A rational interpretation of Zeno's paradoxes

Rajib Kumar Bandopadhyay

Abstract:

Many have attempted a solution to the Zeno's paradoxes, including Peter Lynds in his papers "Time and Classical and Quantum Mechanics: Indeterminacy vs. Discontinuity" and "Zeno's Paradoxes: A Timely Solution". Peter Lynds' solutions have been widely accepted by the peers. To me, Lynds and others have overlooked the moot point. Even if the uncertainties in measurement of distance and time interval were removed Zeno's paradoxes would still stand unsolved. The actual events satisfying Zeno's implicit conditions are totally different – Achilles does never catch the tortoise, it is never possible to complete a journey (the dichotomy problem) or the arrow can never be in motion since it is frozen in a box at any instant. This is because Zeno is, and we are, attempting something that is not permitted in nature – Zeno' paradoxes assume the improbable – that we could change and control our ability to observe an event, spanning an interval of time, arbitrarily, from an infinitesimally small value to a infinitely large value.

A rational interpretation of Zeno's paradoxes

Rajib Bandopadhyay

In his papers[1][4] Peter Lynds exhaustively deals with the paradoxes and their apparent solutions, along with references to earlier works on the Zeno's paradoxes. I will briefly write down the three main paradoxes here:

Achilles and the Tortoise:

Here, the Greek warrior Achilles is to run a race with a tortoise. Because the tortoise is the slower of the two, he is allowed to begin at a point some distance ahead. Once the race has started however, Achilles can never overtake his opponent. For to do so, he must first reach the point from where the tortoise began. But by the time Achilles reaches that point, the tortoise will have advanced further yet. It is obvious, Zeno maintains, that the process is never ending: There will always be some distance, however small, between the two contestants. More specifically, it is impossible for Achilles to preform an infinite number of acts in a finite time.

Distance behind the Tortoise: 5, 2.5, 1.25, 0.625, 0.3125, 0.015625, ..

Time: 1, 0.5, 0.25, 0.125, 0.625, 0.03125, ..

The Dichotomy:

It is not possible to complete any journey, because in order to do so, you must firstly travel half the distance to your goal, and then half the remaining distance, and again of what remains, and so on. However close you get to the place you want to go, there is always some distance left.

Distance: 1, 1.5, 1.75, 1.875, 1.9375, 1.96885, 1.984425, ..

Time: 1, 1.5, 1.75, 1.875, 1.9375, 1.96885, 1.984425, ..

Furthermore, it is not even possible to get started. After all, before the second half of the distance can be travelled, one must cover the first half. But before that distance can be travelled, the first quarter must be completed, and before that can be done, one must traverse the first eight, and so on, and so on to infinitum.

Distance: 2, 1, 0.5, 0.25, 0.125, 0.625, 0.03125, 0.015425, ..

Time: 2, 1, 0.5, 0.25, 0.125, 0.625, 0.03125, 0.015425, ..

The Arrow:

All motion is impossible, since at any given instant in time an apparently moving body (the arrow) occupies just one block of space. Since it can occupy no more than one block of space at a time, it must be stationary at that instant. The arrow cannot therefore ever be in motion as at each and every instant it is frozen still.

Is there any fundamental difference in quantum mechanical and classical measurements?

Before embarking on the quest to provide actual solutions to the above paradoxes I would like to emphasise uncertainty in measurement is inseparable in either classical or quantum mechanical experiments, only the degree or the order of the uncertainty is different.

In the measurements concerning classical mechanics we employ media whose masses are negligible in comparison to that of the macroscopic bodies whose attributes we had set out to measure. For example, while measuring the speed of a ball we use a radar gun, which in turn employs photon beams to measure the velocity of the ball. But, if we leave aside the interaction between the photons of the beam with the electrons of the ball, the relativistic mass of a photon beam is negligible in comparison to the mass of the ball in question. So the uncertainty in measurement is negligible.

In the measurements concerning quantum mechanical problems we employ media whose inertial mass is not negligible in comparison to the microscopic bodies in question. I would ask the readers to refer to any standard text book of quantum mechanics which deals with the explanation of the Heisenberg's uncertainty principle[2][3].

The implicit assumptions of Zeno in formulating his paradoxes:

If we could visualise the problems in our mind we find that the most important condition required by Zeno is ignoring our main goal and focus instead on our ability to perceive any arbitrary interval of time between two observations. Particularly, playing the movies in our mind to check the veracity of the paradoxes leads us to a conclusion that we are progressively watching smaller and smaller time intervals, i.e., we are essentially slowing down the passage of the event, or speeding ourselves up. **Zeno wants us to forget our primary aim of determining the outcome of the events as they naturally end, and instead be the supreme masters of Space-Time, making us assume that we have the ability to discern changes within any arbitrary time interval and discern any arbitrary distance of separation!**

Assuming that what Zeno stipulates is plausible and possible, such Geometric Order measurements are possible upto at most Planck's Length and Planck's Time, beyond which the Standard Model fails.

The Mathematical Solution to Achilles and the Tortoise problem:

Let us assume the tortoise has an advantage of 10 metres, Achilles runs 10 metres per second, and the tortoise, 5 metres per second. In that case, since the race would be over by a particular time, the equation would be as

$$\frac{Distance\ run\ by\ Achilles}{Acilles'speed} = \frac{Distance\ run\ by\ Tortoise}{Tortoise'sspeed}$$

follows:

$$\Rightarrow \frac{Distance_{Achilles}}{Acilles'speed} = \frac{Distance_{Tortoise}}{Tortoise'sspeed}$$

$$\Rightarrow \frac{10 metres + Distance_{Tortoise}}{10\frac{m}{s}} = \frac{Distance_{Tortoise}}{5\frac{m}{s}}$$

$$\Rightarrow Distance_{Tortoise} = 10\ \text{metres}$$

Since Achilles runs 10 metres more, we then have,

$$\Rightarrow Distance_{Achilles} = 10\ \text{metres} + Distance_{Tortoise}$$

Hence, Time taken for the Tortoise (or Achilles) would be, $Time_{Tortoise} = Time_{Achilles}$ when they are exactly at the same place on the racing track, before Achilles overtakes the tortoise, ending the race.

$\Rightarrow \quad Time_{Achelles} = \dfrac{10\, metres}{5\frac{m}{s}}$ = 2 seconds, which means the race would be over by two seconds.

Also. Zeno's proposition, if treated mathematically, would yield the expected result. Time taken for Achilles to catch up with the tortoise would be:

$$t = \frac{5 + \frac{5}{2} + \frac{5}{2^2} + \frac{5}{2^4} + \frac{5}{2^8} + \ldots metres}{5\frac{m}{s}}$$

$$\Rightarrow \quad t = 5\, X \frac{\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^8} + \ldots metres\right)}{5\frac{m}{s}}$$

$$\Rightarrow \quad t=\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^8}+\ldots seconds\right)$$

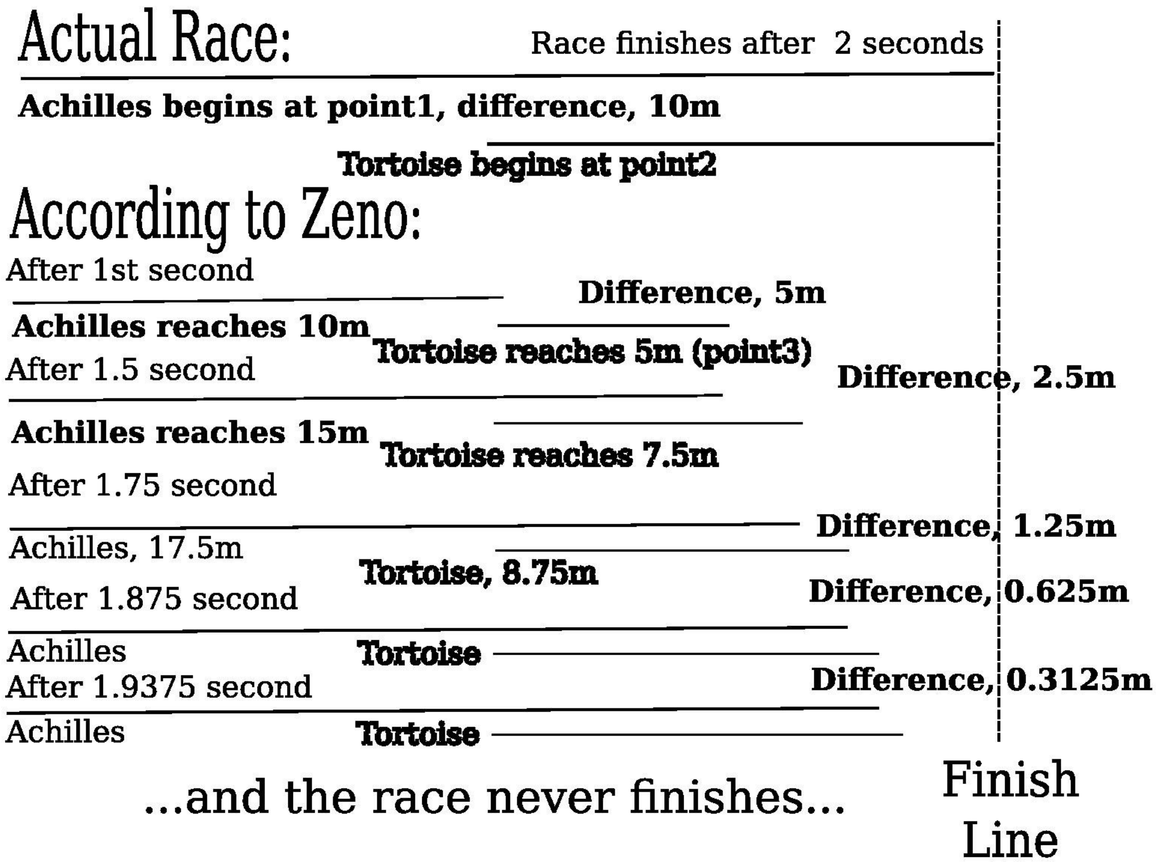

Figure 1

Which is an infinite series. Why infinite? It takes a large number of terms in the Geometric progression to reach Planck's Time. At the first glance, the fault in the logic in Zeno's proposition may appear to be that the sum of an infinite series is an infinite number.

But it is not. The sum of an infinite series is finite, if the series is convergent.

For example, `lim {n→∞},` $\sum_{n=1}^{n} S_n$ = 2, i.e., 2 seconds, as we had found earlier.

The easy complete solution to Zeno paradox is as follows:
Assuming that stopping a continuous event arbitrarily is plausible and possible, then we proceed to measure at distinct intervals of length and time during the races.

We ignore the implications of measuring time while following Zeno's pre-conditions, that is, first, Achilles has to complete the first half of the rest, then he has to complete the next half of half, i.e., the next one-forth, then the next half of half of half, i.e., one-eightth, and so on, ad nauseum, ad infinitum.
We are to remember that first we have to speed up our time-measuring ability and overcome the limits of the Reference Instruments' Resolutions of distance and time, then, after each stoppage of the race, we have to measure the distance covered. This measuring, our human endeavour, either manually or digitally, takes time.
So the series becomes:

$$t = (1+ \Delta x_1) + \left(\frac{1}{2} + \Delta x_2\right) + \left(\frac{1}{2^2} + \Delta x_3\right) + \left(\frac{1}{2^3} + \Delta x_4\right) + \left(\frac{1}{2^4} + \Delta x_5\right) + \ldots \infty$$

Where Δx_1, Δx_2, Δx_3, Δx_4, and so on, are the respective measuring time periods taken during each stoppages and measurements, and could be taken equal to Δx for simplicity, following Zeno's conditions.
Mathematically, $\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4, \ldots \approx \Delta x$
which essentially becomes a series as:

$$S_n = \left(1+ \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots \infty\right) + (\Delta x + \Delta x + \Delta x + \Delta x + \ldots \infty)$$

$$\text{or, } S_n = \left(1+ \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots \infty\right) + \infty$$

or,. S_n = (a finite series, solution being 2 seconds) + ∞
or,. $S_n = \infty$

Which makes S_n an infinite series.

In other words, as I had stated herein, the race never ends.

A Story-Teller's Solution to Achilles and the Tortoise problem:

Let us now assume that we are endowed with a glass chamber with two steerings that can rotate both clockwise and anticlockwise, infinitely.

The glass chamber has some unique qualities. It makes us both invisible to the outside world and impervious to the ravages of nature that plagues the outside world. It stops our ageing and makes us immortal. It can take us anywhere and everywhere.

The first steering can help the chamber and us inside, both shrink to become infinitely small so that we can zoom in to arbitrarily smaller detail. It can also blow itself and us inside it up infinitely so we can even zoom out and observe the whole universe fitted into our field of view. Turned anticlockwise it allows us to shrink, and turned clockwise, it can blow us up.

The second steering is unique too. It, when rotated anticlockwise, slows down the flow of time outside the chamber, the slowing down directly dependent on the turn. When turned clockwise, it speeds up the flow of time outside the chamber, the speeding up being dependent on the degree of turn.

Then what can we do with such a potent device? With it we can watch the universe form from the Big Bang and then watch the galaxies and stars form. We can watch life evolving, from single celled organism to man to other superior intelligence. We can see the superstrings vibrating in great detail and see how quarks interact with each other!

We all know that such a fairytale device is impossible to build. True, but it is with this very impossibility that Zeno's paradoxes can be resolved.

In the real race between Achilles and the tortoise, one in which Achilles overtakes the tortoise, as we all know this is inevitable, we assume that looking from where we stand, the race takes place with both Achilles and the tortoise going from left to right to finish the race. Achilles is far away to the left from the tortoise, at point 1. The tortoise is at point 2, much ahead of Achilles, if the direction of the race is considered.

Achilles begins at point 1 to our left. Now let us follow the instructions as stipulated by Zeno, i.e., we must first reach the point, say point 2, from where the tortoise began. Now note that in order to do this we have just to turn the two steerings so that we can approach precisely at the stipulated position near the point 2 and not proceed beyond it. Then by the time we find Achilles had reached the point 2, the tortoise will have advanced further yet, say at point 3. We slow down the time even further and zoom in closer, so that we could reach the point 3 and not supersede it, as the distance covered by Achilles from point 2 to point 3 is lesser than point 1 and point 2. In this way, we find that to keep a tab on the proceedings as desired by Zeno we must progressively reduce the passage of time and augment our distance measuring ability, because Achilles and the tortoise are progressively getting closer.

What essentially we are doing in this case is we are augmenting our ability to perceive changes in progressively smaller and smaller time intervals and distances, and forgetting our main objective - to find out when Achilles overtakes the tortoise. In other words, we have simply overcome the limits of the natural laws, forget about the outcome of the race and focus instead on accuracy. **To us the race will never end.**

Solution to the two dichotomy problems:

The first dichotomy:

The first dichotomy paradox, which states it is not possible to complete any journey, because in order to do so, you must firstly travel half the distance to your goal, and then half the remaining distance, and again of what remains, and so on, can be solved in the same way as above. Again we progressively speed up our ability to observe changes in smaller and smaller time intervals, and tinker with the laws of nature, which of course is not possible, and forget our main objective.

The second dichotomy:

In the same way as above we could show that we could not start the journey at all if we were to meet with Zeno’s stipulation in the dichotomy paradox. This would in turn mean we would stop time, which in turn would mean stopping the perception of our own consciousness in process. This not only rules out our ability to do the experiments but also our awareness of being alive.

Solution to the arrow problem:

Lynds is essentially correct in his resolution to the arrow problem - the arrow does not occupy a fixed space in a fixed point of time. But if we used our abilities to remove, by assumptions and suppositions, the smearing effects of each of the slides of the filmstrip we could approximate - as is common in our solutions to all mathematical problems - that the arrow indeed occupied a fixed position at a precise interval of time. Depending on the accuracy we want – at precisely what time will the arrow hit its target – will we decide how accurately to watch the event, from within our magic glass chamber. Again we arrive at the same situation we had earlier. We would be altering the flow of time and our accuracy to measure distance to satisfy Zeno’s conditions, which again, would in turn mean we were tinkering with the insurmountable natural laws.

Closing comment:

In real life too, especially in science, we are always trying to reach for better answers. As Lynds said in his comments, it took 2500 years to find what he thought was the solutions to the paradoxes originally conceived by Zeno. It would be interesting to see whether I found the solutions at all. But one thing is sure: we can not tinker unaided with our ability to observe two events separated by time, arbitrarily. This is true for all life forms. We humans, unaided by technology, are limited in a narrow band of time-intervals ranging from one fifteenth of a second to at best a few lifetimes. This is what we may call Anthropic Reality, and Zeno actually wants to negate this reality in his paradoxes.

To illustrate this let us assume a charged Pion (half-life at rest ~1.77 $X10^{-8}$ seconds [5]) and a mountain (life time ~ say 15 million years) are alive and receptive to natural events. To the charged Pion Achilles never start, let alone catch the tortoise, because before Achilles even starts it decays and finishes its life. To the Pion its life is what may be called pionic reality, which is distinctly different from our reality.

Similarly, The mountain is oblivious of the momentous event, as to it centuries are essentially what seconds are to us, and it would even fail to register the birth and death of Achilles, let alone the race. The mountain is preoccupied with its unique mountainous reality.

Zeno, by his paradoxes, is exhorting us to forget our main goal and focus instead on progressively smaller time intervals. In a way he asks us to move continuously from the Anthropic Reality to the Pionic Reality and beyond. Zeno's paradoxes could also be manipulated to target the mountainous reality instead.

It is also interesting to note how the peers from the scientific community have appreciated Peter Lynds' solution to the Zeno's paradoxes without the looking at the problems closely [5]. What has happened to the community!

Rajib Kumar Bandopadhyay

References:

[1] Lynds, P. Time and Classical and Quantum Mechanics: Indeterminacy vs. Discontinuity. Foundations of Physics Letters, 15(3), (2003).

[2] John Yarwood's "Atomic physics", Oxford University Press, 9th impression, 1989, pages 234-237

[3] Leonard I. Schiff's "Quantum Mechanics", 3rd edition, McGraw Hill, art: discussion of measurement, pages 9-14

[4] Lynds, Peter (2003) Zeno's Paradoxes: A Timely Solution, deposited on 15th September 2003, ID code 1197, Philosophy of science. This paper is closely related to a physics paper titled "Time and Classical and Quantum Mechanics: Indetermincy vs. Discontinuity" which has recently been published in the August edition of "Foundations of Physics Letters" 16(4) (2003).

[5] Robert Resnick's "Introduction to special relativity", 11th Wiley Eastern reprint, 1989, pages 75-76

[6] http://www.eurekalert.org/pub_releases/2003-07/icc-gwi072703.php