

**PROOF OF THE LEMMA**  
**ABOUT THE ABSENCE OF NONTRIVIAL CYCLES**  
**COLLATZ SEQUENCES**

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*Abstract: the article provides an outline of the proof of the absence of nontrivial cycles in the Collatz sequence.*

*The described lemma about the absence of nontrivial sequence cycles is an independent part of the proof of the Collatz conjecture*

**1. The concepts used.**

**1.1** *The numbers  $a, b, c, \dots$  are an interconnected series of odd numbers of a continuous Collatz sequence in the positive range of the natural series.*

**1.2** *The numbers  $A, B, C, \dots$  are a transformation of the numbers of the Collatz sequence, respectively, which are also an interconnected series of odd numbers (hereinafter referred to as "series X"). Transformation of the form  $A=3a$  and so on.*

**1.3** *The numbers  $A', B', C', \dots$  are a transformation of the numbers of the X series ( $3n-1$ ), making up an interconnected series of even numbers (hereinafter referred to as the "Y series"). The transformation of the form is performed by  $A-1=A'$  and so on.*

**1.4** *A series  $\{2/3\}$  is a set of numbers such that the ratio of even and odd numbers is  $\frac{2^h}{3^m}$ , (where,  $h, m$  are integers). The symbol  $\equiv$  indicates that the numbers belong to a series.*

**2. Proof.**

**2.1** *The presence of a nontrivial cycle in the Collatz sequence corresponds to 1.1. Accordingly, the series X must correspond to 1.2*

$$\frac{(3a+1)(3b+1)(3c+1)}{abc} = 2^h \quad (1.1)$$

$$\frac{(A+1)(B+1)(C+1)}{ABC} = \frac{2^h}{3^m} \equiv \{2/3\} \quad (1.2)$$

**2.2** *The series X cannot be transformed into the series Y while preserving the integer ratio (2), therefore, for example, no sections of these series can simultaneously belong to the series {2/3}, all parts of which are connected by a common integer ratio  $\frac{2^h}{3^m}$ , that is, cycles in the series X and Y are mutually excluded.*

$$\left\{ \begin{array}{l} 2 \\ 3 \end{array} \right\} \equiv \frac{(A'+1)(B'+1)(C'+1)}{A'B'C'} \neq \frac{ABC}{(A+1)(B+1)(C+1)} \quad (2)$$

**2.3** *It is known that the Collatz sequence in the negative range, that is, the sequence (3n-1), has nontrivial cycles. Accordingly (3), nontrivial cycles can have a series Y, which excludes nontrivial cycles in the series X.*

$$\frac{(A'+1)(B'+1)(C'+1)}{A'B'C'} = \frac{ABC}{(A-1)(B-1)(C-1)} \equiv \{2/3\} \quad (3)$$