Equivalence Principle and Lorentz Covariant Gravitation Contradiction

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Abstract

We show a Lorentz covariant gravitation does not satisfy the equivalence principle.

1 Introduction

We will restrict to a Lorentz covariant gravitation that has only constants c and G with dimension. General relativity [1] is an example. Units are chosen so that $c = G = 1$.

Let $m_A > 0, E_\gamma > 0$, and $0 \leq v < 1$ and define M'_A and E'_γ by

$$M'_A = \sqrt{1-v^2}m_A \qquad E'_\gamma = \sqrt{\frac{1+v}{1-v}}E_\gamma \tag{1}$$

Let $\mathcal{F}$ be a frame of reference with coordintes t, x, y, z and $\mathcal{F}'$ be a frame of reference with coordinates t', x', y, z'. The coordinates of the frames being related by the Lorentz transformation

$$t = \frac{t' + vx'}{\sqrt{1-v^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-v^2}} \qquad y = y' \qquad z = z' \tag{2}$$

With respect to $\mathcal{F}'$ let there be a zero rest mass particle γ moving from positive x' infinity towards the origin along the x' axis. Let $t'_\gamma(x'_\gamma)$ be the path of γ. Also let there be a point mass A such that when γ is at at infinity A is at rest at the origin. When γ is at infinity let M'_A be the mass of A and E'_γ be the energy of γ. When γ is at infinity let P'^μ_A be the components of the energy-momentum four-vector of A and P'^μ_γ be the components of the energy-momentum four-vector of γ. With respect $\mathcal{F}'$ when γ is at infinity

$$\begin{aligned} P'^0_A &= M'_A \qquad P'^1_A = P'^2_A = P'^3_A = 0 \\ P'^0_\gamma &= E'_\gamma \qquad P'^1_\gamma = -E'_\gamma \qquad P'^2_\gamma = P'^3_\gamma = 0 \end{aligned} \tag{3}$$

With respect to $\mathcal{F}$ when γ is at infinity the energy of A is using (1) and (3) and the formula for transformation of energy

$$P^0_A = \frac{P'^0_A + vP'^1_A}{\sqrt{1-v^2}} = \frac{M'_A}{\sqrt{1-v^2}} = \frac{\sqrt{1-v^2}m_A}{\sqrt{1-v^2}} = m_A \tag{4}$$

and the energy of γ is

$$P^0_\gamma = \frac{P'^0_\gamma + vP'^1_\gamma}{\sqrt{1-v^2}} = \frac{E'_\gamma + v(-E'_\gamma)}{\sqrt{1-v^2}} = \sqrt{\frac{1-v}{1+v}}E'_\gamma = \sqrt{\frac{1-v}{1+v}}\sqrt{\frac{1+v}{1-v}}E_\gamma = E_\gamma \tag{5}$$

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2 Energy and momentum functions

With respect $\mathcal{F}'$ let the functions $p'^{\mu}_{\gamma}(x'_{\gamma})$ be the components of the energy-momentum four-vector of γ. The values of M'_A, E'_{γ}, and x'_{γ} completely determines the system with respect to $\mathcal{F}'$. A component of $p'^{\mu}_{\gamma}(x'_{\gamma})$ is then a function of M'_A, E'_{γ}, and x'_{γ} and no other variables. Since we are considering Lorentz covariant graviation with only c and G as constants with dimension we have $p'^{\mu}_{\gamma}(x'_{\gamma})/E'_{\gamma}$ will be a dimensionless function of the dimensionless variables M'_A/x'_{γ} and E'_{γ}/x'_{γ}. Note $M'_A/E'_{\gamma} = (M'_A/x'_{\gamma})(1/(E'_{\gamma}/x'_{\gamma}))$. There is then a dimensionless function C of M'_A/x'_{γ} and E'_{γ}/x'_{γ} such that [2]

$$p'^{0}_{\gamma}(x'_{\gamma}) = E'_{\gamma} + \frac{M'_A E'_{\gamma}}{x'_{\gamma}} C\left(\frac{M'_A}{x'_{\gamma}}, \frac{E'_{\gamma}}{x'_{\gamma}}\right) \tag{6}$$

Similarly for the x' component of momentum there is a dimsionless function D such that

$$p'^{1}_{\gamma}(x'_{\gamma}) = -E'_{\gamma} + \frac{M'_A E'_{\gamma}}{x'_{\gamma}} D\left(\frac{M'_A}{x'_{\gamma}}, \frac{E'_{\gamma}}{x'_{\gamma}}\right) \tag{7}$$

With respect to $\mathcal{F}'$ if γ is at the point $(t'_{\gamma}, x'_{\gamma}, 0, 0)$ where $t'_{\gamma}(x'_{\gamma})$ then with respect to $\mathcal{F}$ it is at the point $(t_{\gamma}, x_{\gamma}, 0, 0)$ where

$$t_{\gamma} = \frac{t'_{\gamma} + v x'_{\gamma}}{\sqrt{1-v^2}} \qquad x_{\gamma} = \frac{x'_{\gamma} + v t'_{\gamma}}{\sqrt{1-v^2}} \tag{8}$$

and $t_{\gamma}(x_{\gamma})$. With respect to $\mathcal{F}$ let the functions $p^{\mu}_{\gamma}(x_{\gamma})$ be the components of the energy-momentum four-vector of γ at x_{γ}. The energy of γ at time t_{γ} is using (1), (6)-(8)

$$\begin{aligned} p^0_{\gamma}(x_{\gamma}) &= \frac{p'^0_{\gamma}(x'_{\gamma}) + v p'^1_{\gamma}(x'_{\gamma})}{\sqrt{1-v^2}} = \frac{E'_{\gamma} + \frac{M'_A E'_{\gamma}}{x'_{\gamma}} C\left(\frac{M'_A}{x'_{\gamma}}, \frac{E'_{\gamma}}{x'_{\gamma}}\right) + v\left[-E'_{\gamma} + \frac{M'_A E''_{\gamma}}{x'_{\gamma}} D\left(\frac{M'_A}{x'_{\gamma}}, \frac{E'_{\gamma}}{x'_{\gamma}}\right)\right]}{\sqrt{1-v^2}} \\ &= \frac{E'_{\gamma}}{\sqrt{1-v^2}}\left\{1 - v + \frac{M'_A}{x'_{\gamma}}\left[C\left(\frac{M'_A}{x'_{\gamma}}, \frac{E'_{\gamma}}{x'_{\gamma}}\right) + vD\left(\frac{M'_A}{x'_{\gamma}}, \frac{E'_{\gamma}}{x'_{\gamma}}\right)\right]\right\} \\ &= E_{\gamma} + \frac{(1+v)m_A E_{\gamma}}{x_{\gamma} - v t_{\gamma}}\left[C\left(\frac{(1-v^2)m_A}{x_{\gamma} - v t_{\gamma}}, \frac{(1+v)E_{\gamma}}{x_{\gamma} - v t_{\gamma}}\right) + vD\left(\frac{(1-v^2)m_A}{x_{\gamma} - v t_{\gamma}}, \frac{(1+v)E_{\gamma}}{x_{\gamma} - v t_{\gamma}}\right)\right] \end{aligned} \tag{9}$$

The $v \to 1$ limit of (9) is

$$p^0_{\gamma}(x_{\gamma}) = E_{\gamma} + \frac{2m_A E_{\gamma}}{x_{\gamma} - t_{\gamma}}\left[C\left(0, \frac{2E_{\gamma}}{x_{\gamma} - t_{\gamma}}\right) + D\left(0, \frac{2E_{\gamma}}{x_{\gamma} - t_{\gamma}}\right)\right] \tag{10}$$

Similarily for the x component of momentum the $v \to 1$ limit is

$$p^1_{\gamma}(x_{\gamma}) = -E_{\gamma} + \frac{2m_A E_{\gamma}}{x_{\gamma} - t_{\gamma}}\left[C\left(0, \frac{2E_{\gamma}}{x_{\gamma} - t_{\gamma}}\right) + D\left(0, \frac{2E_{\gamma}}{x_{\gamma} - t_{\gamma}}\right)\right] \tag{11}$$

Subtracting (10) and (11) gives

$$p^0_{\gamma}(x_{\gamma}) - p^1_{\gamma}(x_{\gamma}) = 2E_{\gamma} \tag{12}$$

3 Velocity of γ

We will assume, of a Lorentz covariant gravitation, that the velocity of γ does not depend on its energy. Consequently with respect to $\mathcal{F}'$ the velocity $dx'_{\gamma}/dt'_{\gamma}$ of γ will then be a function of M'_A and x'_{γ} and not E'_{γ}. We then have $dx'_{\gamma}/dt'_{\gamma}$ will be a dimensionless function of the dimensionless variable M'_A/x'_{γ}. There is then a dimensionless function S such that

$$\frac{dx'_{\gamma}}{dt'_{\gamma}} = -1 + \frac{M'_A}{x'_{\gamma}} S\left(\frac{M'_A}{x'_{\gamma}}\right) \tag{13}$$

The speed of γ decreases as γ moves towards the origin hence $S(0) > 0$. With respect to $\mathcal{F}$ the velocity of γ is using (1), (8), (13) and the velocity addition formula

$$\frac{dx_\gamma}{dt_\gamma} = \frac{\frac{dx'_\gamma}{dt'_\gamma} + v}{1 + v\frac{dx'_\gamma}{dt'_\gamma}} = \frac{-1 + \frac{M'_A}{x'_\gamma} S\left(\frac{M'_A}{x'_\gamma}\right) + v}{1 + v\left[-1 + \frac{M'_A}{x'_\gamma} S\left(\frac{M'_A}{x'_\gamma}\right)\right]} = \frac{-1 + \frac{(1+v)m_A}{x_\gamma - vt_\gamma} S\left(\frac{(1-v^2)m_A}{x_\gamma - vt_\gamma}\right)}{1 + \frac{v(1+v)m_A}{x_\gamma - vt_\gamma} S\left(\frac{(1-v^2)m_A}{x_\gamma - vt_\gamma}\right)} \tag{14}$$

The $v \to 1$ limit of (14) is

$$\frac{dx_\gamma}{dt_\gamma} = \frac{-1 + \frac{2m_A}{x_\gamma - t_\gamma} S(0)}{1 + \frac{2m_A}{x_\gamma - t_\gamma} S(0)} \tag{15}$$

Solving this differential equation gives

$$(x_\gamma - t_\gamma) - (x_{\gamma 1} - t_{\gamma 1}) + 2m_A S(0) \ln \frac{x_\gamma - t_\gamma}{x_{\gamma 1} - t_{\gamma 1}} = 2(t_{\gamma 1} - t_\gamma) \tag{16}$$

where the point $(t_{\gamma 1}, x_{\gamma 1}, 0, 0)$ with $x_{\gamma 1} - t_{\gamma 1} > 0$ is on the path of γ . There is no point $(t_{\gamma 2}, x_{\gamma 2}, 0, 0)$ with $x_{\gamma 2} - t_{\gamma 2} = 0$ on the path of γ hence all points on the path $x_\gamma - t_\gamma > 0$. From (16) then $x_\gamma - t_\gamma \to 0$ as $t_\gamma \to \infty$. Now

$$p^1_\gamma(x_\gamma) = \frac{dx_\gamma}{dt_\gamma}(x_\gamma) p^0_\gamma(x_\gamma) \tag{17}$$

By (12), (15), and (17) we have

$$p^0_\gamma(x_\gamma) = \left[1 + \frac{2m_A}{x_\gamma - t_\gamma} S(0)\right] E_\gamma \tag{18}$$

4 Contradiction

Let $T^{\mu\nu}_\gamma$ be the $v \to 1$ limit of the energy-momentum tensor of γ. For points having $t - x$ aproximately zero and t large positive we have $T^{\mu\nu}_\gamma$ will have approximately a $x - t$ functional dependence. By (15) and (17) we have $T^{00}_\gamma \to T^{01}_\gamma$ as $t_\gamma \to \infty$. Also $T^{02}_\gamma = T^{03}_\gamma = 0$. Consequently $\partial_\mu T^{0\mu}_\gamma \to 0$ as $t_\gamma \to \infty$. We can then conclude p^0_γ is approximately constant in time as $t_\gamma \to \infty$ but (18) has p^0_γ going to infinity as $t_\gamma \to \infty$. This is a contradiction. The velocity of γ must then have a dependence on the energy of γ. By the equivalence principal the velocity of γ does not depend on the energy of γ. We have a contradiction. A Lorentz covariant gravitation does not satisfy the equivalence principle.

References

[1] A. Einstein, *Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes*, Annalen der Physik, 35, (1911), translated in *The Principle of Relativity*, (Dover Publications, Inc., New York, NY, 1952)

[2] K. De Paepe, *Applied Physics Research*, Vol. 4 No. 4, November 2012