General and Consistent Explanation of Tunnel Effect Based on Quantum-Statistical Interpretation

Chol Jong[1]* Son-Il Jo[1] Shin-Hyok Jon[1]
Faculty of Physics, Kim Chaek University of Technology, Pyongyang, Democratic People's Republic of Korea
(Dated: February 1, 2024)

We present an new explanation of the mechanism of tunneling from the quantum-statistical perspective. The purpose of our work is to elucidate the true mechanism of tunneling consistent with quantum theory and energy conservation. We explain tunneling on the basis of the thermionic current due to the ensemble of electron, and the current resonance due to the quantum constraint in the barrier and the exchange-correlation interaction of electrons around the barrier. The statistical aspect of tunneling is interpreted as being due to the existence of electrons able to surmount the barrier thanks to the ensemble of electrons or microscopic states of electron. Considering the current resonance of quantum essence in the barrier and the exchange-correlation interaction around the barrier enables a satisfactory explanation of the quantum aspect of tunneling relative to the width of barrier. Eventually, we arrive at a consistent and general explanation of the mechanism and characteristics of tunneling that is essentially a phenomenon of quantum plus statistical causes.

PACS numbers: 03.65.Xp, 03.75.Lm, 75.55.+v, 03.65.Pm, 66.35.+a

I. INTRODUCTION

Tunneling is famous for the understanding that electrons with energy insufficient to surmount a potential barrier can cross it. One of two typical tunneling experiments is the tunneling between two superconductors separated by an insulator. As the other type of tunneling, the Giaever tunneling (single-particle tunneling) is the tunneling of a single quasi-electron from an ordinary metal to a superconducting metal [1]. Tunneling that is regarded as a pure quantum-mechanical phenomenon, above all, is significant for elucidating the physical processes in microscopic scales. Physical phenomena of semiconductor device including new quantum Hall devices and tunneling diode are explained based on tunneling [2, 3]. It is recognized that tunneling lies behind the α decay and the cold-field emission as well.

Moreover, tunneling is significant for studying a wide range of physical world beyond microscopic world. Several kinds of tunneling and their subtle properties which extend the coverage of tunneling have been discovered and reported. Tunneling rises as an interesting subject even in cosmology, since tunneling is regarded as an indispensable element for solving the problem of black hole [4, 5]. As a phenomenon of significance, the evaporation of black holes as a result of the Hawking radiations has also been considered to be due to tunneling of particles from the horizons of black holes. Brabec's theory of tunnel ionization in complex systems yielded reasonable results in agreement with experiments [6]. Thus, tunneling leads to an understanding of the essence of ionization. Vindel-Zandbergen's studies of tunneling-induced electron transfer between separated protons showed the relevance of tunneling in nuclear reactions [7].

The recent researches have confirmed that tunneling depends on several factors purely beyond quantum realm. Jürgensen reported the observation of density-induced tunneling which breaks the symmetric behavior for attractive and repulsive interactions predicted by the Hubbard model [8]. Perrin proposed that theoretical predictions and models based on other mechanisms such as asymmetric tunneling barriers or asymmetric charging are possible [9]. Tokieda's study on quantum tunneling in a one-dimensional potential in the presence of energy dissipation showed that it was possible to calculate the tunneling probability using a time-dependent wave packet method [10]. An analytical study of relativistic tunneling through opaque barriers demonstrated that it is necessary to consider the relativistic aspect of tunneling [11]. Chuprikov showed a deep relevance of tunneling time and superposition principle, the gist of which is that scattering of a quantum particle on a one-dimensional potential barrier violates the superposition principle and thus the potential barrier and the layered structure play role of nonlinear elements [12].

In order to explain the mechanism of tunneling in an intuitive and reasonable way, there have been proposed and developed several methods. The non-tunnel model for a physical particle consisting of a bare particle and its virtual decay cloud was assumed. In this theory, it is supposed that the barrier is more transparent for virtual particles than for the bare particle and the barrier width is less than the size of virtual cloud. Then tunneling is explained in such a manner that virtual particles regenerate the primary physical particle behind the barrier [13]. Jonson ascribed a mechanism of tunneling to the exchange-correlation potential (image potential) felt by an electron tunneling from a metal through a classically forbidden region into vacuum [14].

The quasi-classical approaches of quantum mechanics

*Chol-Jong: jch59611@star-co.net.kp

in phase space is an important paradigm, which uses the Wigner or Husimi function useful for physical picture [15–23]. The quasi-classical approaches based on trajectory have been used as an effective mean to treat the quantum dynamical processes relevant to tunneling [24, 25]. Meanwhile, the quantum maps method for time-continuous systems based on quasi-classical approach has been developed by Takahashi and Ikeda [25].

As a version of quasi-classical approaches, the entangled-trajectory molecular dynamics (ETMD) has been newly developed based on the quantum phase-space theory [24, 26]. This approach has become a powerful tool for solving the quantum Liouville equation numerically and also elucidating the dynamical processes of quantum characteristics such as the tunnel effect by using entangled trajectories in phase space. Comparing to the classical molecular dynamics, the entangled-trajectory molecular dynamics takes some advantages of getting the quantum effect in evolution and requiring lesser trajectories in finding the convergent value [24]. In various fields of quantum dynamics, the results obtained by applying ETMD approaches have been reported [24, 27]. These approaches are based on utilization of the Wigner function which may produce negative values at some points. This shortcoming precludes resolving quantum-mechanical problem correctly.

On the other hand, the Husimi function-based approach is useful for obtaining the correct information related to tunneling, since the Husimi function is positive at every point and guarantees accuracy. Several works using this approach have been reported [25, 28].

Despite the advances, several difficulties with interpreting the tunnel effect by using clear physical intuition still remain unsolved. Concerning the conventional interpretation, it should be noted that the solution of the Schrödinger equation in the barrier region violates the law of energy conservation and the continuity of probability current density. This big problem makes the conventional explanation of tunneling unreliable.

What is important is to elucidate the mechanism of tunneling in a consistent way. The problem is whether tunneling is a quantum effect by a single electron or quantum statistical effect by an ensemble of electrons or electronic states. If tunneling is due to the ensemble of electrons or ensemble state of an electron, then it should depend on temperature. Noticeably, the conventional interpretation of tunneling is not associated with temperature. However, many studies of tunneling at room temperature in chemical and biochemical systems as well as processes at cryogenic temperature have been reported [29]. Some researchers claim that the most general and exact approach is to apply quantum dynamics, i.e., to solve the time-dependent Schrödinger equation starting from the reactant state ensemble [29].

It is necessary to turn our attention to the fact that actually, experiments on tunneling are related not to an independent single electron but to an ensemble of electrons or of electron states. Moreover, it should be considered that the conventional theory cannot explain as yet in a general way the behavior of current across the barrier in the whole domain of variability of factors such as bias voltage and temperature.

The purpose of our work is to elucidate that tunneling is not a pure quantum mechanical effect but a quantum statistical hybrid effect. Based on quantum statistical approach, we satisfactorily explain the main characteristics of tunneling by embracing all the quantum statistical factors. Importantly, based on the statistical formalism of quantum mechanics presented already [32], we produce clear results showing the quantum statistical nature and the characteristics of current across the barrier.

II. RESULTS

In the case of the absence of bias voltage, the cross flow of thermionic electrons in the barrier balances. In the presence of bias voltage, the balance breaks down, so the drift current in the barrier occurs. Based on quantum statistics, we wrote the current density for this case as

$$j_{st} = 2AT^2 \exp\left(-\frac{\Phi}{kT}\right) \sinh\left(\frac{U_a}{kT}\right). \tag{1}$$

Here, the subscript st signifies the statistical origin, or thermionic. In addition, the bias voltage changes the density of electrons capable of surmounting the barrier on both sides and as a result the gradient of electron density appears. It causes the diffusion current and therefore this effect plays the role of changing the heights of the barrier on both sides. In this case, U_a in Eq. (1) can be replaced simply with $U_a + U_g$ to be

$$j_{st} = 2AT^2 \exp\left(-\frac{\Phi}{kT}\right) \sinh\left(\frac{U_a + U_g}{kT}\right). \tag{2}$$

To obtain the additional voltage U_g, we solved in consideration of the electron density in the barrier region the Poisson equation:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = -\frac{\rho}{\varepsilon_0}.$$

Supposing the linearity of the charge density in the barrier region, we determined the potential difference between the edges of the barrier:

$$\triangle\varphi = -\frac{1}{\varepsilon_0}\left(\frac{\rho_l}{3} + \frac{\rho_0}{6}\right) l^2. \tag{3}$$

As a consequence, the additional voltage due to the gradient of charge density has been determined as

$$U_g = -e\triangle\varphi = \frac{e}{\varepsilon_0}\left(\frac{\rho_l}{3} + \frac{\rho_0}{6}\right) l^2. \tag{4}$$

The obtained result shows that the thinner the width of the barrier is, the larger the current in the barrier, which explains a main character of tunneling as well.

Another effect of quantum origin related to the barrier width has been clarified. Taking into account the general form of the wave function in phase space, we set up the quantum constraint in the barrier:

$$\int_0^l p(x)\mathrm{d}x = \int_0^l \sqrt{2m\left[E - U(x)\right]}\mathrm{d}x = nh, \qquad (5)$$

where E is the total energy of an electron determined by applied voltage and $U(x)$, the potential in the barrier. In terms of this constraint, quantum characteristic of tunneling is clarified. By introducing the definition of the mean value of momentum of an electron in the barrier:

$$\bar{p} = \frac{\int_0^l p(x)\mathrm{d}x}{l} = \frac{\int_0^l \sqrt{2m[E - U(x)]}\mathrm{d}x}{l}, \qquad (6)$$

the quantum constraint is represented as

$$\bar{p}l = nh.$$

An electron satisfying this condition is to be in a quantum resonance state in the barrier, in which case the current is increased.

We explained this fact in a semi-quantitative way. The current density obtained by the statistical consideration is represented with respect to momentum p as

$$j_{\mathrm{st}} \propto \sinh(aU_a) = \sinh(bp^2). \qquad (7)$$

The current-voltage characteristic is depicted in Fig. (1). The axes of coordinate are marked in a relative measure.

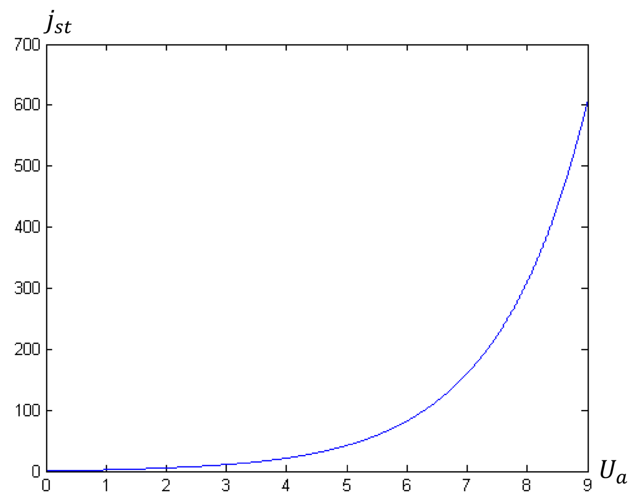

FIG. 1: Current density of statistical origin in the barrier

On the other hand, the current of statistical origin undergoes a kind of filtering due to the quantum constraint in the barrier to be resonated. This filtering effect modulates the statistical current density. We represented the modulation coefficient characterizing the filtering as the following mathematically modelled relation:

$$\eta = \sum_n C_n \exp\left(-\beta_n\left(U_a - U_n\right)^2\right). \qquad (8)$$

Eq. (8) is visualized in Fig. 2, taking one peak in view of the short-range characteric of the exchange-correlation interaction. In addition, we include in U_g the effect due to

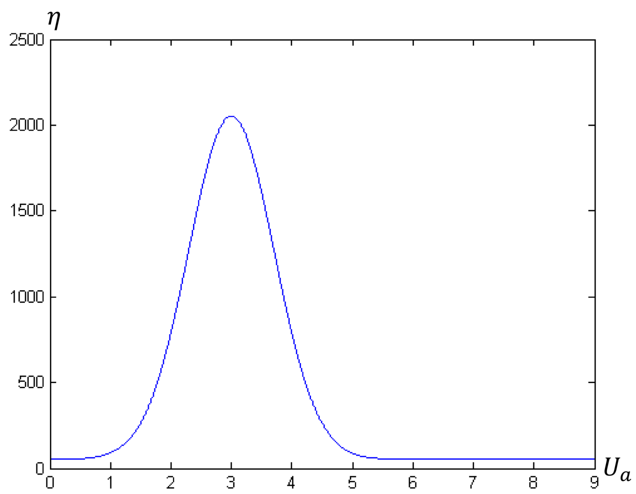

FIG. 2: modulation-voltage due to condition of quantum resonance in barrier

the exchange-correlation interaction of electrons, which effect inherent to many-electron systems plays the role of lowering the barrier height. The exchange-correlation interaction is evaluated by

$$E_{xc} = -\frac{Ae^2\left\{1 - \cos[(k-K)l]\right\}}{2l(k-K)^2}.$$

In the end, the resultant current density filtered according to the condition of quantum constraint is represented as

$$j = \eta j_{\mathrm{st}}. \qquad (9)$$

This must be just the tunnel current density in the conventional sense. The resultant current density according to applied voltage is depicted in Fig. 3.

Fig. 3 is in good agreement with the current-voltage characteristic of the tunneling diode that exhibits a maximum followed by a minimum and subsequently an exponential increase [3]. Ultimately, all the properties of tunneling have been explained consistently without violating energy conservation. Based on our work, hereafter, tunneling is understood to be the phenomenon related to the current across the barrier occurring in case the bias is lower than the barrier height. In this case, the current resonance due to the quantum constraint and electronic correlation plays a key role in the increase in current across the barrier.

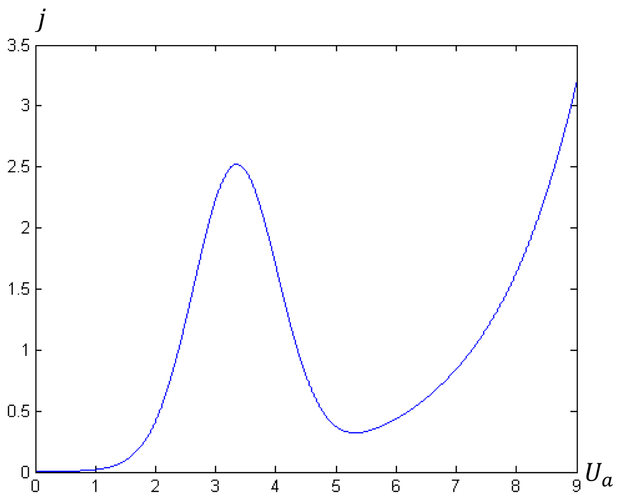

FIG. 3: Resultant current density according to applied voltage

III. DISCUSSION

A. Necessity of improving understanding of tunneling

Obviously, there is a diversity of views on the mechanism of tunneling. Since these views are not in harmony with one another, it is necessary to achieve consensus on the mechanism of tunneling. The conventional explanation of the tunnel effect is based on a solution of the Schrödinger equation subject to boundary conditions related to a barrier region and two free-motion regions. Fig. 4 shows these regions in the case of the square barrier. The transmission factor for tunneling through the square barrier:

$$T(E) = \left\{ 1 + \frac{1}{4} \left[\frac{\Phi^2}{E(\Phi - E)} \right] \sinh^2 \alpha l \right\}^{-1} \tag{10}$$

gives a plausible explanation of main characteristics of the tunnel effect. Here, $\alpha = \frac{\sqrt{2m(\Phi - E)}}{\hbar}$. For high, wide barriers, we simplify the formula as

$$T = T_0 e^{-\frac{2}{\hbar}\sqrt{2m(\Phi - E)}\ l}. \tag{11}$$

Thus, for barriers of an arbitrary form, we have

$$T = T_0 e^{-\frac{2}{\hbar}\int_0^l \sqrt{2m[U(x) - E]} \mathrm{d}x}.$$

However, the conventional interpretation of the tunnel effect encounters serious problems related to physical foundations. This is because it violates the fundamentals of quantum mechanics and the law of energy conservation as a universal law of physics. To review this problem, it is necessary to reconsider the solution of the Schrödinger equation in the region of potential barrier.

From Fig. 4, the Schrödinger equation in a square barrier region is represented as

$$-\frac{\hbar^2}{2m}\Delta\psi + \Phi\psi = E\psi, \tag{12}$$

where Φ is the barrier height. This equation reflects the correspondence principle for the energy relation:

$$E = \frac{p^2}{2m} + \Phi. \tag{13}$$

In other words, the total energy of an incident electron before a barrier should equal the sum of the kinetic energy of the electron in the barrier and the energy which the electron loses in the barrier. Therefore, the kinetic energy must be positive also in the barrier. If the barrier height is higher than the total energy of a particle, its kinetic energy in the barrier should be negative. For the negative kinetic energy, two cases are possible: one is the case where the momentum of a particle is purely imaginary and the other is the case where the mass of the particle is negative.

But both the cases are not allowed because they are physically meaningless. Above all, the momentum of a particle is never imaginary because of the Hermitian property of the momentum operator. Thus, the kinetic energy of negative value is inconsistent with quantum theory. This fact is enough to understand that the conventional method for describing tunneling has an incorrect starting point.

Let us consider the problem of negative kinetic energy in more detail. The wave functions for the regions in Fig.

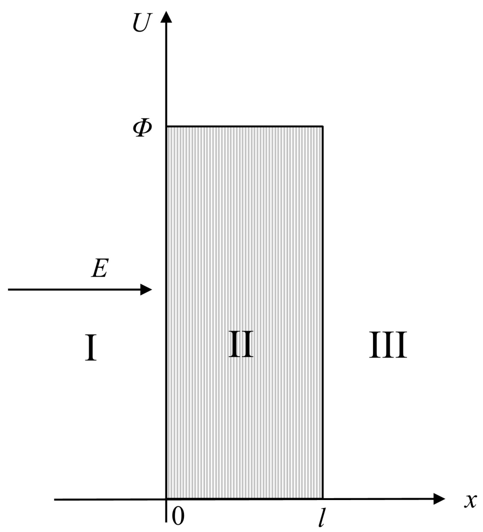

FIG. 4: A barrier (II) and two free-motion regions (I and III): A free electron of total energy E in region I is incident upon the barrier of potential Φ, i.e., region II . The transmitted electron moves freely in region III.

4 are represented respectively as

$$\left.\begin{array}{l}\psi_{\rm I}=e^{{\rm i}kx}+B_1e^{-{\rm i}kx}\\ \psi_{\rm II}=A_2e^{{\rm i}Kx}+B_2e^{-{\rm i}Kx}\\ \psi_{\rm III}=A_3e^{{\rm i}kx}\end{array}\right\}, \tag{14}$$

where $k=\frac{\sqrt{2mE}}{\hbar}$ and $K=\frac{\sqrt{2m(E-\Phi)}}{\hbar}$. All things considered, Eq. (14) states that particles do not lose their energy in the barrier and the barrier performs only the role of reflecting particles.

At its face value, Fig. 4 says that at point 0, an electron loses energy as much as Φ, and then at point l, it gains energy as much as Φ. Therefore, the energy of the electron remains unchanged after having passed through the barrier. In the end, the barrier has no effect on the dynamical state of the electron besides changing the probability of transmission. $\psi_{\rm III}$ in Eq. (14) illustrates this fact. In fact, k is the same before and after the barrier.

But in the true sense, an electron in region III should possess kinetic energy $E-\Phi$, since it has already passed through the barrier. To describe this process correctly, Fig. 4 should be replaced by Fig. 5. Correspondingly,

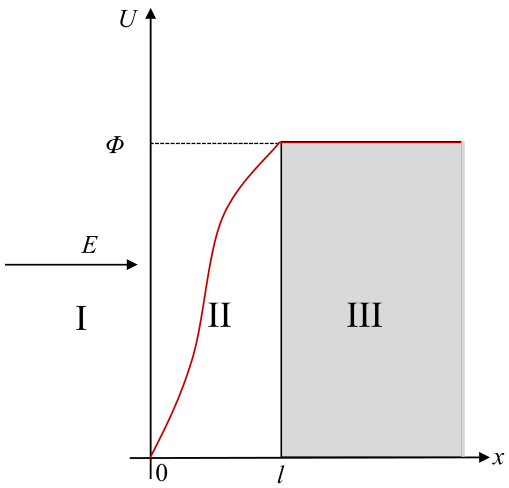

FIG. 5: A barrier (II) and two free-motion regions (I and III): A free electron of total energy E in region I is incident upon the barrier of potential Φ, i.e., region II . The transmitted electron moves freely with kinetic energy $E-\Phi$ in region III. The red line denotes the potential energy.

Eq. (14) should be replaced by

$$\left.\begin{array}{l}\psi_{\rm I}=e^{{\rm i}kx}+B_1e^{-{\rm i}kx}\\ \psi_{\rm II}=A_2e^{{\rm i}Kx}+B_2e^{-{\rm i}Kx}\\ \psi_{\rm III}=A_3e^{{\rm i}Kx}\end{array}\right\}. \tag{15}$$

On the other hand, it is necessary to reconsider the continuity condition. We conventionally takes the continuity condition as

$$\left.\begin{array}{l}\psi_{\rm I}|_{x=0}=\psi_{\rm II}|_{x=0}\\ \psi_{\rm II}|_{x=l}=\psi_{\rm III}|_{x=l}\\ \psi_{\rm I}^{'}|_{x=0}=\psi_{\rm II}^{'}|_{x=0}\\ \psi_{\rm II}^{'}|_{x=l}=\psi_{\rm III}^{'}|_{x=l}\end{array}\right\}. \tag{16}$$

However, it is more reasonable to impose the continuity condition on the boundaries in terms of the continuity of the wave function and probability current density. Then we can write the continuity condition otherwise as

$$\left.\begin{array}{l}\psi_{\rm I}|_{x=0}=\psi_{\rm II}|_{x=0}\\ \psi_{\rm II}|_{x=l}=\psi_{\rm III}|_{x=l}\\ j_{\rm I}|_{x=0}=j_{\rm II}|_{x=0}\\ j_{\rm II}|_{x=l}=j_{\rm III}|_{x=l}\end{array}\right\}. \tag{17}$$

Evidently, Eqs. (16) and (17) are not identical. Really, Eq. (17) is more reasonable than Eq. (16). This shows that the starting point of the conventional interpretation is not correct.

The negative kinetic energy implies that in the Schrödinger equation in the barrier:

$$-\frac{\hbar^2}{2m}\Delta\psi=(E-\Phi)\psi,$$

$E-\Phi$ is negative. According to the law of energy conservation, the total energy of the electron behind the barrier, i.e., in region III should become $E-\Phi$. This indicates that the kinetic energy of the electron behind the barrier should be negative. However, the conventional theory supposes that the kinetic energy of the electron in region III is equal to that before the barrier. Obviously, it is the violation of the law of energy conservation, thus physically inconsistent.

It is instructive to recall the following fact. If negative $E-\Phi$ were permitted, for the wave equation for a free particle,

$$-\frac{\hbar^2}{2m}\Delta\psi=E\psi,$$

a negative E should be admitted. Obviously, it is physically impossible because it means that the de Broglie wave damps out or increases spontaneously in a free space. Therefore, the Schrödinger equation has physical meaning only for positive $E-\Phi$. Of course, purely from the mathematical point of view, we may obtain a solution of the Schrödinger equation violating the energy conservation law. However, the case is no more than a purely mathematical instance, so we must necessarily examine whether obtained solutions satisfy physical requirements.

On the other hand, the fact that the Hamilton operator is commuted with itself is sufficient to understand that the energy conservation law should hold exactly at every instant in microscopic systems. In fact, for a stationary state, we have

$$\frac{{\rm d}\hat{H}}{{\rm d}t}=\frac{1}{{\rm i}\hbar}[\hat{H},\hat{H}]=0.$$

This indicates energy conservation.

Next, it is unreasonable to justify the violation of energy conservation by means of the uncertainty relation. The two laws, i.e., the law of energy conservation and the uncertainty relation must hold together with each other.

In this connection, it is necessary to recall the conventional interpretation. The conventional interpretation is described as follows [30]. The kinetic energy $T = p^2/2m$ is a function of momentum. Therefore, according to the uncertainty relation $\triangle x \cdot \triangle p \geq \hbar$, it is impossible to split the energy precisely into $E = T + U$. It would seem that localizing a particle beyond the classically permissible region implies the violation of energy conservation. However, this is not the case. If we try to localize the particle (i.e., concentrate its wave function) in the small tails of the function ψ, the uncertainty of momentum increases to a point where the new total energy exceeds the value of the potential energy $U(x)$. Thus, from the point of view of energy, the particle is allowed to take a x value beyond the classically permitted region. In any case, it is the wave character that allows the penetration into potential barriers and, finally, its tunneling [30].

The following reasoning refutes the above description. Contrary to the above argument, it is possible to divide the energy precisely into $E = T + U$. This is because the total energy E is constant according to the Schrödinger equation for stationary state and the potential energy $U(x)$ is a definite function of x. So in the case of a finite square barrier, the potential energy is constant. Thus, the kinetic energy has a definite value. What varies is the probability density depending on position, which is due to the wave property. From the above argument, it is impossible to imagine a quantum effect which causes the energy of an electron to surpass the barrier height. We understand the uncertainty relation based on the ensemble interpretation, specifically, in terms of the relation between the distribution of position and that of momentum [31, 32]. This understanding denies the conventional interpretation of tunneling based on the uncertainty relation. Since the conventional theory permits purely imaginary momentum inconsistent with the Hermitian property of the momentum operator, any subsequent discussions are unnecessary. Evidently, any quantum observables should always be real.

It is necessary to examine the physical meaning of the square potential barrier. As mentioned above, the square potential barrier carries the meaning of the loss of energy at the beginning of the barrier and then the gain of energy at the end of the barrier. This indicates the lack of the change in dynamical state of transmitted electrons. In the end, the square barrier is meaningless. On the other hand, the square potential barrier purports a constant potential inside the barrier. Since the constant potential does not generate force, it cannot affect the energy of a particle inside the barrier regardless of the width of potential barrier. However, according to the conventional interpretation, the probability of tunneling depends on the width of barrier. The possible effect of the width of barrier in the quantum sense is the constraint relevant to the de Broglie wavelength of an electron. In fact, the width of barrier plays only the role of selecting the de Broglie wavelengths of electrons commensurate to the barrier. Consequently, an electron in the potential barrier should be considered to be in a state of free motion except for the quantum constraint due to the wave nature. But the conventional theory does not describe such a mechanism. It is reasonable to consider that the potential in the barrier in general is dependent on coordinate.

According to the conventional theory, the wave function inside the barrier has the form of $\psi_{\text{II}} = A_2 e^{-\chi x} + B_2 e^{\chi x}$ where $K = \frac{\sqrt{2m(E-\Phi)}}{\hbar} = \mathrm{i}\chi$. Hence, it follows that since χ is real, the probability current density inside the barrier vanishes according to

$$j_{\text{II}} = \frac{\mathrm{i}\hbar}{2m}\left(\psi_{\text{II}}\frac{d\psi_{\text{II}}^*}{dx} - \psi_{\text{II}}^*\frac{d\psi_{\text{II}}}{dx}\right).$$

It violates the continuity condition.

The conventional interpretation of tunneling cannot give the reasonable explanation for electron of higher energy than the top of barrier. Actually, with the transmission factor for tunneling through the barrier, i.e., Eqs. (10) and (11), it is not possible to give reasonable interpretation of tunneling. This is because in case E is higher than Φ, the transmission factor becomes complex number. In fact, Eqs. (10) and (11) are derived from the ratio of transmitted current density j_t to incident current density j_i:

$$D = \frac{j_t}{j_i},$$

so they should be real. Eventually, D of a complex value is inconsistent from the mathematical aspect.

As an important problem, it is necessary to reveal whether the tunnel effect is associated with a single electron or statistical ensemble of electrons. In this connection, we should note that we always make an experiment with electron ensemble but not one electron. Even for one electron confined in a certain region, such an electron cannot be considered a free particle, since the electron interacts ceaselessly with surrounding system, i.e., many-particle system, so we should consider that it exists in a set of statistical states undergoing quantum fluctuation. Such a state should be represented with the help of the density matrix, $\Psi = \sum_n W_n |\psi_n\rangle\langle\psi_n|$. Even in the case of cryogenic phenomena such as the tunnel effect in superconductors, it means not exactly the absolute zero-point state.

It is important to consider that every electron does not possess the total energy as determined only by a bias voltage. Electrons constitute a statistical ensemble by interacting with one another and therefore, it is natural to consider that in the sense of probability there exist the electrons that possess higher energy than that given by the applied voltage. In contrast to this, there exist the electrons of lower energy than that by applied voltage. It is both classical statistics and quantum statistics that explain this fact. Therefore, temperature as a quantity determining the statistical distribution of velocity of electrons signifies nothing but the existence of electrons capable of surmounting the potential barrier. In fact, the number of electrons possessing higher total energy than the barrier height is determined by temperature. Therefore, for problems relevant to ensemble of electrons, temperature should be necessarily considered, but the conventional theory on tunneling is limited purely to the realm of quantum mechanics. The above arguments lead us to a reasonable interpretation of tunneling. First of all, we can give clear answer to the question on whether or not the conventional interpretation is correct. Obviously, tunneling is attributed to the quantum factor. But the wave property of electrons can never result in a miracle such that electrons might surmount a barrier with lower energy than the height of the barrier. If tunneling is possible, it means that work is performed but there is no energy consumption. In the conventional interpretation, one uses the solution to the Schrödinger equation with negative kinetic energy and purely imaginary momentum in the barrier region. This is inconsistent with the fundamentals of quantum mechanics, e.g., including the real property of observables. In any case, the Schrödinger equation must satisfy the energy conservation law and fundamentals of quantum mechanics.

B. Consistent and general results

The conventional theory on the tunnel effect as a peculiar quantum phenomenon seems to give a possible explanation of the main features of the tunnel effect relative to the height and width of barrier. However, more detailed consideration leads to the understanding that the conventional theory is not perfect. Obviously, the mechanism of tunneling still remains unclear.

To explain the mechanism of tunneling in a consistent way, we adopted the quantum statistical approach. Tunneling attracts peculiar attention on the grounds that electrons have a probability of passing through the barrier even if the height of barrier is higher than the energy due to an applied voltage. The smaller the difference between the barrier height and the total energy of electron is and the thinner the barrier width is, the larger the tunneling current. This shows the two main features of the tunnel effect. For the purpose of explaining this phenomenon, the conventional theory on tunneling states that although the energy of an electron is lower than the barrier height, the electron can tunnel through the barrier with a definite probability thanks to the wave-like property of electron. As it is, this implies that we should take the physically meaningless solution of the Schrödinger equation admitting of even the negative kinetic energy. For this reason, this interpretation is not consistent with the quantum theory and violates energy conservation.

In this work, we have satisfactorily explained the two important characteristics of tunneling in line with the law of energy conservation and quantum theory. Considering that tunneling pertains to the statistical ensemble of electrons, we treated the tunneling current based on the thermionic emission, current resonance in the barrier and collective behavior of correlated electrons around the barrier.

Our view is that tunneling is due to not only quantum but also statistical characteristics of electrons. In this connection, our method should be considered to belong to such a category of picture as presented in [15–23].

According to our interpretation, the current across the barrier depends on three factors. The statistical ensemble of electrons is in an equilibrium state before applying voltage. The electrons passing through the barrier exist with a definite probability because the electron ensemble corresponds to a statistical distribution. In an equilibrium state, the cross flows from both sides of a barrier balance. If a voltage is applied, then the barrier heights on both sides which electrons feel become different. Consequently, the momentum distributions of electrons able to surmount the barrier on both sides change, and as a result the current in the forward direction occurs. As shown in Eq. (1), the current increases with the behavior of the exponential function according as the difference between the barrier height and the energy of electron due to applied voltage decreases. This is the first origin responsible for the current phenomenon in the barrier, which does not assume quantum.

Meanwhile, a bias voltage results in the change in the density of electrons which can surmount the barrier. Therefore, the gradient of electric potential appears. This gradient of electric potential makes the barrier lower on the left side whereas higher on the right side. Thus, the influence of applied voltage on the statistical distribution of electron density increases the current through the barrier. This is the second origin of the current phenomenon in the barrier, which is not quantum as well.

On the other hand, the barrier imposes the quantum constraint on the current across the barrier. The quantum constraint which is determined by the relation between the electron momentum passing through the barrier and the barrier width plays the role of making the momentum distribution of the passing electron be raised up. In other words, the thinner the width of the barrier is, the higher the mean velocity of electrons. For this reason, the barrier can be considered a quantum filter to make the current increase. Based on the proposed qualitative interpretation, we can describe the effect of barrier width on the current across the barrier. On the other hand, as a quantum effect, the exchange-correlation interaction around the barrier which combines electrons to form a chain of electrons behaving collectively results in lowering the barrier height. These two effects are the third origin of the current phenomenon in the barrier.

If temperature is very high, the current across the barrier related to the difference $\Phi - U_a$ is insignificant, since

Eq. (1) is reduced to

$$j_{st} = AT^2 \exp\left(-\frac{\Phi - U_a}{kT}\right)\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right]$$
$$\approx AT^2\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \approx AT^2\frac{2U_a}{kT} = \frac{2ATU_a}{k}. \quad (18)$$

Therefore, the current across the barrier increases with temperature and the role of barrier is insignificant. This means that in this case, the characteristic current phenomenon due to the relation between the energy by applied voltage and height of barrier vanishes completely as a consequence of being quenched by the increasing drift current due to barrier-surmountable electrons.

In the case of very low temperature too, the characteristic current related to the difference $\Phi - U_a$ vanishes, since as $\frac{\Phi - U_a}{kT} \to \infty$, the current becomes

$$j_{st} = AT^2 \exp\left(-\frac{\Phi - U_a}{kT}\right)\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \to 0. \quad (19)$$

From Eq. (19), it is evident that the current across the barrier of statistical origin would decrease with lowering temperature, thereby approaching zero near absolute zero. But in this case, the current of quantum origin across the barrier, which is temperature independent, gets rather higher.

Eqs. (1) and (2) shows that according as the applied voltage gets higher, the influence of barrier gradually vanishes and thus in the case of $U_a \gg \Phi$ the total current is dominated purely by the current determined only by applied voltage, i.e.,

$$j_{st} = AT^2\left[\exp\left(-\frac{\Phi - U_a}{kT}\right) - \exp\left(-\frac{\Phi + U_a}{kT}\right)\right]$$
$$\approx AT^2 \exp\left(\frac{U_a}{kT}\right)\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \approx AT^2 \exp\left(\frac{U_a}{kT}\right). \quad (20)$$

The hitherto described contents are limited to the statistical consideration. The main feature of tunneling is due to quantum origin. The quantum condition given by Eq. (5) and the exchange-correlation interaction cause the effect increasing the current across the barrier. Essentially, the quantum constraint reflects the periodicity of quantum phase given by the barrier. This condition is in common with studies for interpreting tunneling by applying the quasi-classical approach in terms of Wigner or Husimi function [15–17, 24, 25]. What should be emphasized here is that this periodicity condition is given by the fundamental equation of the statistical formalism in phase space presented previously by the authors. The current of statistical origin across the barrier is filtered and amplified by the ensemble of electrons entangled by the exchange-correlation interaction.

The cold-field emission which occurs when applying a strong electric field to a metal is considered to be due to tunneling. For this case too, our approach is consistent, if it is considered that electrons in metal should be in a statistical state. Before an electric field is applied to a metal, near the surface of metal there is a cross flow responsible for thermionic emission and electrons returning to the metal by the image charge attraction. The barrier is caused by the image charge attraction. This cross flow is formed within a definite distance from the surface, which is determined by the distribution of electrons in the metal. By the application of an electric field, the work function of electrons in the metal gets smaller, so the distribution of electrons according to distance from the surface of metal shifts to a new equilibrium state. Thus, the range of cross flow is expanded. When this range is stretched to the electrode, the current by the emitted electrons emerges. This phenomenon can be explained quantitatively based on our theory. Furthermore, the tunneling in ionization is explained in the same way based on the ensemble of states as well.

According to our theory, α decay can be attributed to a preformed particle rattling around within the nucleus of the radioactive (parent) element, eventually surmounting the potential barrier to escape as a detectable decay product. While inside the parent nucleus, the α is virtually free, but nonetheless confined to the nuclear potential well by the nuclear force. But as a result of the interaction between nuclear particles, the concentration of energy on a particle inside the nucleus may arise and thus this possibility provides the opportunity for the electron to escape out of the nucleus. In this case, the escaping α particle should have energies of narrow range related to the height of the potential well, since the energies near the top of the barrier have high probability. Such a view leads to the reasonable explanation of the fact that all particles emitted from any one source have nearly the same energy and, for all known emitters, emerge with kinetic energies in the same narrow range, from about 4 to 9 MeV. Once outside the nucleus, the particle experiences only the Coulomb repulsion of the emitting nucleus. The nuclear force on the α outside the nucleus is insignificant due to its extremely short range. Due to the stochastic process inside nucleus, α decay should exhibit a wide range of half-life of the emitter. This is compatible with the fact that in contrast to the uniformity of energies, the half-life of the emitter varies over an enormous range-more than 20 orders of magnitude.

The main purpose of this work has been to demonstrate that the phenomenon of current across the barrier called tunneling is actually a quantum statistical hybrid phenomenon beyond pure quantum realm. Our work has unraveled some imperfect aspects of the conventional interpretation and has offered a consistent and general formulation for explaining the true mechanism of tunneling.

IV. METHOD

A. Current of statistical origin

1. Bias current in barrier

Let us consider the tunnel effect from the point of view of quantum statistics. Electrons, for instance, in metals can be thought of to be an interacting system or open system, and therefore its velocity distribution depends on temperature. Consequently, there exist electrons which have the energies able to surmount the barrier with a certain probability. Before applying a bias voltage, currents toward both sides of barrier are in equilibrium. Fig. 6 shows merely the changes in the heights on both sides of the barrier without considering a particular shape of the barrier. Suppose that one applies the voltage, U_a making

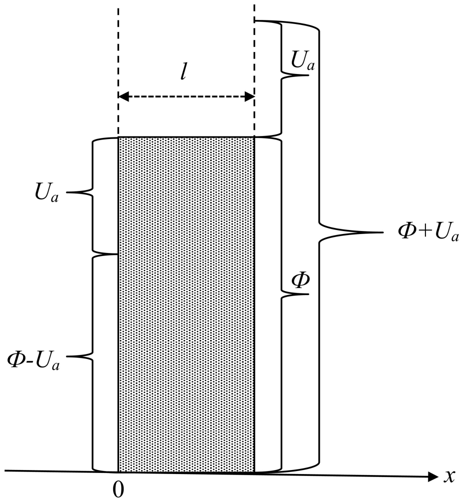

FIG. 6: Variation in height of barrier on both sides due to applied voltage. U_a denotes the variation in height of the barrier by an bias voltage.

electrons move in the forward direction, which conveys a lower energy than the height of barrier to the electrons.

Then for the electrons on the left side of the barrier, the height of barrier becomes $\Phi - U_a$ and for the electrons on the right side of barrier, $\Phi + U_a$. Fig. 6 shows the changes in the barrier heights on both sides, respectively. If a bias voltage is applied, then the currents across the barrier from both sides are shifted to a new equilibrium, so the total current is not balanced. Since the current for this case is thermionic, we calculate it by means of the Richardson-Dushmann equation for current density for the thermionic emission:

$$j_t = AT^2 \exp\left(-\frac{\Phi}{kT}\right). \tag{21}$$

In case the forward bias is applied, the current density in the forward direction is

$$j_{\Rightarrow} = AT^2 \exp\left(-\frac{\Phi - U_a}{kT}\right), \tag{22}$$

while the current density in the backward direction,

$$j_{\Leftarrow} = AT^2 \exp\left(-\frac{\Phi + U_a}{kT}\right). \tag{23}$$

Then the resultant current density in the forward direction can be evaluated from

$$\begin{aligned} j_{st} &= j_{\Rightarrow} - j_{\Leftarrow} \\ &= AT^2 \left[\exp\left(-\frac{\Phi - U_a}{kT}\right) - \exp\left(-\frac{\Phi + U_a}{kT}\right)\right] \\ &= AT^2 \exp\left(-\frac{\Phi}{kT}\right)\left[\exp\left(\frac{U_a}{kT}\right) - \exp\left(-\frac{U_a}{kT}\right)\right] \\ &= 2AT^2 \exp\left(-\frac{\Phi}{kT}\right) \sinh\left(\frac{U_a}{kT}\right). \end{aligned} \tag{24}$$

This is Eq. 1 that has been already shown.

2. Diffusion current

In addition to the drift current of statistical origin, another physical effect should be taken into consideration. A bias voltage changes the density of electrons capable of surmounting the barrier on both sides and as a result the gradient of electron density appears. It causes the diffusion current and therefore this effect additionally change the heights of the barrier on both sides. Fig. 7 shows the variation in the barrier heights on both sides due to the gradient of electron density.

In this case, U_a in Eq. (24) should be replaced with $U_a + U_g$, so that

$$j_{st} = 2AT^2 \exp\left(-\frac{\Phi}{kT}\right) \sinh\left(\frac{U_a + U_g}{kT}\right). \tag{25}$$

To obtain U_g, we should solve the Poisson equation in consideration of the electron density in the barrier region. The Poisson equation in the barrier region is expressed as

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = -\frac{\rho}{\varepsilon_0}. \tag{26}$$

Supposing that the charge density in the barrier region has such a linearity as

$$\rho(x) = ax + b, \tag{27}$$

Eq. (26) becomes

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = -\frac{ax + b}{\varepsilon_0}. \tag{28}$$

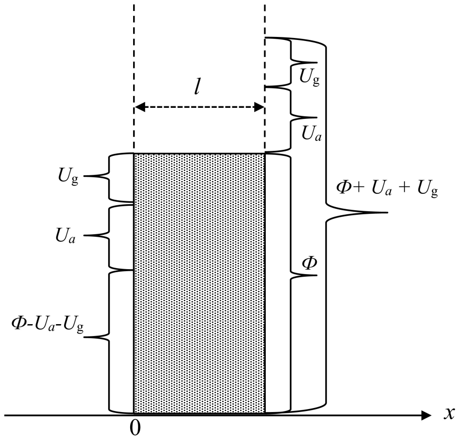

FIG. 7: Variation in barrier heights on both sides of the potential barrier due to applied voltage U_a and density gradient. U_g is the variation in barrier height due to the density gradient.

From Eq. (27), we have

$$\rho_0 = b. \tag{29}$$

At the end point of the barrier, the charge density is determined as

$$\rho_l = al + \rho_0. \tag{30}$$

Thus, we identify

$$a = \frac{\rho_l - \rho_0}{l}. \tag{31}$$

By the Fermi-Dirac distribution, the electron density can be expressed as

$$\begin{aligned} n &= \int_U^\infty f(E)N(E)\mathrm{d}E \\ &= \frac{4\pi(2m)^{3/2}}{h^3}\int_U^\infty \frac{(E-U)^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E, \end{aligned} \tag{32}$$

where E_f is the Fermi energy and U is $U_a + U_g$. Then, integrating the Eq. (28), we obtain

$$\frac{\mathrm{d}\varphi}{\mathrm{d}x} = -\frac{a}{2\varepsilon_0}x^2 - \frac{b}{\varepsilon_0}x + c. \tag{33}$$

Considering the boundary condition, we can determine constant c. Since at the end of the barrier the intensity of electric field due to charge gradient is zero, we get

$$\left.\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right|_{x=l} = -\frac{a}{2\varepsilon_0}l^2 - \frac{b}{\varepsilon_0}l + c = 0. \tag{34}$$

Thus, the integral constant c is

$$c = \frac{a}{2\varepsilon_0}l^2 + \frac{b}{\varepsilon_0}l. \tag{35}$$

Integrating Eq. (33), we obtain

$$\varphi(x) = -\frac{a}{6\varepsilon_0}x^3 - \frac{b}{2\varepsilon_0}x^2 + cx + d. \tag{36}$$

The potential at the end point of barrier is

$$\varphi(l) = -\frac{a}{6\varepsilon_0}l^3 - \frac{b}{2\varepsilon_0}l^2 + cl + d. \tag{37}$$

Therefore, the potential difference between the edges of the barrier is represented as

$$\varphi(0) - \varphi(l) = \frac{a}{6\varepsilon_0}l^3 + \frac{b}{2\varepsilon_0}l^2 - cl. \tag{38}$$

Inserting the determined constants into Eq. (38), we get

$$\begin{aligned} \triangle\varphi &= \frac{\rho_l - \rho_0}{6\varepsilon_0 l}l^3 + \frac{\rho_0}{2\varepsilon_0}l^2 - \left[\frac{1}{\varepsilon_0}\left(\frac{\rho_l-\rho_0}{l}\right)\frac{l^2}{2} + \frac{\rho_0}{\varepsilon_0}l\right]l \\ &= -\frac{1}{3\varepsilon_0}(\rho_l - \rho_0)l^2 - \frac{\rho_0}{2\varepsilon_0}l^2 = -\frac{1}{\varepsilon_0}\left(\frac{\rho_l}{3} + \frac{\rho_0}{6}\right)l^2. \end{aligned} \tag{39}$$

As a consequence, the additional voltage due to the gradient of charge density is written as

$$U_g = -e\triangle\varphi = \frac{e}{\varepsilon_0}\left(\frac{\rho_l}{3} + \frac{\rho_0}{6}\right)l^2. \tag{40}$$

Thus, Eq. (25) has been completely determined. From Eq. (32), the charge densities at the edges of barrier are expressed respectively as

$$\rho_0 = -en_0, \quad \rho_l = -en_l, \tag{41}$$

where the electron densities are determined respectively by

$$\begin{aligned} n_0 &= \int_{\Phi-U_a}^\infty f(E)N(E)\mathrm{d}E \\ &= \frac{4\pi(2m)^{3/2}}{h^3}\int_{\Phi-U_a}^\infty \frac{(E-\Phi+U_a)^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E, \end{aligned} \tag{42}$$

$$\begin{aligned} n_l &= \int_{\Phi+U_a}^\infty f(E)N(E)\mathrm{d}E \\ &= \frac{4\pi(2m)^{3/2}}{h^3}\int_{\Phi+U_a}^\infty \frac{(E-\Phi-U_a)^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E. \end{aligned} \tag{43}$$

As an example of using the approximate expression of carrier density in completely degenerated semiconductor, the electron densities can be expressed respectively as

$$n_0 = \frac{8\pi}{3h^3}(2m)^{3/2}(E_f - \Phi + U_a)^{3/2}, \tag{44}$$

$$n_l = \frac{8\pi}{3h^3}(2m)^{3/2}(E_f - \Phi - U_a)^{3/2}. \tag{45}$$

With the help of Eqs. (41), (44) and (45), ρ_0 and ρ_l are determined respectively as

$$\rho_0 = -\frac{8\pi e}{3h^3}(2m)^{3/2}(E_f - \Phi + U_a)^{3/2}, \quad (46)$$

$$\rho_l = -\frac{8\pi e}{3h^3}(2m)^{3/2}(E_f - \Phi - U_a)^{3/2}. \quad (47)$$

Thus, we have obtained the relation of the height variation of barrier to the difference between the charge densities at the edges of barrier caused by an applied voltage and have explained the dependence of current across the barrier on barrier width. Inserting Eqs. (46) and (47) into Eq. (40), we identify that the thinner the barrier is, the larger the current. In this way, we can explain the dependence of the current across the barrier on the barrier width. In the end, it is Eq. 2 that has been explained.

B. Quantum effects in barrier

1. *Current resonance due to quantum constraint in barrier*

Based on the formalism of quantum mechanics in phase space, the quantum characteristic of current across the barrier can be satisfactorily interpreted. This temperature independent, purely quantum effect should be considered to correspond to the so-called tunneling. Let us start by using the fundamental equation of quantum mechanics in phase space. According to the statistical formalism of quantum mechanics in phase space [32], the equation for probability density is

$$\frac{\partial \rho}{\partial t} = -\frac{1}{2}\sum_i \left[\frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial p_i}\right], \quad (48)$$

where H is the Hamilton function. We suppose that the current across the barrier is stationary. In this case, the probability density is not dependent on time and thus, the above equation is reduced to

$$\sum_i \left[\frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial p_i}\right] = 0. \quad (49)$$

This equation holds for the constant probability density. Generally, if the probability density is a function of the Hamiltonian function, then the equation is satisfied. The case is verified by

$$\begin{aligned}&\sum_i \left[\frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial p_i}\right]\\ &= \sum_i \left[\frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial H}\frac{\partial H}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial H}\frac{\partial H}{\partial p_i}\right]\\ &= \frac{\partial \rho}{\partial H}\sum_i \left[\frac{\partial H}{\partial p_i}\frac{\partial H}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial H}{\partial p_i}\right] = 0. \quad (50)\end{aligned}$$

Therefore, the probability density in a stationary state is expressed as

$$\rho = \psi^*\psi = f(H), \quad (51)$$

where ψ is the wave function defined in phase space.

Hence, the probabilistic amplitude is represented as

$$\psi_0 = \sqrt{\psi^*\psi} = \sqrt{f(H)}. \quad (52)$$

As a result, a solution to the wave equation for stationary states in general takes the following form:

$$\psi(x,p) = \psi_0(H(x,p)) \exp\left[\frac{\mathrm{i}}{\hbar}\int_0^x p(x')\mathrm{d}x'\right]. \quad (53)$$

It is necessary to note that the phase part of this equation is identified with that of the wave function in quasi-classical approximation. From Eq. (53), it is obvious that the barrier region should satisfy the following constraint:

$$\int_0^l p(x)\mathrm{d}x = \int_0^l \sqrt{2m\left[E - U(x)\right]}\mathrm{d}x = nh, \quad (54)$$

where E is the total energy of an electron determined by applied voltage and $U(x)$, the barrier potential. This constraint stands for the periodicity condition of the wave function within the barrier, which is distinguished from the Wentzel-Kramer-Brillouin approximation (WKB approximation) according to which the transmission factor of a barrier is proportional to $\exp\left(-\frac{2}{\hbar}\int\sqrt{U(x)-E}\mathrm{d}x\right)$.

With the help of this constraint, the quantum characteristic of tunneling can be clarified. From Eq. (54), it follows that the kinetic energy of an electron passing through the barrier should be higher than the height of barrier. For the purpose of a qualitative consideration, we define the mean value of momentum of an electron in a barrier region as

$$\bar{p} = \frac{\int_0^l p(x)\mathrm{d}x}{l} = \frac{\int_0^l \sqrt{2m[E-U(x)]}\mathrm{d}x}{l}.$$

According to Eq. (54), the following condition should be satisfied.

$$\bar{p}l = nh.$$

From this, the mean momentum of an electron surmounting the barrier should be quantized as

$$\bar{p}_n = \frac{h}{l}n. \quad (55)$$

This condition relevant to momentum characterizes the current resonance in the barrier. The minimum momentum of an electron able to pass through the barrier is represented as

$$\bar{p} = \frac{h}{l}. \quad (56)$$

Hence, it follows that the thinner a barrier is, the higher the velocity of an electron passing through it should be. A barrier can be regarded as a quantum filter which connects the two free-motion regions and defines quantized momenta necessary to pass through the barrier. With a good reason, it can be considered that the electron density in a barrier region is almost in a saturation state and thus, is constant. This is because the interaction between electrons and the applied voltage functions as a source complementing the electrons satisfying the quantum constraint in the barrier.

From the relation of current density:

$$j = \rho \upsilon, \tag{57}$$

the quantity contributing to the current density is assessed principally as the velocity of electron, υ. According to the quantum constraint, Eq. (55), the thinner the width of barrier is, the larger the momentum which the barrier filters, and as a consequence the current is increased.

2. *Exchange-correlation interaction and collective behavior of electrons around barrier*

We assume that the exchange-correlation interaction is an important factor contributing to lowering the barrier height. Two ensembles of electrons before and after the barrier are correlated, or entangled, so the interaction between them helps electrons of low energy to acquire the additional energy necessary to overcome the barrier from the correlated electrons of high energy after the barrier. Then it depends on the distance between the incident electrons and the transmitted electrons, i.e., the width of the barrier: the thinner the width of the barrier is, the stronger the exchange interaction. Thus, electrons as an integrated entity around the barrier cooperate with one another to overcome the barrier. In this case, the quantum constraint in the barrier is indispensable. This mechanism of the resonant current across the barrier attributed purely to quantum property satisfies energy conservation unlike the conventional interpretation of tunneling.

Based on Ref. [33], it is possible to evaluate the exchange-correlation energy combining electrons around the barrier as

$$E_{xc} = -\frac{1}{2}\sum_{j\neq i}\left[\delta\left(m_{s_i}, m_{s_j}\right)\cdot \int \psi_i^*\left(\mathbf{r}_1\right)\psi_j^*\left(\mathbf{r}_2\right)\hat{H}_{12}\psi_i\left(\mathbf{r}_2\right)\psi_j\left(\mathbf{r}_1\right)\theta(\mathbf{r}_1,\mathbf{r}_2)\mathrm{d}\tau_1\tau_2\right], \tag{58}$$

where ψ_i and ψ_j are the one-electron wave functions for states before and after the barrier, m_{s_i}, m_{s_j} the spin quantum numbers for states before and after the barrier, respectively, $\hat{H}_{12}$ the operator of interaction between two electrons, and $\theta(\mathbf{r}_1, \mathbf{r}_2)$ the correlation hole function. Setting $\psi_i(\mathbf{r}) = e^{\mathrm{i}kx}, \psi_j(\mathbf{r}) = e^{\mathrm{i}Kx}, \hat{H}_{12} = \frac{e^2}{x_2 - x_1}$ where k and K are the wave number of electron before and after the barrier, we write the energy of the exchange-correlation interaction as

$$E_{xc} = -\frac{1}{2}\sum_{j\neq i}\left[\delta\left(m_{s_i}, m_{s_j}\right)\cdot \int\int e^{-\mathrm{i}kx_1}e^{-\mathrm{i}Kx_2}\frac{e^2}{x_2 - x_1}e^{\mathrm{i}kx_2}e^{\mathrm{i}Kx_1}\mathrm{d}x_1\mathrm{d}x_2\right].$$

Introducing parameter A taking synthetically into consideration the spin correlation of the many-electron system, we write

$$E_{xc} = -\frac{Ae^2}{2}\int\int\frac{e^{\mathrm{i}(k-K)(x_2-x_1)}}{x_2 - x_1}\mathrm{d}x_1\mathrm{d}x_2.$$

Taking into consideration that the exchange-correlation interaction is the interaction between electrons which are spaced about the width of the barrier apart, we set $x_2 - x_1$ approximately as the width of the barrier, l to rewrite

$$E_{xc} \approx -\frac{Ae^2}{2l}\int_0^l\int_0^l e^{\mathrm{i}(k-K)(x_2-x_1)}\mathrm{d}x_1\mathrm{d}x_2.$$

By computing the integral and making A embrace all approximate factors, we obtain the final result,

$$E_{xc} = -\frac{Ae^2\left\{1 - \cos[(k-K)l]\right\}}{2l(k-K)^2}.$$

This outcome manifestly shows that the smaller the barrier width and the difference between the energy of electrons and the height of barrier are, the larger the exchange-correlation interaction. Therefore, electrons are to behave collectively before and after the barrier by combining one another. This effect should be considered to perform the role of lowering the height of the barrier. Then bias U_a includes E_{xc}.

Meanwhile, it is possible to explain the current-voltage characteristic of the tunnel effect. Originally, according as the momentum of electrons due to the bias voltage approaches $\bar{p}$, the current through the barrier increases in compliance with the quantum constraint, Eq. (55) but at the next stage according as the increasing momentum of electrons gets away from $\bar{p}$ the current across the barrier decreases gradually as a result of the breach of the quantum constraint. This gives a good explanation to the question on why a peak occurs on the current-voltage characteristics for tunneling.

It is possible to show this fact in a semi-quantitative way. The current density obtained through the consideration of the statistical relation can be represented with respect to momentum as

$$j_{st} \propto \sinh(aU_a) = \sinh(bp^2). \tag{59}$$

This reflects the fact that the increase in kinetic energy of an electron is proportional to applied voltage, i.e.,

$U_a \propto p^2$, where U_a indicates $-eV$. The current-voltage characteristic is depicted in Fig. (1).

On the other hand, the current of statistical origin undergoes a kind of filtering due to the quantum constraint in the barrier as determined by Eq. (54). This filtering effect modulates the statistical current density. In line with physical meaning, one can suppose the modulation coefficient by which the statistical current is multiplied. The modulation coefficient characterizing the filtering of statistical current can be represented as the following mathematically modelled relation:

$$\eta = \sum_n C_n \exp\left(-\alpha_n \left(p^2 - p_n^2\right)^2\right),$$

where p_n denotes the extreme values of momentum of electron corresponding to the quantum constraint. Considering $U \propto p^2$, the above relation can be recast as modulation-voltage relation

$$\eta = \sum_n C_n \exp\left(-\beta_n \left(U_a - U_n\right)^2\right), \tag{60}$$

Eq. (60) is sketched in Fig. 2. Considering that the exchange-correlation interaction is effective within the range comparable to the de Broglie wavelength, we can take one peak for the modulation corresponding to current resonances. Then the resultant current density across the barrier, filtered by the condition of current resonance, is evaluated from

$$j = \eta j_{st}. \tag{61}$$

The current density according to applied voltage is depicted in Fig. 3. The figure is in good agreement with the current-voltage characteristic of the tunnel diode. This demonstrates that our quantum statistical analysis is reasonable. Thus, it is revealed that we have referred to the current phenomena due to such a quantum cause as tunneling. Unlike the conventional theory, our approach reasonably explains the characteristic of current change in case the energy by applied voltage is higher than the barrier height as well.

This approach gives a reasonable explanation for the resonant tunnel effect as well. From the quantum constraint, Eq. (54), it is obvious that in the region where barriers and wells alternate periodically, the quantum constraints for every region should be satisfied, i.e.,

$$\int_{l_i^{(k)}}^{l_f^{(k)}} p(x)\mathrm{d}x = \int_{l_i^{(k)}}^{l_f^{(k)}} \sqrt{2m[E - U(x)]}\mathrm{d}x = n_k h, \tag{62}$$

where $l_i^{(k)}$ is the first boundary of the kth barrier or well and $l_f^{(k)}$, its final boundary. Only electrons satisfying these constraints all together can pass through the whole region of barriers and wells. Therefore, barriers and wells should be regarded as playing the role of a kind of resonance which filters electrons in agreement with the quantum constraint for passing through. For this reason, it is possible that in a multiple barrier-well region, the quantum-selective effect, i.e., the sharp resonance effect occurs. Ultimately, all the properties of tunneling have been explained in a consistent way.

V. CONCLUSION

Our aim has been to elucidate the mechanism of tunneling in a consistent way and to explain general characteristics of tunneling based on the quantum statistical theory. For this reason, we adopted a quantum statistical approach distinct from the conventional one. Based on quantum statistical interpretation, we have satisfactorily explained the statistical and quantum aspects of the current phenomenon in the barrier. The adopted approach encompasses the thermionic emission due to statistical ensemble of electrons or electron states and the quantum resonant current and the exchange-correlation interaction.

As a main motivation of our work, it has been analyzed that the conventional explanation of the tunnel effect violates the universal law of energy conservation and is incompatible with quantum theory. Our theory satisfies the energy conservation and quantum theory together. This is because we took into consideration the statistical aspect of the current phenomenon in the barrier.

The main conclusion of our work is that the phenomenon of the current across the barrier should be explained based on quantum statistics as well as quantum mechanics. Such a perspective has enabled us to elucidate the statistical and quantum aspects of the current across the barrier, thus making us unravel the essence of tunneling. Moreover, it should be emphasized that our work has successfully explained the current phenomenon in the barrier in a general way in the whole range of applied voltage.

Ultimately, our work makes it possible to solve some intractable problems such as the violation of the law of energy conservation arising from the conventional approach to tunneling and the direct contradiction to quantum theory, and offers the possibility of leading to the innovation in the picture of tunneling. We expect our work to substantially contribute to the elucidation of the physical nature of tunneling and also to the researches on tunneling of complex systems including the resonant tunneling effect.

Acknowledgements

We thank Professor Byong-Il Ri for help and comment.

[1] Patterson J. D. and Bailey B. C, Solid-State Physics, Ch. 8 (Springer, Berlin · Heidelberg · New York, 2007).
[2] Sasaki S., Tunneling Effect in Quantum Hall Device, e-J. Surf. Sci. Nanotech. Vol. **8**, 121 (2010).
[3] Grundmann M., The Physics of Semiconductors, Ch. 18 (Springer, Berlin · Heidelberg · New York).
[4] Belinski V. A., On tunneling through the black hole horizon, Phys. Lett. A **376**, 207 (2012).
[5] Ejaz A., Gohar H., Lin H., Saifullah K. and Yau S.-T., Quantum tunneling from three-dimensional black holes, Phys. Lett. B **726**, 827 (2013).
[6] Brabec T., Côte M., Boulanger P. and Ramunno L., Theory of Tunnel Ionization in Complex Systems, Phys. Rev. Lett. **95**, 073001 (2005).
[7] Vindel-Zandbergen P., Meier C. and Sola I. R., Tunneling induced electron transfer between separated protons, Chem. Phys. Lett. **698**, 152 (2018).
[8] Jürgensen O., Meinert F., Mark M. J., Nägerl H.-C. and Lühmann D.-S., Observation of Density-Induced Tunneling, Phys. Rev. Lett **113**, 193003 (2014).
[9] Perrin M. L. *et al*, Single-Molecule Resonant Tunneling Diode, J. Phys. Chem. C **119**, 5697 (2015).
[10] Tokieda M. and Hagino K., Quantum tunneling with friction. arXiv:1703.02714v1 [nucl-th] (2017).
[11] Leo S. D. and Leonardi V. Relativistic tunneling through opaque barriers, Phys. Rev. A **83**, 022111 (2011).
[12] N. L. Chuprikov, Tunneling time and superposition principle, arXiv:1705.00897v2 [quant-ph] (2017).
[13] Leison L. B. and Oraevsky V. N., Non-tunnel Effect in Quantum Mechanics, Phys. Lett. Vol. **87** A, No. 5, 207 (1982).
[14] Jonson N., The dynamical image potential for tunneling electrons, Solid State Commun. Vol. **33**, 743 (1980).
[15] Wigner E. P., On the quantum correction for thermodynamic equilibrium, Phys. Rev. **40**, 749 (1932).
[16] Kluksdahl N. C., Kriman A. M., Ringhofer C. and Ferry D. K., Quantum tunneling properties from a Wigner function study, Solid-State Electronics Vol. **31** No. **3/4**, 743 (1988).
[17] González M. F., Bofill J. M, and Giménez X, Bohmian Total Potential View to Quantum Effects III. Tunnelling in Phase Space, J. Phys. Chem. A **113**, 15024 (2009).
[18] Jensen K. L., The Methodology of Simulating Particle Trajectories Through Tunneling Structures Using a Wigner Distribution Approach, IEEE Trancactions on electron devices Vol. **38** No. **10** (1991).
[19] Shifren L., Ringhofer C. and Ferry D. K., A Wigner Function-Based Quantum Ensemble Monte Carlo Study of a Resonant Tunneling Diode, IEEE Trancactions on electron devices Vol. **50**, No. **3**, 769 (2003).
[20] Wagner A. F., Improved Multidimensional Semiclassical Tunneling Theory, J. Phys. Chem. A **117**, 13089 (2013).
[21] Martinez A. *et al*, Phase Space Tunneling in Multistate Scattering, J. Func. Analy. **191**, 297 (2002).
[22] Czirják A., Kopold R., Becker W., Kleber M. and Schleich W.P., The Wigner function for tunneling in a uniform static electric field, Opt. Commun. **179**, 29 (2000).
[23] Han B., Yang H., Zheng Y. and Varandas A. J. C., Quasiclassical trajectory and quantum mechanics study of the reaction $H(^{2S}) + NH - N(^{4S}) + H_2$, Chem. Phys. Lett. **493**, 225 (2010).
[24] Donoso A. and Martens C. C., Quantum Tunneling Using Entangled Classical Trajectories, Phys. Rev. Lett. **87**, 223202 (2001).
[25] Takahashi K. and Ikeda K. S., Complex-classical mechanism of the tunnelling process in strongly coupled 1.5-dimensional barrier systems, J. Phys. A **36**, 7953 (2003).
[26] Onishi T., Shudo A., Ikeda K. S. and Takahashi K., Tunneling mechanism due to chaos in a complex phase space, Phys. Rev. E **64**, 025201 (2001).
[27] Wang L., Zhang Q., Xu F., Cui X. and Zheng Y., Quantum Tunneling Process for Double Well Potential, Int. J. Quantum Chem. **115**, 208 (2015).
[28] Lin W. A. and Ballentine L. E., Quantum tunneling and chaos in a driven anharmonic oscillator, Phys. Rev. Lett. **65**, 2928 (1990).
[29] Álvarez-Barcia S., Flores J. R. and Kästner J., Tunneling Above the Crossover Temperature, J. Phys. Chem. A **118**, 78 (2014).
[30] Greiner W., Quantum mechanics; An introduction Ch. 7 (Springer, Berlin · Heidelberg · New York, 2001).
[31] Home D. and Whitaker A., Einstein's Struggles with Quantum Theory, Ch. 4 (Springer, Berlin, 2007).
[32] Jong C. *et al.* Ensemble in Phase Space: Statistical Formalism of Quantum Mechanics, Pramana J. Phys. Vol. **92** Iss. **5**, No. **83** (2019).
[33] Jong C., Self-consistent one-electron equation for many-electron systems and its general application to ground and excited states, Eur. Phys. J. Plus **135**, 746 (2020).