,t) = \sqrt{\frac{\rho(x,t)}{m}} \exp[iS(x,t)] \tag{3}$$

and substituting it into (1)-(2) leads to

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \tag{4}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{m} \nabla (Q + V) \tag{5}$$

Here, $u(x,t)$ denotes the flow velocity, $\rho = m|z|^2$ stands for the mass density and

$$Q = -\frac{1}{2m}\frac{\nabla^2(\sqrt{\rho})}{\sqrt{\rho}} \tag{6}$$

is the Bohm potential. The flow velocity and its associated probability current are given by, respectively,

$$u(x,t) = \frac{1}{m}\nabla S = -\frac{i}{m}\frac{\nabla z}{z} \tag{7}$$

$$j = \rho u = \frac{1}{2mi}[z^*(\nabla z) - z(\nabla z^*)] \tag{8}$$

Since the Schrödinger equation is conservative, the Madelung transformation naturally leads to the Euler equation, which is exclusively valid for *inviscid flows*. To account for fluid viscosity and arrive at the NS equation, one needs to either appeal to an extended version of the NS equation containing non-conservative terms or bring up the concept of *kinematic viscosity* – a concept linked to the mass of quantum particles as in

$$\nu = \frac{1}{2m} \tag{9}$$

By (9) and for incompressible flows, the NS equation that mirrors (5) can be written as,

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla p + \nu \nabla^2 u \tag{10}$$

where p denotes the pressure.

4. From the NS equation to gravitational dynamics

According to the gauge-gravity duality, Einstein's equations in $D = d + 1$ spacetime dimensions contain a negative cosmological constant Λ and are written as [14-15]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{11}$$

in which,

$$\Lambda = -\frac{d(d-1)}{2R_{AdS}^2}; \; d = 0,1,2,... \tag{12}$$

with R_{AdS} denoting the AdS curvature radius, a parameter that can be conveniently set to unity. On a *minimal fractal spacetime* defined in 1+1 dimensions ($\mu,\nu=0,1$), the spatial dimension flows with the Renormalization Group (RG) scale and spans a continuous range of values as in

$$d(\mu_{RG})=1-\varepsilon(\mu_{RG})\propto 1-O[\,m^2(\mu_{RG})/\Lambda_{UV}^2\,]\,;\ \varepsilon<<1 \tag{13}$$

where μ_{RG} stands for the RG scale and Λ_{UV} is the ultraviolet cutoff. In contrast with the conventional gauge-gravity duality, it follows from (13) that (12) turns into a *positive* cosmological constant, that is,

$$\overline{\Lambda}=\Lambda R_{AdS}^2=O(\varepsilon)>0 \tag{14}$$

Following (13) and [14], in the high temperature / long wavelength limit of gravitational dynamics, Einstein's equations reduce to the NS equations (10) in one-dimensional space ($d=1$).

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