Bangiya Sabdakosh and The Graphical Law

Anindya Kumar Biswas*

Department of Physics;

North-Eastern Hill University,

Mawkynroh-Umshing, Shillong-793022.

(Dated: February 13, 2023)

Abstract

We study the Bangiya Sabdakosh: A Bengali-Bengali lexicon compiled by the Late Haricharan Bandyopadhyay. We draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that the dictionary can be characterised by BW(c=0.01), the magnetisation curve of the Ising Model in the Bragg-Williams approximation in the presence of external magnetic field, H. $c = \frac{H}{\gamma\epsilon} = 0.01$ with ϵ being the strength of coupling between two neighbouring spins in the Ising Model, γ representing the number of nearest neighbours of a spin, which is very large.

* anindya@nehu.ac.in

a	*á*	i	í	u	*ú*	ṛi	ṛí	li	lí	e	ei	o	*ó*	ou	ka	kha	ga	gha	gna	cha	chha	ja	jha
5962	2879	577	83	1631	146	98	3	4	5	578	50	190	0	66	4445	990	1823	429	2	1347	481	1151	364
nya	ṭa	ṭha	ḍa	ḍha	ṇa	ta	tha	da	dha	na	pa	pha	ba	bha	ma	ya	ra	la	va	sha	ṣha	sa	ha
6	354	179	303	158	3	1693	160	1902	639	2112	4613	573	5267	1365	3750	882	1659	1073	0	2502	191	6620	1991

TABLE I. Bangiya Sabdakosh words: the odd rows represent letters of the "Kannada" alphabet,[4], in the serial order, omitting mostly non-zero words, the even rows represent the number of words of the Bangiya Sabdakosh, [1].

I. INTRODUCTION

"....Moter upore, erup abhidan bangala bhaṣhay itipurbe bahir hoy nai."—-Suniti Kumar Chattopadhyay, an eminent linguist.

The abhidan(dictionary) is the Bangiya Sabdakosh: A Bengali-Bengali lexicon compiled by the Late Haricharan Bandyopadhyay, [1]. This is unique among the others of its folks, was published part be part over a span of fourteen years to get at the end a forward from Rabindranath Tagore. There is a clear cut separation between the set of words and the set of explanations appearing with the meaning of the words. The explanations were drawn through thorough researches from literature s, mythologies, folklore s, histories, geographies, cultures etc. This dictionary can stand as the standard for a dictionary of a language of a particular age. This mammoth dictionary spreads over two thousand four hundred pages.

The other bengali-bengali dictionaries, we have studied are Samsad Bangla Abhidan compiled by Sailendra Biswas, the fifth edition, [2] and Chalantika, [3], before. Also we have studied embedding the bengali letters in the Kannada alphabet ala, [4]. We do that in this paper also, almost. We replace the Kannada letters lu and lú by li and lí respectively. We count each and every word of the Bangiya Sabdakosh, [1]. We have excluded the Parishishta(addendum) from counting. The result is the table, I. To visualise we plot the number of words against the respective letters in the dictionary sequence,[1],[4], in the adjoining figure, fig.1. We put the Samsad Bangla Abhidan compiled by Sailendra Biswas, the fifth edition, [2] and Chalantika, [3] in the context in the following in the tables, II III and pictorially represent the number of words, entries against the respective letters in the dictionary sequence,[4], in the adjoining figure, fig.2.

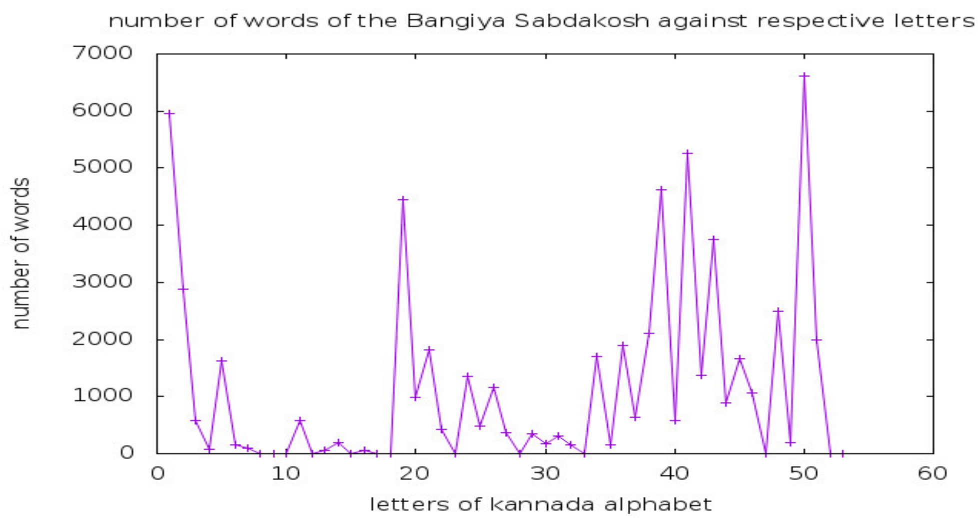

FIG. 1. The vertical axis is the number of words of the Bangiya Sabdakosh, [1]. The horizontal axis is the letters of the "Kannada" alphabet. Letters are represented by the sequence number in the alphabet as it appears in the dictionary, [4].

a	*á*	i	í	u	*ú*	ṛi	e	*é*	ei	o	*ó*	ou	ka	kha	ga	gha	gna	cha	chha	ja	jha	nya	ṭa
4295	2029	271	59	1205	53	47	494	0	50	173	0	46	3864	835	1725	480	0	1531	495	1139	318	0	401
ṭha	ḍa	ḍha	ṇa	ta	tha	da	dha	na	pa	pha	ba	bha	ma	ya	ra	la	va	sha	ṣha	sa	ha	ḷa	kṣha
172	287	155	7	1653	153	2103	788	2263	4503	565	4745	1091	2902	577	1324	796	0	1586	88	4182	1119	0	0

TABLE II. Samsad Bangla Abhidan words: the odd rows represent letters of the "Kannada" alphabet,[4], in the serial order, omitting mostly non-zero entries, the even rows represent the number of entries of the Samsad Bangla Abhidan, [2].

a	*á*	i	í	u	*ú*	ṛi	e	*é*	ei	o	*ó*	ou	ka	kha	ga	gha	gna	cha	chha	ja	jha	nya	ṭa
2595	1397	177	35	1034	30	25	237	0	28	113	0	30	2314	599	1157	316	0	988	350	895	235	0	236
ṭha	ḍa	ḍha	ṇa	ta	tha	da	dha	na	pa	pha	ba	bha	ma	ya	ra	la	va	sha	ṣha	sa	ha	ḷa	kṣha
137	191	134	0	1078	102	1392	515	1463	3196	392	3170	791	1773	356	737	434	0	955	47	2530	629	0	0

TABLE III. Chalantika words: the odd rows represent letters of the "Kannada" alphabet,[4], in the serial order, omitting mostly non-zero entries, the even rows represent the number of the Chalantika words, [3].

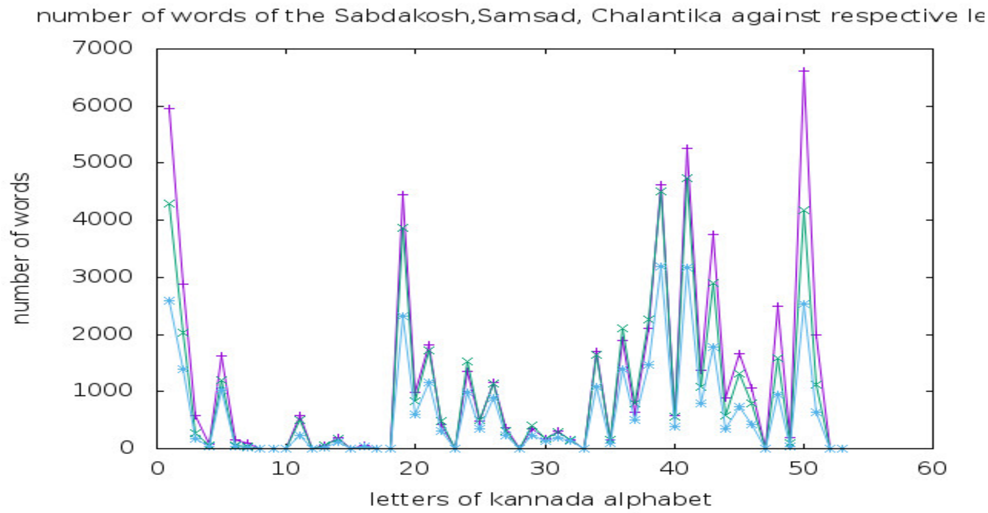

FIG. 2. The vertical axis is the number of words(entries), in red(green, blue), of the Sabdakosh(Samsad, Chalantika), Bengali-Bengali dictionary, [1]([2],[3]). The horizontal axis is the letters of the "Kannada" alphabet. Letters are represented by the sequence number in the alphabet as it appears in the dictionary, [4].

Next on to the Graphical Law, we proceed in the rest of the paper. We have started considering magnetic field pattern in [5], in the languages we converse with. We have studied there, a set of natural languages, [5] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law.

Then, we moved on to investigate into, [6], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[7] and the basque language[8]. This was pursued by finding of the graphical law behind the Romanian language, [9], five more disciplines of knowledge, [10], Onsager core of Abor-Miri, Mising languages,[11], Onsager Core of Romanised Bengali language,[12], the graphical law behind the Little Oxford English Dictionary, [13], the Oxford Dictionary of Social Work and Social Care, [14], the Visayan-English Dictionary, [15], Garo to English School Dictionary, [16], Mursi-English-Amharic Dictionary, [17] and Names of Minor Planets, [18], A Dictionary of Tibetan and English, [19], Khasi English Dictionary, [20], Turkmen-English Dictionary, [21], Websters Universal Spanish-English Dictionary, [22], A Dictionary of Modern Italian, [23], Langenscheidt's German-English Dictionary, [24], Essential Dutch dictionary by G. Quist and D. Strik, [25], Swahili-English dictionary by C. W. Rechenbach, [26], Larousse Dictionnaire De Poche for the French, [27], the Onsager's solution behind the Arabic, [28], the graphical law behind Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung, [29], the graphical law behind the NTC's Hebrew and English Dictionary by Arie Comey and Naomi Tsur, [30], the graphical law behind the Oxford Dictionary Of Media and Communication, [31], the graphical law behind the Oxford Dictionary Of Mathematics, Penguin Dictionary Of Mathematics, [32], the Onsager's solution behind the Arabic Second part, [33], the graphical law behind the Penguin Dictionary Of Sociology, [34], behind the Concise Oxford Dictionary Of Politics, [35], a Dictionary Of Critical Theory by Ian Buchanan, [36], the Penguin Dictionary Of Economics, [37], the Concise Gojri-English Dictionary by Dr. Rafeeq Anjum, [38], A Dictionary of the Kachin Language by Rev.O.Hanson, [39], A Dictionary Of World History by Edmund Wright, [40], Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool, [41], A Dictionary of Plant Sciences by Michael Allaby, [42], respectively. The graphical law was pursued more in Along the side of the Onsager's solution, the Ekagi language ,[43], Along the side of the Onsager's solution, the Ekagi language-Part Three, [44], Oxford Dictionary of Biology by Robert S. Hine and the Graphical law, [45], A Dictionary of the Mikir Language by G. D. Walker and the Graphical law, [46], A Dictionary of Zoology by Michael Allaby and the Graphical Law, [47], Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical Law, [48], Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law, [49], A Greek and English Lexicon by H.G.Liddle et al simplified by Didier Fontaine and the Graphical law, [50], Learner's Mongol-English Dictionary and the Graphical law, [51], Complete Bulgarian-English Dictionary and the Graphical law, [52], A Dictionary of Sindhi Literature by Dr. Motilal Jotwani and the Graphical Law, [53], Penguin Dictionary of Physics, the Fourth Edition, by John Cullerne, and the Graphical law, [54], Oxford Dictionary of Chemistry, the seventh edition and the Graphical Law, [55], A Burmese-English Dictionary, Part I-Part V, by J. A. Stewart and C. W. Dunn et al, head entries and the Graphical Law, [56], The Graphical Law behind the head words of Dictionary Kannada and English written by W. Reeve, revised, corrected and enlarged by Daniel Sanderson, [57], Sanchayita and the Graphical Law, [58], Samsad Bangla Abhidan and The Graphical Law, [59], respectively.

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the words of the Bangiya Sabdakosh: A Bengali-Bengali lexicon compiled by the Late Haricharan Bandyopadhyay, [1]. The section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II. THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, [66], denoted by k. k is a positive integer starting from one. Moreover, the minimum non-zero number of words is two. Hence, we attach a limiting word number one. The limiting rank is maximum rank plus one, here it is forty six. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,IV, and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.3. We then ignore the letter with the highest of words, tabulate in the adjoining table,IV,and redo the plot, normalising the lnfs with lnf_{n-max}, and starting from $k=2$ in the figure fig.4. This programme we continue to get up to the figure fig.7

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{n-max}	lnf/lnf_{2n-max}	lnf/lnf_{3n-max}
1	0	0	6620	8.798	1	Blank	Blank	Blank
2	0.69	0.180	5962	8.693	0.988	1	Blank	Blank
3	1.10	0.287	5267	8.569	0.974	0.986	1	Blank
4	1.39	0.363	4613	8.437	0.959	0.971	0.985	1
5	1.61	0.420	4445	8.400	0.955	0.966	0.980	0.996
6	1.79	0.467	3750	8.230	0.935	0.947	0.960	0.975
7	1.95	0.509	2879	7.965	0.905	0.916	0.930	0.944
8	2.08	0.543	2502	7.825	0.889	0.900	0.913	0.927
9	2.20	0.574	2112	7.655	0.870	0.881	0.893	0.907
10	2.30	0.601	1991	7.596	0.863	0.874	0.886	0.900
11	2.40	0.627	1902	7.551	0.858	0.869	0.881	0.895
12	2.48	0.648	1823	7.508	0.853	0.864	0.876	0.890
13	2.56	0.668	1693	7.434	0.845	0.855	0.868	0.881
14	2.64	0.689	1659	7.414	0.843	0.853	0.865	0.879
15	2.71	0.708	1631	7.397	0.841	0.851	0.863	0.877
16	2.77	0.723	1365	7.219	0.821	0.830	0.842	0.856
17	2.83	0.739	1347	7.206	0.819	0.829	0.841	0.854
18	2.89	0.755	1151	7.048	0.801	0.811	0.822	0.835
19	2.94	0.768	1073	6.978	0.793	0.803	0.814	0.827
20	3.00	0.783	990	6.898	0.784	0.794	0.805	0.818
21	3.04	0.794	882	6.782	0.771	0.780	0.791	0.804
22	3.09	0.807	639	6.460	0.734	0.743	0.754	0.766
23	3.14	0.820	578	6.360	0.723	0.732	0.742	0.754
24	3.18	0.830	577	6.358	0.723	0.731	0.742	0.754
25	3.22	0.841	573	6.351	0.722	0.731	0.741	0.753
26	3.26	0.851	481	6.176	0.702	0.710	0.721	0.732
27	3.30	0.862	429	6.061	0.689	0.697	0.707	0.718
28	3.33	0.869	364	5.897	0.670	0.678	0.688	0.699
29	3.37	0.880	354	5.869	0.667	0.675	0.685	0.696
30	3.40	0.888	303	5.714	0.649	0.657	0.667	0.677
31	3.43	0.896	191	5.252	0.597	0.604	0.613	0.622
32	3.47	0.906	190	5.247	0.596	0.604	0.612	0.622
33	3.50	0.914	179	5.187	0.590	0.597	0.605	0.615
34	3.53	0.922	160	5.075	0.577	0.584	0.592	0.602
35	3.56	0.930	158	5.063	0.575	0.582	0.591	0.600
36	3.58	0.935	146	4.984	0.566	0.573	0.582	0.591
37	3.61	0.943	98	4.585	0.521	0.527	0.535	0.543
38	3.64	0.950	83	4.419	0.502	0.508	0.516	0.524
39	3.66	0.956	66	4.190	0.476	0.482	0.489	0.497
40	3.69	0.963	50	3.912	0.445	0.450	0.457	0.464
41	3.71	0.969	6	1.792	0.204	0.206	0.209	0.212
42	3.74	0.977	5	1.609	0.183	0.185	0.188	0.191
43	3.76	0.982	4	1.386	0.158	0.159	0.162	0.164
44	3.78	0.987	3	1.099	0.125	0.126	0.128	0.130
45	3.81	0.995	2	0.693	0.079	0.080	0.081	0.082
46	3.83	1	1	0	0	0	0	0

TABLE IV. Bangiya Sabdakosh words: ranking,natural logarithm, normalisations

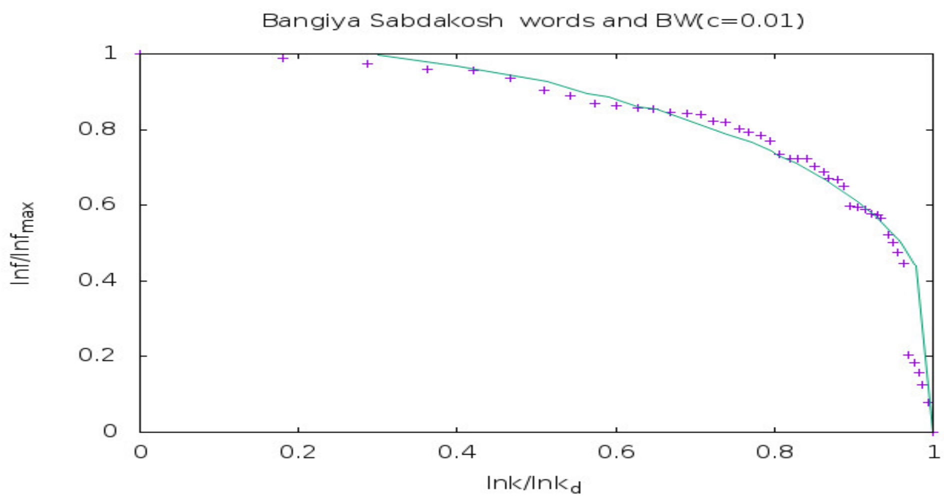

FIG. 3. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Bangiya Sabdakosh words, with the fit curve being the Bragg-Williams curve in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$.

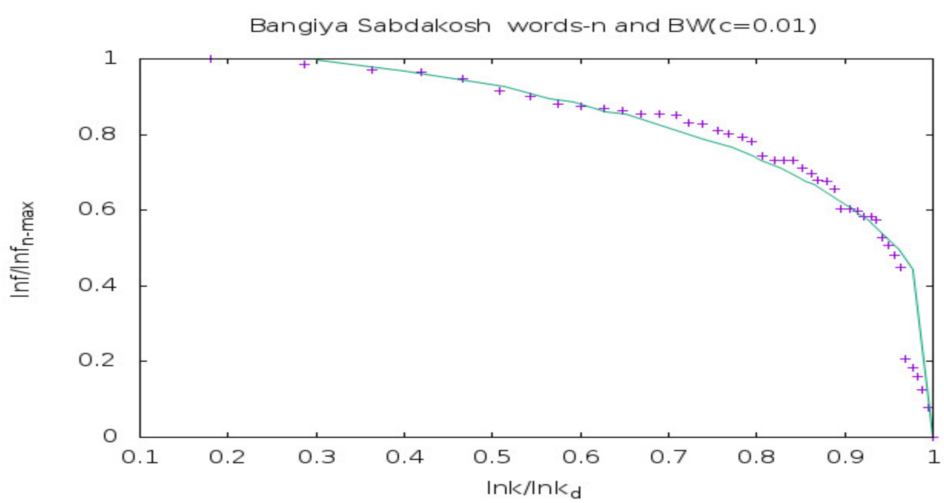

FIG. 4. The vertical axis is $\frac{lnf}{lnf_{n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Bangiya Sabdakosh words, with the fit curve being the Bragg-Williams curve in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$.

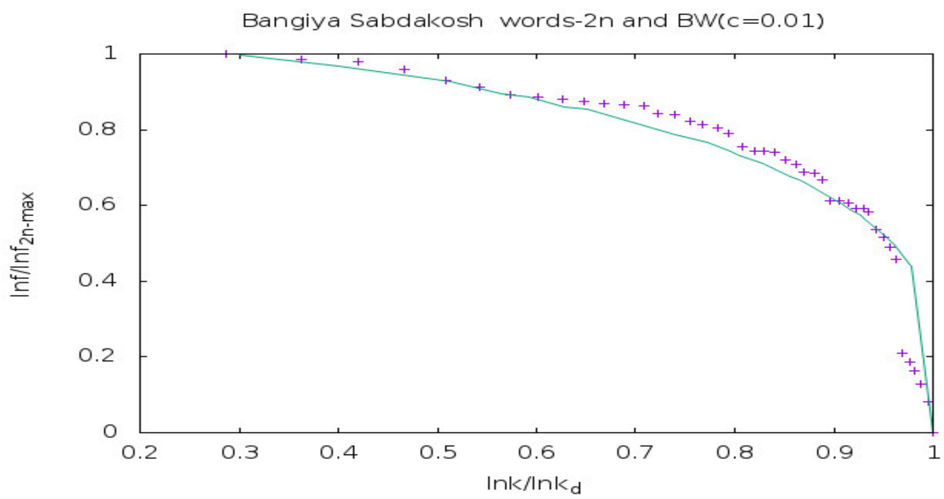

FIG. 5. The vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Bangiya Sabdakosh words, with the fit curve, being the Bragg-Williams curve in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$.

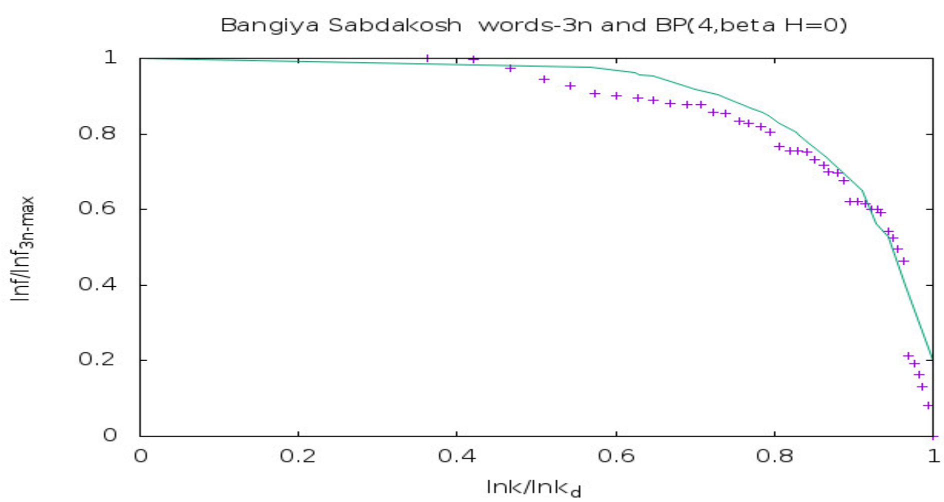

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Bangiya Sabdakosh words, with the fit curve, BP(4,$\beta H = 0$), being the Bethe-Peierls curve in the presence of four nearest neighbours and no external magnetic field, $m = 0$ or, $\beta H = 0$.

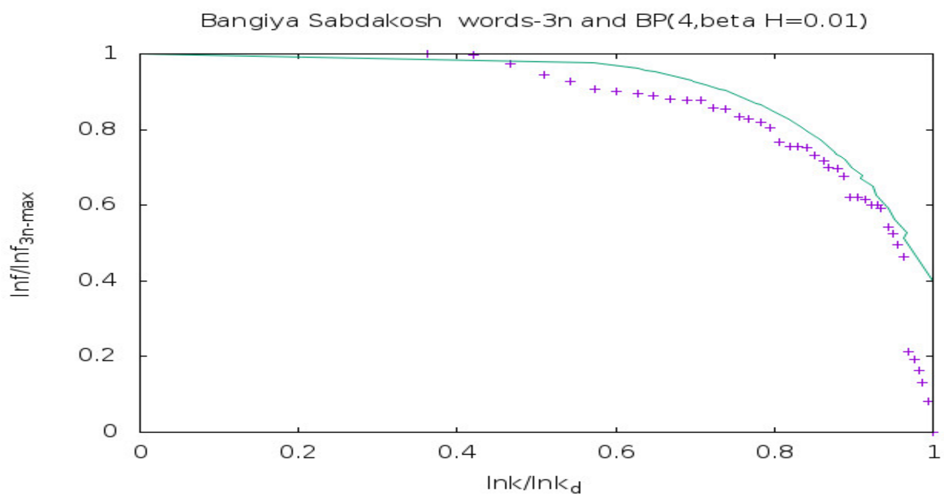

FIG. 7. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Bangiya Sabdakosh words, with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.

A. conclusion

From the figures (fig.3-fig.7), we observe that there is a curve of magnetisation, behind the Bangiya Sabdakosh words,[1]. This is the magnetisation curve in the Bragg-Williams approximation of the Ising model, in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$. Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [67].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[60], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [61], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [62]. In the Bragg-Williams approximation,[63], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \tag{1}$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [64]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [61]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [60],[61],[62],[63],[64], due to Bethe-Peierls, [65], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

BW	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE V. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours.

data s generated from the equation(1) and the equation(2) in the table, V, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, V, mean corresponding point pairs were not used for plotting a line.

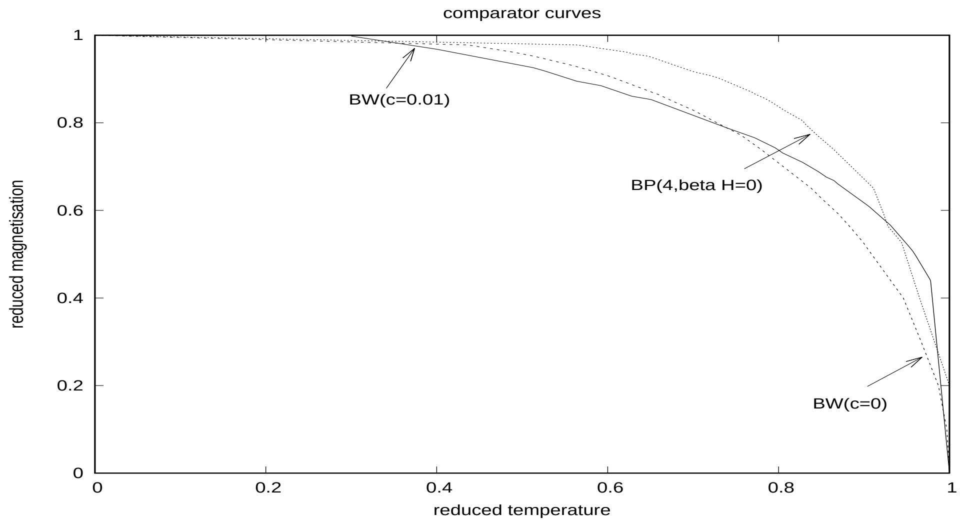

FIG. 8. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [65], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula ala [65] is given in the appendix of [10].
$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe datas in the table, VI, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that

$\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.2. Empty spaces in the table, VI, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE VI. Bethe-Peierls approx. in presence of little external magnetic fields

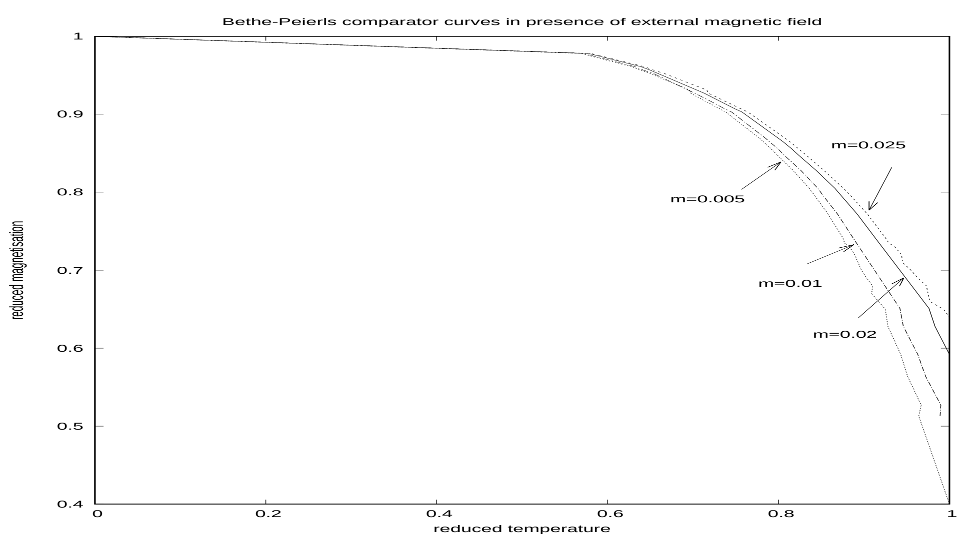

FIG. 9. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

IV. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

---

[1] "Bangiya Sabdakosh": A Bengali-Bengali lexicon compiled by the Late Haricharan Bandyopadhyay, first published 1340-1353 Bongabda; Eighth printing 2011, Sahitya Akademi, Printer: Jayshree Press, 91/1B Baithakkhana Road, Kolikata 700 009; ISBN 978-81-260-2661-6.

[2] Sailendra Biswas, "Samsad Bangla Abhidan(A Dictionary of the Bengali Langugae)", the fifth edition, revised by Subhas Bhattacharya; published in October 2000, by Debojyoti Dutta, Shishu Sahitya Samsad Pvt. Ltd., 32A Acharya Prafulla Chandra Road, Kolkata-700009; the eighth printing, March 2013. ISBN 978-81-86806-92-X.

[3] Rajsekhar Basu Sankalita(edited), "Chalantika, Adhunik Bangabhashar Abhidhan", thirteenth edition(teroso unonobboi bongabda (1982)), M. C. Sarkar and Sons private limited, 14, Bankim Chatterjee Street, Kolkata, © Dipankar Basu.

[4] Rev. W. Reeve, "Dictionary Kannada and English", 1858; revised, corrected and enlarged by Daniel Sanderson, 1979, Wesleyan Missionary, Asian Educational Services, New Delhi; c-2/15, SDA, New Delhi-110016.

[5] Anindya Kumar Biswas, "Graphical Law beneath each written natural language", arXiv:1307.6235v3[physics.gen-ph]. A preliminary study of words of dictionaries of twenty six languages, more accurate study of words of dictionary of Chinese usage and all parts of speech of dictionary of Lakher(Mara) language and of verbs, adverbs and adjectives of dictionaries of six languages are included.

[6] Anindya Kumar Biswas, "A discipline of knowledge and the graphical law", IJARPS Volume 1(4), p 21, 2014; viXra: 1908:0090[Linguistics].

[7] Anindya Kumar Biswas, "Bengali language and Graphical law", viXra: 1908:0090[Linguistics].

[8] Anindya Kumar Biswas, "Basque language and the Graphical Law", viXra: 1908:0414[Linguistics].

[9] Anindya Kumar Biswas, "Romanian language, the Graphical Law and More", viXra: 1909:0071[Linguistics].

[10] Anindya Kumar Biswas, "Discipline of knowledge and the graphical law, part II", viXra:1912.0243 [Condensed Matter],International Journal of Arts Humanities and Social Sciences Studies Volume 5 Issue 2 February 2020.

[11] Anindya Kumar Biswas, "Onsager Core of Abor-Miri and Mising Languages", viXra: 2003.0343[Condensed Matter].

[12] Anindya Kumar Biswas, "Bengali language, Romanisation and Onsager Core", viXra: 2003.0563[Linguistics].

[13] Anindya Kumar Biswas, "Little Oxford English Dictionary and the Graphical Law", viXra: 2008.0041[Linguistics].

[14] Anindya Kumar Biswas, "Oxford Dictionary Of Social Work and Social Care and the Graphical law", viXra: 2008.0077[Condensed Matter].

[15] Anindya Kumar Biswas, "Visayan-English Dictionary and the Graphical law", viXra: 2009.0014[Linguistics].

[16] Anindya Kumar Biswas, "Garo to English School Dictionary and the Graphical law", viXra: 2009.0056[Condensed Matter].

[17] Anindya Kumar Biswas, "Mursi-English-Amharic Dictionary and the Graphical law", viXra: 2009.0100[Linguistics].

[18] Anindya Kumar Biswas, "Names of Minor Planets and the Graphical law", viXra: 2009.0158[History and Philosophy of Physics].

[19] Anindya Kumar Biswas, "A Dictionary of Tibetan and English and the Graphical law", viXra: 2010.0237[Condensed Matter].

[20] Anindya Kumar Biswas, "Khasi English Dictionary and the Graphical law", viXra: 2011.0011[Linguistics].

[21] Anindya Kumar Biswas, "Turkmen-English Dictionary and the Graphical law", viXra: 2011.0069[Linguistics].

[22] Anindya Kumar Biswas, " Webster's Universal Spanish-English Dictionary, the Graphical law and A Dictionary of Geography of Oxford University Press", viXra: 2103.0175[Condensed Matter].

[23] Anindya Kumar Biswas, "A Dictionary of Modern Italian, the Graphical law and Dictionary of Law and Administration, 2000, National Law Development Foundation", viXra: 2107.0171[Condensed Matter].

[24] Anindya Kumar Biswas, "Langenscheidt's German-English English-German Dictionary and the Graphical law", viXra: 2107.0179[Linguistics].

[25] Anindya Kumar Biswas, "Essential Dutch dictionary by G. Quist and D. Strik, the Graphical law Classification", viXra: 2108.0040[Linguistics].

[26] Anindya Kumar Biswas, "Swahili, a lingua franca, Swahili-English Dictionary by C. W. Rechenbach and the Graphical law", viXra: 2108.0101[Linguistics].

[27] Anindya Kumar Biswas, "The French, Larousse Dictionnaire De Poche and the Graphical law", viXra: 2109.0080[Linguistics].

[28] Anindya Kumar Biswas, "An Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution", viXra: 2109.0119[Condensed Matter].

[29] Anindya Kumar Biswas, "Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung and the Graphical law", viXra: 2109.0141[Linguistics].

[30] Anindya Kumar Biswas, Bawansuk Lyngkhoi, "The Graphical law behind the NTC's Hebrew and English Dictionary by Arie Comey and Naomi Tsur", viXra: 2109.0164[Linguistics].

[31] Anindya Kumar Biswas, "Oxford Dictionary Of Media and Communication and the Graphical law", viXra: 2109.0202[Social Science].

[32] Anindya Kumar Biswas, "Oxford Concise Dictionary Of Mathematics, Penguin Dictionary Of Mathematics and the Graphical law", viXra: 2112.0054[Social Science].

[33] Anindya Kumar Biswas, "An Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution Second part", viXra: 2201.0021[Condensed Matter].

[34] Anindya Kumar Biswas, "The Penguin Dictionary Of Sociology and the Graphical law", viXra: 2201.0046[Social Science].

[35] Anindya Kumar Biswas, "The Concise Oxford Dictionary Of Politics and the Graphical law", viXra: 2201.0069[Social Science].

[36] Anindya Kumar Biswas, "A Dictionary Of Critical Theory by Ian Buchanan and the Graphical law", viXra: 2201.0136[Social Science].

[37] Anindya Kumar Biswas, "The Penguin Dictionary Of Economics and the Graphical law", viXra: 2201.0169[Economics and Finance].

[38] Anindya Kumar Biswas, "The Concise Gojri-English Dictionary by Dr. Rafeeq Anjum and the Graphical law", viXra: 2201.0205[Linguistics].

[39] Anindya Kumar Biswas, "A Dictionary of the Kachin Language by Rev.O.Hanson and the Graphical law" ("A Dictionary of the Kachin Language by Rev.o.Hanson and the Graphical law", viXra: 2202.0030[Linguistics]).

[40] Anindya Kumar Biswas, "A Dictionary Of World History by Edmund Wright and the Graphical law", viXra: 2202.0130[History and Philosophy of Physics].

[41] Anindya Kumar Biswas, "Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool and the Onsager's solution", viXra: 2202.0157[Condensed Matter].

[42] Anindya Kumar Biswas, "A Dictionary of Plant Sciences by Michael Allaby and the Graphical law", viXra: 2203.0011[Mind Science].

[43] Anindya Kumar Biswas, "Along the side of the Onsager's solution, the Ekagi language", viXra: 2205.0065[Condensed Matter].

[44] Anindya Kumar Biswas, "Along the side of the Onsager's solution, the Ekagi language-Part Three", viXra: 2205.0137[Condensed Matter].

[45] Anindya Kumar Biswas, "Oxford Dictionary of Biology by Robert S. Hine and the Graphical law", viXra: 2207.0089[Phyiscs of Biology].

[46] Anindya Kumar Biswas, "A Dictionary of the Mikir Language by G. D. Walker and the Graphical law", viXra: 2207.0165[Linguistics].

[47] Anindya Kumar Biswas, "A Dictionary of Zoology by Michael Allaby and the Graphical law", viXra: 2208.0075[Phyiscs of Biology].

[48] Anindya Kumar Biswas, "Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical law", viXra: 2208.0093[Religion and Spiritualism].

[49] Anindya Kumar Biswas, "Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law", viXra: 2211.0061[Social Sciences].

[50] Anindya Kumar Biswas, "A Greek and English Lexicon by H.g.liddle et al simplified by Didier Fontaine and the Graphical law", viXra: 2211.0087[Linguistics].

[51] Anindya Kumar Biswas, "Learner's Mongol-English Dictionary and the Graphical law", viXra: 2211.0101[Linguistics].

[52] Anindya Kumar Biswas, "Complete Bulgarian-English Dictionary and the Graphical law", viXra: 2212.0009[Linguistics].

[53] Anindya Kumar Biswas, "A Dictionary of Sindhi Literature by Dr. Motilal Jotwani and the Graphical Law", viXra: 2212.0015[Social Sciences].

[54] Anindya Kumar Biswas, "Penguin Dictionary of Physics, the Fourth Edition, by John Cullerne, and the Graphical law", viXra: 2212.0072[History and Philosophy of Physics].

[55] Anindya Kumar Biswas, "Oxford Dictionary of Chemistry, the seventh edition and the Graphical Law", viXra: 2212.0113[Chemistry].

[56] Anindya Kumar Biswas, "A Burmese-English Dictionary, Part I-Part V, by J. A. Stewart and C. W. Dunn et al, head entries and the Graphical Law", viXra: 2212.0127[Linguistics].

[57] Anindya Kumar Biswas, "The Graphical Law behind the head words of Dictionary Kannada and English written by W. Reeve, revised, corrected and enlarged by Daniel Sanderson", viXra: 2212.0185[Linguistics].

[58] Anindya Kumar Biswas, "Sanchayita and the Graphical Law", viXra: 2301.0075[Social Science].

[59] Anindya Kumar Biswas, "Samsad Bangla Abhidan and The Graphical Law", viXra: 2302.0026[Linguistics].

[60] E. Ising, Z.Physik 31,253(1925).

[61] R. K. Pathria, Statistical Mechanics, p400-403, 1993 reprint, Pergamon Press,© 1972 R. K. Pathria.

[62] C. Kittel, Introduction to Solid State Physics, p. 438, Fifth edition, thirteenth Wiley Eastern Reprint, May 1994, Wiley Eastern Limited, New Delhi, India.

[63] W. L. Bragg and E. J. Williams, Proc. Roy. Soc. A, vol.145, p. 699(1934);

[64] P. M. Chaikin and T. C. Lubensky, Principles of Condensed Matter Physics, p. 148, first edition, Cambridge University Press India Pvt. Ltd, New Delhi.

[65] Kerson Huang, Statistical Mechanics, second edition, John Wiley and Sons(Asia) Pte Ltd.

[66] A. M. Gun, M. K. Gupta and B. Dasgupta, Fundamentals of Statistics Vol 1, Chapter 12, eighth edition, 2012, The World Press Private Limited, Kolkata.

[67] Sonntag, Borgnakke and Van Wylen, Fundamentals of Thermodynamics, p206-207, fifth edition, John Wiley and Sons Inc.

[68] Alexander M. Petersen, Joel N. Tenenbaum, Shlomo Havlin, H. Eugene Stanley, and Matjaž Perc, "Languages cool as they expand: Allometric scaling and the decreasing need for new words", Sci. Rep.2(2012) 943, arXiv:1212.2616v1. and references therein.