An alternative explanation of tunnel effect based on statistical interpretation and condition of quantum resonance by barrier

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Abstract

We present an alternative quantum-statistical interpretation of the electron tunneling through the potential barrier, which is distinguished from the conventional interpretation because in our interpretation the tunnel effect is considered to be due to statistical characteristics of electron ensemble in addition to the wave nature of electrons. The conventional interpretation of the tunnel effect is based purely on the wave nature of a single electron. This approach cannot elucidate satisfactorily the dynamics of electron motion in the potential barrier because the interpretation violates the universal law of energy conservation, just as the subtle term "tunnel effect" implies. In this work, we clarify the fact that the tunnel effect is ascribed substantially to a statistical ensemble of electrons, and explain it both by applying the statistical method and by considering the quantum restriction by the potential barrier on passing of electron rather than tunneling. Our interpretation satisfies the law of energy conservation and naturally explains all the characteristics of tunneling including the influence of temperature as well. The quantum restriction condition that is determined by potential barrier clarifies the quantum properties of tunneling. Finally, we offer a complete and general explanation of the tunnel effect, thus demonstrating that the physically not intuitive "tunneling" substantially connotes quantum plus statistical phenomenon.

Key words: Quantum tunneling dynamics, Entangled trajectory, Husimi function, Quasi-classical approach to tunneling

1. Introduction

Tunneling experiments are famous for the claim that electrons with energy insufficient to surmount a potential barrier can cross it. One of two types of tunneling experiments is the tunneling from a superconductor to a superconductor with a thin insulator separating the two superconductors. As the other type of tunneling, the Giaever tunneling (single-particle tunneling) is the tunneling of single quasi-electrons from an ordinary metal to a superconducting metal [1]. As a pure quantum-mechanical phenomenon, tunneling plays an important role in elucidating the physical processes in microscopic scales.

The conventional description of the tunnel effect is based on solution of the Schrödinger equation subject to boundary conditions related to a barrier region and two free-motion regions. The expression for the transmission coefficient of tunneling through the rectangular potential barrier:

$$D = D_0 e^{-\frac{2}{\hbar}\sqrt{2m(U_0-E)}\,l},$$

or, that through an arbitrary form of barrier:

$$D = D_0 e^{-\frac{2}{\hbar}\int_0^l \sqrt{2m[U(x)-E]}\,\mathrm{d}x}$$

gives a plausible explanation of main characteristics of the tunnel effect.

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However, the pure quantum interpretation of the tunnel effect encounters a serious problem concerning physical concepts. This is because this interpretation violates the law of energy conservation as a universal law of physics and does not reflect the temperature dependence at all, and cannot explain the characteristics of the tunneling current in a whole range of applied voltage.

In order to make dynamics of the tunneling intuitive and intelligible, there have been developed alternative methods. The quasi-classical approaches of quantum mechanics in phase space have been investigated, which aim to overcome the above-mentioned difficulties by making use of the Wigner or Husimi function useful for physical picture [2, 3, 4]. The trajectory-based quasi-classical approaches have been used as an effective mean to treat the quantum dynamical processes relevant to tunneling [5, 6, 7, 8, 9]. Miller had first introduced an approach to treat a collision of diatom with an atom by means of complex classical dynamical method [7].

The quasi-classical approach based on the Husimi function has been widely used in the numerical simulation of tunneling and further developed by Adachi. Tunneling in systems which is not integrable too has been investigated by Shuodo and his coworkers [8].

Meanwhile, the quantum maps method for time-continuous systems based on quasi-classical approach has been developed by Takahashi and Ikeda [9].

The entangled-trajectory molecular dynamics (ETMD) is a quasi-classical approach which has been newly developed based on the quantum phase-space theory [6, 10, 11, 12, 13, 14, 15, 16]. This approach is a powerful tool for solving the quantum Liouville equation numerically and also is effective in elucidating the dynamical processes of quantum characteristics such as the tunnel effect by using entangled trajectories in phase space. Comparing to the classical molecular dynamics, the entangled-trajectory molecular dynamics takes some advantages, i.e., it gets the quantum effect in evolution and requires lesser trajectories in finding the convergent value [12, 13, 14, 15, 16]. In various fields of quantum dynamics, the results obtained by applying ETMD approaches have been reported [6, 10, 11, 17, 18]. These approaches are based on utilization of the Wigner function which may produce negative values at some points. This shortcoming is an obstacle to resolving quantum-mechanical problem perfectly and correctly [19].

On the other hand, the Husimi function-based approach is a new resolution proposed in an attempt to find the correct information related to tunneling, since the Husimi function is positive at every point and guarantees accuracy. Several works making use of this approach have been reported [20, 21, 22].

In spite of the advances, several difficulties with interpreting the tunnel effect by using physical intuition still remain uncontrolled. For the conventional interpretation highly questionable is that the solution of the Schrödinger equation in the barrier region violates the law of energy conservation. As far as this problem has not been resolved, we are not justified in saying that a physically reasonable interpretation of tunneling has been offered.

Next, another problem is whether the tunneling is a quantum effect by a single electron or quantum-statistical effect by electron ensemble. If tunneling is attributed also to the electron ensemble, then it should depend on temperature. Noticeably, the conventional interpretation of tunneling is independent of temperature, so the effect is considered to be most important at low temperature. However, many cases of tunneling at room temperature in chemical and biochemical systems have been reported as well as processes at cryogenic temperature [23, 24, 25]. The most general and rigorous approach is to perform quantum dynamics, i.e., to solve the time-dependent Schrödinger equation starting from the reactant state ensemble [25]. This indicates the significance of the time dependence of the tunnel effect.

It is necessary to turn our attention to the fact that actually, experiments on tunneling is not related definitely to a single electron but deals with an ensemble of electrons. Moreover, it should be considered that the conventional interpretation cannot explain as yet in a general way the overall behavior of current through the barrier in the domain of variability of factors relevant to tunneling such as voltage and temperature.

The purpose of our work is to elucidate that tunneling is not a pure quantum-mechanical effect but a quantum-statistical hybrid effect. On the basis of quantum-statistical approach, we satisfactorily explain the main characteristics of tunneling by embracing all the quantum-statistical factors. Importantly, based on the statistical formalism of quantum mechanics presented already [26], we produce clear results showing the quantum nature and characteristics of the tunneling current through the barrier.

2. Review of the conventional interpretation of tunneling

Obviously, the conventional interpretation of tunneling violates the universal law of the energy conservation. To begin with, in order to review this point, it is necessary to assess whether the solution of the Schrödinger equation in the region of potential barrier is physically meaningful.

Let us consider this point in detail.

The Schrödinger equation in a barrier region is represented as

$$-\frac{\hbar^2}{2m}\Delta\psi + U\psi = E\psi, \tag{1}$$

where U is the potential energy in the barrier region. This equation is obtained by applying the correspondence principle to the energy relation:

$$\frac{p^2}{2m} + U = E. \tag{2}$$

This fact shows that the law of energy conservation must hold also in the microscopic world. If the height of barrier is higher than the total energy of a particle, its kinetic energy proves to be negative. For the negative kinetic energy, two cases are possible: one is the case that the momentum of a particle is purely imaginary and another is the case that the mass of the particle is negative. But both are not allowed in any physical sense and thus such physically meaningless motions in the potential barrier are impossible.

Indeed, an electron of negative kinetic energy moving through the barrier cannot be imagined because the case is not consistent with the physical meaning of the Schrödinger equation and leads to the violation of the universal conservation law of physics. In fact, if negative kinetic energy were possible, then in the Schrödinger equation for a free particle,

$$-\frac{\hbar^2}{2m}\Delta\psi = E\psi,$$

negative E should be allowed, which is physically inconsistent.

In any case, the Schrödinger equation should satisfy the energy conservation law. Recalling the fact that the Hamilton operator is commuted with itself is sufficient to understand that the energy conservation law should be satisfied at every instant in the microscopic system. In fact, for a stationary state, we have

$$\frac{\mathrm{d}\hat{H}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[\hat{H},\hat{H}] = 0.$$

This relation tells us that just at every instant the energy of microscopic particles should be conserved.

Thus, it is obvious that the universal conservation laws should be satisfied for microscopic systems as well as macroscopic ones. Of course, purely from the mathematical point of view, it is possible to obtain a certain solution to the Schrödinger equation violating the energy conservation law. However, the case is no more than a purely mathematical instance, so we must necessarily examine obtained solutions in view of physical requirements.

It is impossible to imagine negative mass or purely imaginary momentum inasmuch as quantum mechanics too should obey the universal conservation laws. Indeed, in any event we cannot assume such a solution to the Schrödinger equation which is physically meaningless even if it may be mathematically possible. We make use of the Schrödinger equation exactly on condition that the law of energy conservation is satisfied.

On the other hand, it is unreasonable to justify the violation of the law of energy conservation by dint of the uncertainty relation. The two laws should not exclude or dominate each other, and must be compatible with each other.

In this connection, it is necessary to recall the conventional interpretation. The conventional interpretation is as follows. The kinetic energy $T = p^2/2m$ is a function of momentum. Therefore, according to the uncertainty relation $\triangle x \cdot \triangle p \geq \hbar$, it is impossible to split the energy precisely into $E = T + U$. It would seem that localizing the particle beyond the classically permissible region implies a violation of energy conservation: however, this is not the case. If we try to localize the particle (i.e., concentrate its wave function) in the small tails of the function ψ, the uncertainty of momentum increases to a point where the new total energy exceeds the value of the potential energy $U(x)$. Thus, from the point of view of energy, the particle is allowed to take an x value beyond the classically permitted region. In any case, it is the wave character of the quantum-mechanical wave function which allows the penetration into potential barriers and, finally, its tunneling [27].

Now, we can refute the above description as follows. it is possible to divide the energy precisely into $E = T + U$. This is because the total energy E is a constant according to the law of energy conservation and the potential energy

$U(x)$ is a definite function of x as seen from Eq. (1). Therefore, the momentum of a particle is determined definitely by position in the potential barrier. We understand the uncertainty relation based on the ensemble interpretation, specifically, in terms of the relation between the distribution of position and that of momentum [26, 28]. According to the conventional interpretation, beyond the classically permitted region, i.e., in the potential barrier the total energy of an electron increases, thereby overcoming the potential barrier. It signifies the creation of energy purely from quantum origin, which is nothing but the violation of the law of energy conservation. In this connection, we should not miss the fact that we treat the Schrödinger equation for stationary state where the total energy is constant. Obviously, the conventional description based on the uncertainty relation cannot give clear answer to the problem of the negative kinetic energy and purely imaginary momentum.

For the purpose of overcoming such a difficulty, some researchers try to explain the tunnel effect by using the time-dependent Schrödinger equation. In this case too, the tunneling through the barrier is not allowed because tunneling signifies a negative kinetic energy of an electron. As a possible case, an external force can lower the height of the barrier or can increase the kinetic energy of the electron, so the barrier may oscillate, which indicates just the overcoming of barrier rather than tunneling.

True, it is inconsistent to introduce the uncertainty relation to validate the meaningless solution of the Schrödinger equation allowing purely imaginary momentum. If purely imaginary momentum were to be allowed, we could not guarantee the Hermitian property of the momentum operator too attributed to the real number property of quantum observables.

Obviously, if such an interpretation of tunneling is permitted, our argument inevitably cannot but go astray, thus losing even the significance of the Schrödinger equation as the starting point of discussion.

Next, we should consider the physical significance of the rectangular potential barrier. The rectangular potential barrier purports a constant potential. It is common knowledge that a constant potential cannot affect physical processes, and thus the width of potential barrier should not have an effect on the motion of electrons merely from the point of view of dynamics.

However, as the conventional interpretation is concerned, the tunneling depends on the width of barrier. In other words, it means that the constant potential takes effect on the motion of electrons, so to speak, in the quantum sense. This is because due to the wave nature of electron, the width of barrier imposes the limitation relevant to periodicity of wave on the electron motion. In fact, according to the conventional interpretation the width of barrier plays only the role which defines the periodicity condition at the boundaries of barrier. Consequently, an electron in a barrier region is in a state of free motion except for the periodic condition relevant to the wave nature. But it is reasonable to consider that the potential in a region of barrier in general is dependent on coordinate.

Next, it is important to interpret the temperature dependence of tunneling. We assume that the tunnel effect is dependent on temperature. It is necessary to reveal whether the tunnel effect is associated with a single electron or statistical ensemble of electrons. In this connection, we should remember that we always make an experiment with electron ensemble and cannot do with only one electron. Even for one electron confined in a certain region, such an electron cannot be considered a free particle, since the electron interacts ceaselessly with surrounding system, i.e., many-particle system, so we should consider that it exists in a set of statistical states undergoing quantum fluctuation. Such a state should be represented with the help of the density matrix, $\Psi = \sum_n W_n|\psi_n\rangle\langle\psi_n|$. Even in the case of cryogenic phenomena such as the tunnel effect in superconductors, it means not exactly the absolute zero-point state.

It is a key point to consider that every electron does not possess the total energy as determined only by a bias voltage. Electrons constitute a statistical ensemble by interacting with one another and therefore, it is natural to consider that in the sense of probability there exist the electrons that possess higher energy than that given by the applied voltage. In contrast to this, there exist the electrons of lower energy than that by applied voltage. It is both classical statistics and quantum statistics that explain this fact.

Temperature as a quantity determining the statistical distribution of velocity of electrons signifies nothing but the existence of electrons capable of surmounting the potential barrier. True, the number of electrons possessing higher total energy than the height of barrier is determined by temperature. Therefore, for problems relevant to ensemble of electrons, temperature should be necessarily considered, but the conventional interpretation of tunneling is limited purely to the realm of quantum mechanics.

3. Interpretation of tunneling based on quantum–statistical approach

3.1. Interpretation of current in barrier based on statistical consideration

Let us consider the tunnel effect from the physical aspect. Electrons concerning tunneling can be thought of as an interacting system or open system, and therefore its velocity distribution is dependent on temperature. Consequently there exist electrons which have the energies able to surmount the barrier with a certain probability. Before applying a bias voltage, currents toward the both sides of barrier are in a dynamic equilibrium state. As seen in Fig. 1, suppose

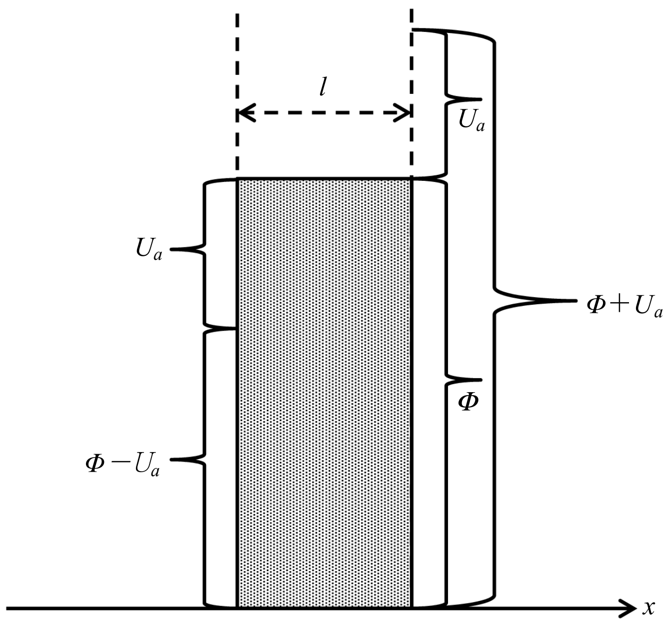

Figure 1: Variation in height of barrier in both sides due to applied voltage

that one applies the voltage conveying to an electron an energy lower than the height of barrier, Φ directed from the left to the right.

Then for the electrons in the left side of barrier the height of barrier becomes $\Phi - U_a$ and for the electrons in the right side of barrier, $\Phi + U_a$. Fig. 1 shows simply the two heights of barrier, respectively. If a bias voltage is applied, then the currents through the barrier from the both sides are shifted to a new equilibrium, so the total current is not balanced. To calculate this current density quantitatively, we use the Richardson-Dushmann equation for current density for the thermionic emission:

$$j = AT^2 \exp\left(-\frac{\Phi}{kT}\right). \tag{3}$$

In case the bias voltage is applied from the right to the left, the current density from the left to the right is

$$j|_{x=0} = AT^2 \exp\left(-\frac{\Phi - U_a}{kT}\right), \tag{4}$$

while the current density from the right to the left,

$$j|_{x=l} = AT^2 \exp\left(-\frac{\Phi + U_a}{kT}\right), \tag{5}$$

where U_a is the variation in the height of barrier due to the applied voltage. Fig. 2 shows the variation in the heights of barrier in both sides due to applied voltage.

Then the total current density from the left to the right can be expressed as

$$\begin{aligned} j = j|_{x=0} - j|_{x=l} &= AT^2 \left[\exp\left(-\frac{\Phi - U_a}{kT}\right) - \exp\left(-\frac{\Phi + U_a}{kT}\right) \right] \\ &= AT^2 \exp\left(-\frac{\Phi}{kT}\right) \left[\exp\left(\frac{U_a}{kT}\right) - \exp\left(-\frac{U_a}{kT}\right) \right] \\ &= 2AT^2 \exp\left(-\frac{\Phi}{kT}\right) \sinh\left(\frac{U_a}{kT}\right). \end{aligned} \tag{6}$$

In addition, another physical effect should be taken into consideration. An applied voltage changes the density of electrons capable of surmounting the barrier in both sides and as a result the gradient of electron density appears. It causes the diffusion current and therefore this effect can be considered to make the electric field due to the potential difference lower the height of barrier. Fig. 2 shows the variation in the heights of barrier in both sides due to the gradient of electron density.

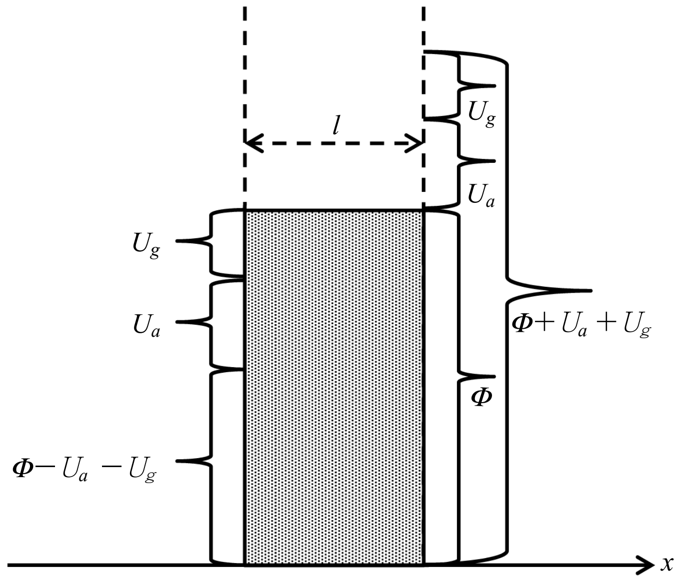

Figure 2: Variation of the heights of potential barrier in both sides of due to applied voltage U_a and density gradient. U_g is the variation in potential due to the density gradient.

In this case, U_a in Eq. (6) can be replaced with $U_a + U_g$ such that

$$j = 2AT^2 \exp\left(-\frac{\Phi}{kT}\right) \sinh\left(\frac{U_a + U_g}{kT}\right). \tag{7}$$

To obtain U_g, we should solve the Poisson equation in consideration of the electron density in the barrier region. The Poisson equation in the barrier region is expressed as

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = -\frac{\rho}{\varepsilon_0}. \tag{8}$$

Supposing that the charge density in the barrier region has such a linearity as

$$\rho(x) = ax + b, \tag{9}$$

Eq. (8) becomes

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = -\frac{ax+b}{\varepsilon_0}. \tag{10}$$

From the expression of the charge density (9), we have

$$\rho_0 = b. \tag{11}$$

At the end point of the barrier, the charge density is determined from

$$\rho_l = al + \rho_0. \tag{12}$$

Thus, we obtain

$$a = \frac{\rho_l - \rho_0}{l}. \tag{13}$$

By the Fermi-Dirac distribution, the electron density can be expressed as

$$n = \int_U^\infty f(E)N(E)\mathrm{d}E = \frac{4\pi(2m)^{3/2}}{h^3}\int_U^\infty \frac{(E-U)^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E, \tag{14}$$

where E_f is the Fermi energy and U is $U_a + U_g$. Then, integrating the Eq. (10) we obtain

$$\frac{\mathrm{d}\varphi}{\mathrm{d}x} = -\frac{a}{2\varepsilon_0}x^2 - \frac{b}{\varepsilon_0}x + c. \tag{15}$$

Considering the boundary condition, we can determine constant c. Since at the end of the barrier the intensity of electric field due to charge gradient is zero, we get

$$\left.\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right|_{x=l} = -\frac{a}{2\varepsilon_0}l^2 - \frac{b}{\varepsilon_0}l + c = 0. \tag{16}$$

Thus, the integral constant c is

$$c = \frac{a}{2\varepsilon_0}l^2 + \frac{b}{\varepsilon_0}l. \tag{17}$$

Integrating Eq. (15), we obtain

$$\varphi(x) = -\frac{a}{6\varepsilon_0}x^3 - \frac{b}{2\varepsilon_0}x^2 + cx + d. \tag{18}$$

The potential at the end of barrier is

$$\varphi(l) = -\frac{a}{6\varepsilon_0}l^3 - \frac{b}{2\varepsilon_0}l^2 + cl + d. \tag{19}$$

Therefore, the potential difference between the beginning point and the end point of the barrier is represented as

$$\varphi(0) - \varphi(l) = \frac{a}{6\varepsilon_0}l^3 + \frac{b}{2\varepsilon_0}l^2 - cl. \tag{20}$$

Substituting the determined constants into Eq. (18), the potential difference is given as

$$\begin{aligned} \triangle\varphi &= \frac{\rho_l - \rho_0}{6\varepsilon_0 l}l^3 + \frac{\rho_0}{2\varepsilon_0}l^2 - \left[\frac{1}{\varepsilon_0}\left(\frac{\rho_l-\rho_0}{l}\right)\frac{l^2}{2} + \frac{\rho_0}{\varepsilon_0}l\right]l \\ &= -\frac{1}{3\varepsilon_0}(\rho_l - \rho_0)l^2 - \frac{\rho_0}{2\varepsilon_0}l^2 = -\frac{1}{\varepsilon_0}\left(\frac{\rho_l}{3} + \frac{\rho_0}{6}\right)l^2. \end{aligned} \tag{21}$$

As a consequence, the additional potential energy due to the gradient of charge density is written as

$$U_g = -e\triangle\varphi = \frac{e}{\varepsilon_0}\left(\frac{\rho_l}{3} + \frac{\rho_0}{6}\right)l^2. \tag{22}$$

Thus, Eq. (7) has been completely determined. From Eq. (14), the charge densities at the beginning and end points of barrier can be expressed respectively as

$$\rho_0 = -en_0, \quad \rho_l = -en_l, \tag{23}$$

where the electron densities are represented respectively as

$$n_0 = \int_{\Phi-U_a}^{\infty} f(e)N(E)\mathrm{d}E = \frac{4\pi(2m)^{3/2}}{h^3}\int_{\Phi-U_a}^{\infty}\frac{(E-\Phi+U_a)^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E, \tag{24}$$

$$n_l = \int_{\Phi+U_a}^{\infty} f(E)N(E)\mathrm{d}E = \frac{4\pi(2m)^{3/2}}{h^3}\int_{\Phi+U_a}^{\infty}\frac{(E-\Phi-U_a)^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E. \tag{25}$$

As an example using the approximate expression of carrier density in completely degenerated semiconductor, the electron density can be expressed respectively as

$$n_0 = \frac{8\pi}{3h^3}(2m)^{3/2}(E_f-\Phi+U_a)^{3/2}, \tag{26}$$

$$n_l = \frac{8\pi}{3h^3}(2m)^{3/2}(E_f-\Phi-U_a)^{3/2}. \tag{27}$$

With Eqs. (23), (26) and (27), ρ_0 and ρ_l are expressed respectively as

$$\rho_0 = -\frac{8\pi e}{3h^3}(2m)^{3/2}(E_f-\Phi+U_a)^{3/2}, \tag{28}$$

$$\rho_l = -\frac{8\pi e}{3h^3}(2m)^{3/2}(E_f-\Phi-U_a)^{3/2}. \tag{29}$$

Thus, we have obtained the relation of the height variation of barrier to the difference between the charge densities in both sides of barrier caused by an applied voltage and have explained the dependence of barrier width on current through barrier. Inserting Eqs. (28) and (29) into Eq. (22), we can see that the thinner the barrier is, the higher the current is. In this way, we can explain the dependence of the barrier current on the barrier width.

3.2. filtering effect of statistical current by condition of quantum resonance in barrier

Using the formalism of quantum mechanics in phase space, quantum characteristic of electron motion in the barrier region can be satisfactorily interpreted.

Let us consider the case using the fundamental equation of quantum mechanics in phase space. According to the statistical formalism of quantum mechanics in phase space [26], the equation for probability density is

$$\frac{\partial\rho}{\partial t} = -\frac{1}{2}\sum_i\left[\frac{\partial H}{\partial p_i}\frac{\partial\rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial\rho}{\partial p_i}\right]. \tag{30}$$

We assume the barrier current to be stationary. In this case, the probability density is not dependent on time and thus, the above equation is reduced to

$$\sum_i\left[\frac{\partial H}{\partial p_i}\frac{\partial\rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial\rho}{\partial p_i}\right] = 0. \tag{31}$$

A solution of this equation is obtained in case the probability density becomes a constant. Generally, if the probability density is a function of the Hamiltonian function, then the equation is satisfied. Actually, the case is verified by

$$\begin{aligned}\sum_i\left[\frac{\partial H}{\partial p_i}\frac{\partial\rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial\rho}{\partial p_i}\right] &= \sum_i\left[\frac{\partial H}{\partial p_i}\frac{\partial\rho}{\partial H}\frac{\partial H}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial\rho}{\partial H}\frac{\partial H}{\partial p_i}\right]\\ &= \frac{\partial\rho}{\partial H}\sum_i\left[\frac{\partial H}{\partial p_i}\frac{\partial H}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial H}{\partial p_i}\right] = 0.\end{aligned} \tag{32}$$

Therefore, the probability density in a stationary state is expressed as

$$\rho = \psi^* \psi = f(H), \tag{33}$$

where ψ is the wave function defined in phase space.

Hence, the probabilistic amplitude is represented as

$$\psi_0 = \sqrt{\psi^* \psi} = \sqrt{f(H)}. \tag{34}$$

As a result, a solution to the wave equation for stationary states in general takes the following form:

$$\psi(x, p) = \psi_0[H(x, p)] \exp\left[\frac{\mathrm{i}}{\hbar} \int_0^x p(x')\mathrm{d}x'\right]. \tag{35}$$

It is necessary to note that the phase part of this equation is identified with that of the wave function in quasi-classical approximation. From Eq. (35), it is obvious that the barrier region should satisfy the following periodicity condition:

$$\int_0^l p(x)\mathrm{d}x = \int_0^l \sqrt{2m[E - U(x)]}\mathrm{d}x = nh, \tag{36}$$

where E is the total energy of an electron determined by applied voltage and $U(x)$, the barrier potential. With this periodicity condition, quantum characteristic of tunneling can be clarified. From Eq. (36), it follows that the kinetic energy of an electron crossing the barrier should be higher than the height of barrier. For a qualitative consideration, we shall define the mean value of momentum of an electron in a barrier region as

$$\bar{p} = \frac{\int_0^l p(x)\mathrm{d}x}{l} = \frac{\int_0^l \sqrt{2m[E - U(x)]}\mathrm{d}x}{l}.$$

According to Eq. (36), the following condition should be satisfied.

$$\bar{p}l = nh.$$

Accordingly, it can be seen that the momentum of an electron crossing over the barrier region should be quantized as

$$\bar{p}_n = \frac{h}{l}n. \tag{37}$$

The minimum momentum of an electron able to cross over the barrier is represented as

$$\bar{p} = \frac{h}{l}. \tag{38}$$

Hence, we can see that the thinner a barrier is, the higher the velocity of an electron crossing it should be. A barrier can be regarded as a "sack neck" to connect two free-motion regions or a "quantum filter" permitting only the quantized velocities defined for crossing. With a good reason, it can be considered that the electron density in a barrier region is almost in a saturation state and thus, is constant. This is because the interaction between electrons and the applied voltage function as a source complementing the electrons satisfying the quantum condition for electron crossing.

From the relation of current density:

$$j = n\upsilon, \tag{39}$$

we can see that the quantity contributing to the current density is principally the velocity of electron, υ. According to the quantum condition for barrier, i.e., Eq. (36), the thinner the width of barrier is, the greater the momentum which the barrier filters is, and as a consequence the current is increased.

Meanwhile, it is possible to explain the volt–ampere characteristic of the tunnel effect. Originally, according as the momentum of electrons due to the bias voltage approaches $\bar{p}$, the current through the barrier increases in compliance with the quantum condition, i.e., Eq. (36), but at the next stage according as the increasing momentum of electrons gets away from $\bar{p}$ the barrier current decreases gradually despite of the increase in bias voltage as a result of the breach of the quantum condition. This gives a good explanation to the question on why a peak occurs on the volt-ampere curve for tunnel phenomenon.

It is possible to show the fact in a semi-quantitative way.

The current density obtained through the consideration of the statistical relation can be represented with respect to momentum as

$$j_{st} \propto \sinh(aU) = \sinh(bp^2). \tag{40}$$

This reflects the fact that the increase in kinetic energy of an electron is in proportion to applied voltage. Setting $U \propto p^2$, the volt-ampere characteristic is depicted. The axes of coordinate are marked in a relative measure.

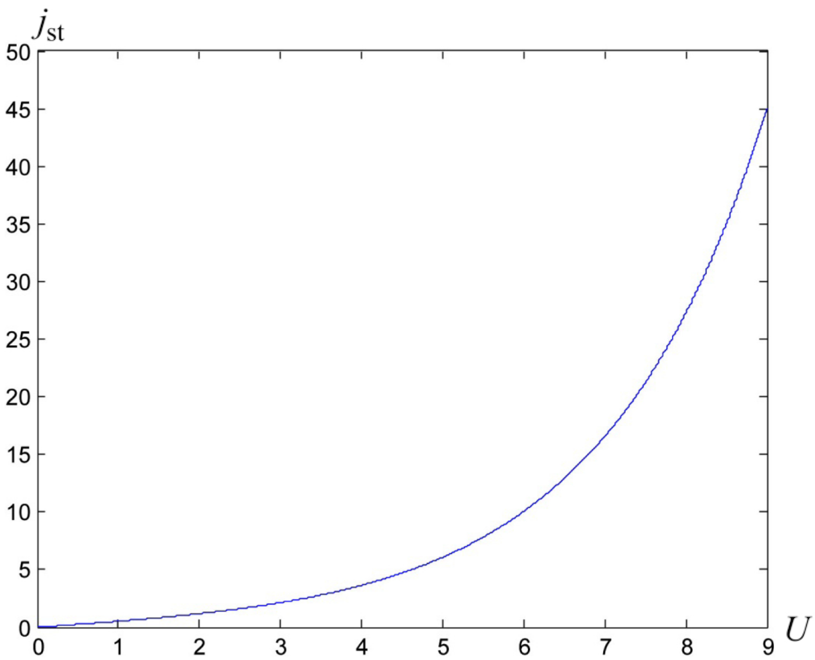

Figure 3: Current density of statistical origin in the barrier

On the other hand, the current of statistical origin undergoes a kind of filtering due to the condition of quantum resonance in the barrier. The condition is represented as mathematical relation

$$\eta = \sum_i C_i \exp(-\alpha_i (p - p_i)^2), \tag{41}$$

where p_i denotes the extreme values of momentum corresponding to the condition of quantum resonance.

Considering that a barrier is extremely thin, one can give the limitation condition that 2~3 peaks of voltage corresponding to extreme currents are effective. The volt-ampere relation has the quantum nature similar to that of the Frank-Herz experiment. Then the barrier current filtered according to the condition of quantum resonance is represented as

$$j = \eta j_{st}. \tag{42}$$

This resultant current should be the tunnel current, which is depicted in Fig. 5.

Fig. 5 is in good agreement with the volt-ampere characteristic of the tunnel diode. This demonstrates that our quantum-statistical analysis is reasonable. Thus, it is revealed that we have referred to the current phenomena due to such a quantum cause as the tunnel effect. Since only the electrons fulfilling Eq. (36) can surmount the barrier, Eq. (7) is an approximate expression. In fact, the exact expression should be given not by the integral in momentum

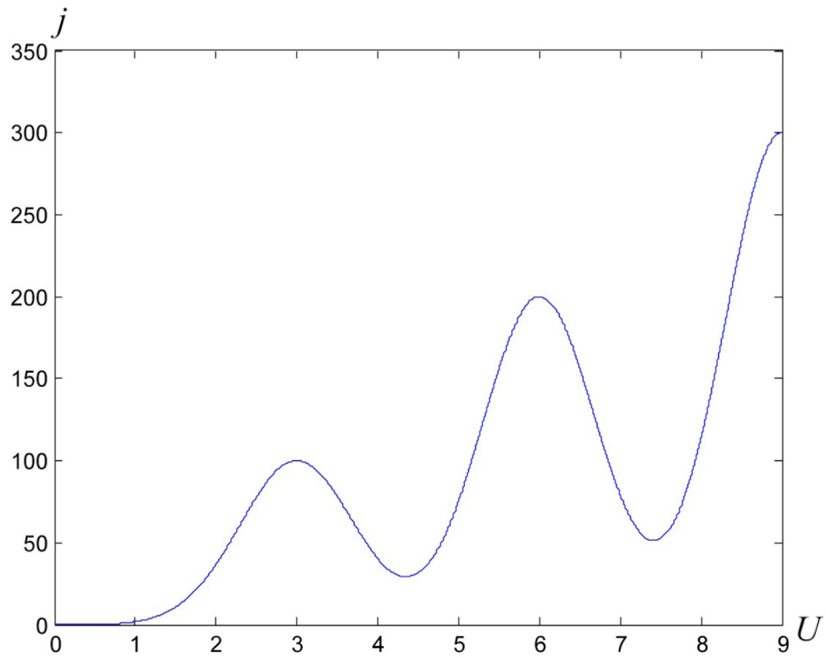

Figure 4: Modulation due to condition of quantum resonance in barrier

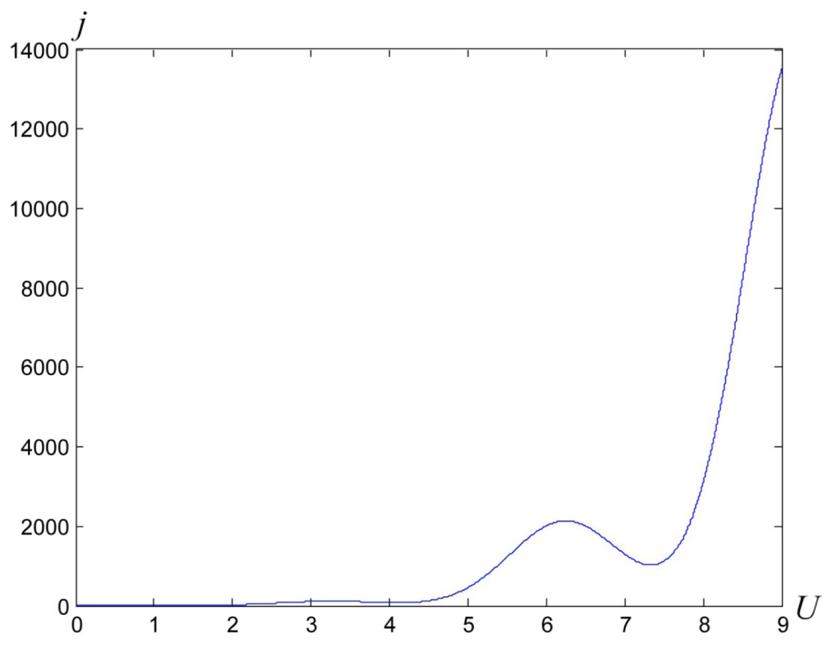

Figure 5: Resultant current density according to applied voltage

space but by the sum with respect to quantized momentum satisfying quantum condition (36). However, Eq. (7) can be considered to explain general characteristics of the barrier current.

The following equation for the current density:

$$j = AT^2 \exp\left(-\frac{\Phi}{kT}\right)\left[\exp\left(\frac{U_a}{kT}\right) - \exp\left(-\frac{U_a}{kT}\right)\right]$$
$$= 2AT^2 \exp\left(-\frac{\Phi}{kT}\right)\sinh(U_a/kT) \quad (43)$$

explains why the tunnel current is significantly increased according as the energy by applied voltage approaches the height of barrier.

According as U_a is increased, the tunnel current behaves in the manner of function $\sinh(U_a/kT)$. Therefore, the current is increased. Unlike the conventional interpretation, this approach reasonably elucidates the characteristic of current change in case the energy by applied voltage is higher than the height of barrier.

This approach gives a reasonable explanation for the resonant tunnel effect as well. From the periodicity condition (36) it is obvious that in the region where barriers and wells alternate periodically, the periodicity condition in every region should be satisfied, i.e.,

$$\int_{l_i^{(k)}}^{l_f^{(k)}} p(x)\mathrm{d}x = \int_{l_i^{(k)}}^{l_f^{(k)}} \sqrt{2m[E - U(x)]}\mathrm{d}x = n_k h, \quad (44)$$

where $l_i^{(k)}$ is the first boundary of the kth barrier or well and $l_f^{(k)}$, the final boundary. Only the electrons satisfying these periodicity conditions all together can pass through the whole region of barriers and wells. Therefore, barriers and wells are regarded as playing the role of a kind of resonance which filters the crossing of electrons in conformity with quantum condition for electron. For this reason, it is possible that in a multiple barrier-well region, the quantum-selective effect, i.e., the resonance effect occurs. Ultimately, all the properties of tunneling have been explained consistently.

4. Results and discussion

Our work has explained the characteristics of tunneling by applying the quantum-statistical approach. Tunneling attracts peculiar attention on the ground that electrons have a probability of crossing the barrier even if the height of barrier is higher than the energy due to an applied voltage. The smaller the difference between the height of barrier and the total energy of electron is and the thinner the barrier width is, the higher the probability of tunneling. For the purpose of explaining this experimental result, the conventional interpretation of tunneling states that although the energy of an electron is lower than the height of barrier, the electron can pass through the barrier with a certain probability thanks to the wave nature of electron. As it is, this implies that we take the physically meaningless solution of the Schrödinger equation admitting of even the negative kinetic energy. For this reason, in this interpretation, the relation of energy conservation is violated.

In this work, the two important characteristics of tunneling are satisfactorily explained on condition that the law of energy conservation is satisfied. Considering that tunneling is related to the statistical ensemble of electrons, we treated the tunneling current in the way similar to the approach to the thermionic emission using the statistical methodology.

According to this interpretation, the barrier current is attributed to three factors. The statistical ensemble of electrons is in an equilibrium state before applying the voltage. The electrons passing through the barrier exist with probability because the electron ensemble is in a statistical distribution state. In an equilibrium state, the currents from both sides of the barrier are identical. If a voltage is applied, then the heights of potential in both sides become different. Consequently, the momentum distributions of electrons able to surmount the barrier in the both sides change, and as a result the current in the direction of the applied voltage occurs. As shown in the barrier current relation, Eq. (6), the current increases with the behavior of the exponential function according as the difference between the height of barrier and the energy by applied voltage decreases. This is the first origin from which the barrier current phenomenon is caused.

Meanwhile, the applied voltage results in the change in the density of electrons which can surmount the barrier. On that account, the electric potential gradient appears. This electric potential gradient makes the barrier lower in the left side whereas higher in the right side. Thus, the influence of applied voltage on the statistical distribution of electron density increases the current through the barrier. This is the second origin of tunneling.

Meanwhile, the barrier imposes the quantum limitation on the tunneling current through the barrier. The periodicity condition which is determined by the relation between the electron momentum passing through the barrier and the barrier width plays the role which makes the momentum distribution of the crossing electron be raised up. In other words, the thinner the width of the barrier is, the higher the mean velocity of electrons. For this reason, a barrier can be considered a quantum filter to make the current increase. Based on the proposed qualitative interpretation, we can describe the effect of barrier width on the tunneling current. This is the third origin of the tunneling current.

Based on the above-mentioned arguments, we can give a reasonable interpretation of tunneling. First of all, we can give clear answer to the question on whether or not the conventional interpretation is correct. Obviously, tunneling is attributed to the quantum factor. But the wave nature of electrons can never result in a miracle such that electrons might surmount a barrier with lower energy than the height of the barrier. We can reach this conclusion by investigating the solution of the Schrödinger equation in the barrier region. In the conventional interpretation the solution to the Schrödinger equation in the barrier region is derived with the approval of negative kinetic energy.

However, the Schrödinger equation itself is derived satisfying the energy conservation law. Thus, the conventional interpretation of tunneling cannot be compatible with the law of energy conservation.

The conventional interpretation of tunneling is associated with one electron. However, we in reality can carry out experiments only with electron ensemble. It is not possible to consider that an electron definitely can have energy determined purely by applied voltage. Since interacting electrons constitute a statistical ensemble, an applied voltage changes the statistical distribution of the electron system and therefore we necessarily cannot but deal with the statistical ensemble in experiments on the electron tunneling. For this reason, tunneling too should depend on temperature, but the conventional interpretation is beyond the discussion with this problem.

However, our work copes with this problem properly and gives reasonable results. If temperature is very high, the characteristic tunneling current related to the difference $\Phi - U_a$ is insignificant, since as $\dfrac{\Phi - U_a}{kT} \to 0$, Eq. (43) becomes

$$\begin{aligned} j &= AT^2 \exp\left(-\frac{\Phi - U_a}{kT}\right)\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \\ &\approx AT^2\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \approx AT^2\frac{2U_a}{kT} = \frac{2ATU_a}{k}. \end{aligned} \tag{45}$$

Therefore, the tunneling current increases with temperature and the role of barrier is insignificant. This means that in this case, the characteristic current phenomenon resulting from the relation between energy by applied voltage and height of barrier vanishes completely as a consequence of being quenched by the intensified drift current due to barrier-surmountable electrons.

In case temperature is very low, the characteristic tunneling current related to the difference $\Phi - U_a$ vanishes, since as $\dfrac{\Phi - U_a}{kT} \to \infty$, the current becomes

$$j = AT^2 \exp\left(-\frac{\Phi - U_a}{kT}\right)\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \to 0. \tag{46}$$

Eq. (46) enables one to predict that the tunneling current would decrease with decrease in temperature, thereby approaching zero near absolute zero.

The conventional interpretation of tunneling cannot give the satisfactory explanation for higher energy by applied voltage than the height of barrier. Actually, with the transmission coefficient for tunneling through the barrier:

$$D = D_0 e^{-\frac{2i}{\hbar}\sqrt{2m(E-U_0)}\,l},$$

we cannot give reasonable physical interpretation of the fact that the exponential part is a pure imaginary number. But there is no limitation like this in our interpretation. Eqs. (6) and (7) are significant for any values of applied voltage. This equation shows that according as the applied voltage gets higher, the influence of barrier gradually vanishes and after all, in the case of $U \gg \Phi$ there is purely the current determined only by applied voltage, i.e.,

$$j = AT^2\left[\exp\left(-\frac{\Phi - U_a}{kT}\right) - \exp\left(-\frac{\Phi + U_a}{kT}\right)\right]$$
$$\approx AT^2\exp\left(\frac{U_a}{kT}\right)\left[1 - \exp\left(-\frac{2U_a}{kT}\right)\right] \approx AT^2\exp\left(\frac{U_a}{kT}\right). \quad (47)$$

Meanwhile, in our interpretation, the quantum characteristics of the barrier current are interpreted satisfactorily. The quantization condition given by Eq. (36) causes the effect increasing the barrier current. Essentially, this condition is the periodicity condition given by the barrier. This condition appears to be in line with studies for interpreting the tunneling by applying the quasi-classical approach in terms of Wigner or Husimi function. What should be emphasized here is that this periodicity condition is given not by the conventional quasi-classical phase space theory but by the fundamental equation of statistical formalism in phase space presented by the authors.

The main purpose of this work is to prove that the barrier current phenomenon called "tunneling" is actually not the tunneling inconsistent with the physical intuition but a quantum-statistical hybrid phenomenon. Our work has elucidated completely the inconsistency of the conventional approach and has suggested a consistent approach to tunneling. It has been clearly demonstrated is that this barrier-current phenomenon is attributed to the quantum-statistical characteristics of microscopic system.

5. Conclusion

To interpret the tunneling in a more reasonable way, we have adopted a quantum–statistical approach distinguished from the conventional one. Based on this interpretation, we have satisfactorily explained the tunneling current through the barrier by using the Richardson-Dushmann equation for the thermionic emission and by taking into consideration the quantum filtering role which the barrier plays in increasing the current through the barrier.

The central feature of our view is to consider that tunneling is due to not only quantum but also statistical characteristics of electrons. The conventional interpretation of tunneling cannot but accept an inconsistent concept which is not allowed in the physical sense, since it ignores the statistical aspect of the physical phenomenon under consideration. In this connection, it is important to ascertain the fact that the conventional interpretation of the tunnel effect violates the universal law of energy conservation. In our interpretation, there is no jump of logic by any non-physical conception and inconsistency, and the relation of energy conservation and the wave nature are in harmony with each other.

The main conclusion of our work is that the phenomenon of current through the barrier should be explained based on the statistical interpretation and the condition of quantum resonance due to barrier. Such a perspective has enabled us to elucidate the temperature dependence of the so-called tunneling current and the influence of barrier, thus making us capture the essence of the tunneling current. Moreover, it should be emphasized that our work successfully explains the tunneling current phenomenon in a general way in the whole range of applied voltage.

In conclusion, our work makes it possible to solve some intractable problem such as violation of the law of energy conservation arising from the conventional approach to the tunneling current phenomenon, and leads to the innovation in the picture of tunneling. We expect our work to substantially contribute to the elucidation of the physical nature of tunneling and also to the researches into the tunneling of complex systems including the resonant tunneling effect.

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