Generalization of the Dirac Equation using Geometric Algebra

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Abstract

In this paper, we will calculate a generalization of the Dirac Equation using Geometric Algebra $Cl_{3,0}$. Apart from the partial derivatives with respect to position and time, also partial derivatives regarding orientation (or angular momentum) will appear.

The reason that these new partial derivatives have not been considered before is probably because their value is very small or directly zero or because they represent an oscillatory movement or value which mean value is zero. Meaning they can influence in local effects (helicoidal movement, rotations etc.) but not in the mean trajectory of the particles.

Two representations of the equations will be shown, being the following one, the one that has a one-to-one map to the standard matrix algebra of the Dirac Equation. In bold the new elements appearing:

$$\begin{aligned}
&\frac{\partial\psi_0}{\partial t}-\frac{\partial\psi_x}{\partial x}-\frac{\partial\psi_y}{\partial y}-\frac{\partial\psi_z}{\partial z}-m\psi_{xy}+\boldsymbol{\frac{\partial\psi_{yz}}{\partial r_{yz}}}+\boldsymbol{\frac{\partial\psi_{zx}}{\partial r_{zx}}}+\boldsymbol{\frac{\partial\psi_{xy}}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_0}}=0\\
&\frac{\partial\psi_x}{\partial t}-\frac{\partial\psi_0}{\partial x}+\frac{\partial\psi_{xy}}{\partial y}-\frac{\partial\psi_{zx}}{\partial z}+m\psi_y+\boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_{yz}}}+\boldsymbol{\frac{\partial\psi_z}{\partial r_{zx}}}-\boldsymbol{\frac{\partial\psi_y}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_{yz}}{\partial r_0}}=0\\
&\frac{\partial\psi_y}{\partial t}-\frac{\partial\psi_{xy}}{\partial x}-\frac{\partial\psi_0}{\partial y}+\frac{\partial\psi_{yz}}{\partial z}-m\psi_x-\boldsymbol{\frac{\partial\psi_z}{\partial r_{yz}}}+\boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_{zx}}}+\boldsymbol{\frac{\partial\psi_x}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_{zx}}{\partial r_0}}=0\\
&\frac{\partial\psi_z}{\partial t}+\frac{\partial\psi_{zx}}{\partial x}-\frac{\partial\psi_{yz}}{\partial y}-\frac{\partial\psi_0}{\partial z}+m\psi_{xyz}+\boldsymbol{\frac{\partial\psi_y}{\partial r_{yz}}}-\boldsymbol{\frac{\partial\psi_x}{\partial r_{zx}}}+\boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_{xy}}{\partial r_0}}=0\\
&-\frac{\partial\psi_{xy}}{\partial t}+\frac{\partial\psi_y}{\partial x}-\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_{xyz}}{\partial z}-m\psi_0-\boldsymbol{\frac{\partial\psi_{zx}}{\partial r_{yz}}}+\boldsymbol{\frac{\partial\psi_{yz}}{\partial r_{zx}}}+\boldsymbol{\frac{\partial\psi_0}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_z}{\partial r_0}}=0\\
&-\frac{\partial\psi_{yz}}{\partial t}+\frac{\partial\psi_{xyz}}{\partial x}+\frac{\partial\psi_z}{\partial y}-\frac{\partial\psi_y}{\partial z}+m\psi_{zx}+\boldsymbol{\frac{\partial\psi_0}{\partial r_{yz}}}-\boldsymbol{\frac{\partial\psi_{xy}}{\partial r_{zx}}}+\boldsymbol{\frac{\partial\psi_{zx}}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_x}{\partial r_0}}=0\\
&-\frac{\partial\psi_{zx}}{\partial t}-\frac{\partial\psi_z}{\partial x}+\frac{\partial\psi_{xyz}}{\partial y}+\frac{\partial\psi_x}{\partial z}-m\psi_{yz}+\boldsymbol{\frac{\partial\psi_{xy}}{\partial r_{yz}}}+\boldsymbol{\frac{\partial\psi_0}{\partial r_{zx}}}-\boldsymbol{\frac{\partial\psi_{yz}}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_y}{\partial r_0}}=0\\
&-\frac{\partial\psi_{xyz}}{\partial t}+\frac{\partial\psi_{yz}}{\partial x}+\frac{\partial\psi_{zx}}{\partial y}+\frac{\partial\psi_{yx}}{\partial z}+m\psi_z+\boldsymbol{\frac{\partial\psi_x}{\partial r_{yz}}}+\boldsymbol{\frac{\partial\psi_y}{\partial r_{zx}}}+\boldsymbol{\frac{\partial\psi_z}{\partial r_{xy}}}+\boldsymbol{\frac{\partial\psi_0}{\partial r_0}}=0
\end{aligned}$$

Being the wavefunction ψ defined as:

$$\psi=\psi_0+\hat{x}\psi_x+\hat{y}\psi_y+\hat{z}\psi_z+\hat{x}\hat{y}\psi_{xy}+\hat{y}\hat{z}\psi_{yz}+\hat{z}\hat{x}\psi_{zx}+\hat{x}\hat{y}\hat{z}\psi_{xyz}$$

Which has a one-to-one map towards the standard representation of the wavefunction in the standard matrix algebra of the Dirac Equation:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \psi_{1r} + i\psi_{1i} \\ \psi_{2r} + i\psi_{2i} \\ \psi_{3r} + i\psi_{3i} \\ \psi_{4r} + i\psi_{4i} \end{pmatrix}$$

As follows:

$$\begin{gathered} \psi_{1r} = -\psi_y \\ \psi_{1i} = -\psi_x \\ \psi_{2r} = \psi_{xyz} \\ \psi_{2i} = \psi_z \\ \psi_{3r} = -\psi_{yz} \\ \psi_{3i} = \psi_{zx} \\ \psi_{4r} = \psi_{xy} \\ \psi_{4i} = \psi_0 \end{gathered}$$

Keywords

Geometric Algebra $Cl_{3,0}$, Generalization of the Dirac Equation

1. Introduction

In this paper, we will calculate a generalization of the Dirac Equation using Geometric Algebra $Cl_{3,0}$. First, we will define the position multivector of a particle or body in Geometric Algebra $Cl_{3,0}$. With that, a new definition (expanded) for the del (∇) operator can be performed. With this new definition of the del operator, we can generalize the Dirac Equation following similar steps as in [6].

2. The position multivector R

To be able to follow the mathematic framework in this paper, I recommend you read the chapters 2 to 8 of [6] or [7] before continuing. There, you will see how to work in Geometric Algebra $Cl_{3,0}$ considering the time as the 8th degree of freedom (the trivector) of the expanded Geometric Algebra created by the three space vectors.

If you do not know what I am talking about, I strongly recommend you check the masterpiece [1] and the best collection of Geometric Algebra knowledge [3].

If we consider an orthonormal frame in Geometric Algebra $Cl_{3,0}$ composed by the three space vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$.

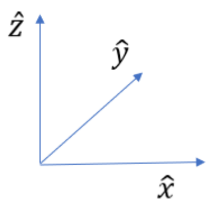

Fig.1 Orthonormal basis vectors in $Cl_{3,0}$

First, as we made in the Annex A1.1 of [2], we will define the position multivector. If we consider a particle or a rigid body as in the Figure 2:

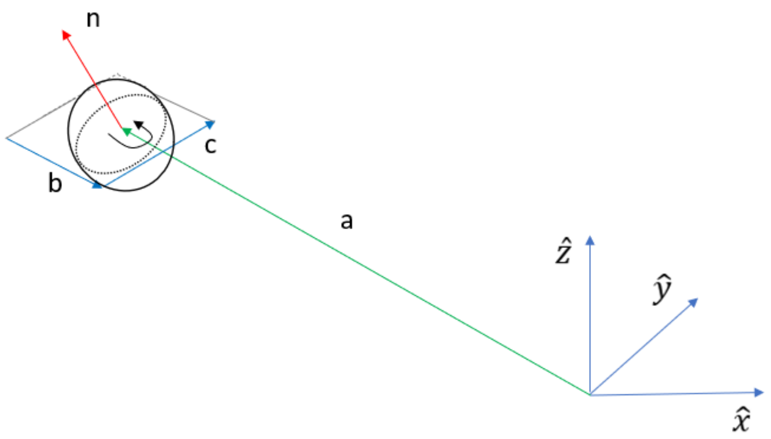

Fig.2 Representation position multivector

This multivector has 8 coordinates (8 degrees of freedom corresponding to the scalar, the three space vectors, the three bivectors and the trivector):

$$R = r_0 + r_x\hat{x} + r_y\hat{y} + r_z\hat{z} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + r_{xyz}\hat{z}\hat{y}\hat{x} \quad (1)$$

If you know something about Geometric Algebra, you will be asking why we have reversed the order of the trivector. You will see this in a minute.

So how this multivector correlated to the Figure 2? First, we see that the vector a corresponds to the linear position of the particle or to the rigid body center of mass:

$$a = r_x\hat{x} + r_y\hat{y} + r_z\hat{z} \quad (2)$$

So, it corresponds to the above elements of the R position multivector. To simplify, we will change the nomenclature of these components to the most classical in literature:

$$r_x = x$$
$$r_y = y$$
$$r_z = z \quad (3)$$

Leading to:

$$a = x\hat{x} + y\hat{y} + z\hat{z} \quad (4)$$

Where the x, y and z without the hat are the spatial coordinates and the $\hat{x}$, $\hat{y}$ and $\hat{z}$ with hat are the space basis vectors.
So, at the moment we are having:

$$R = r_0 + a + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + r_{xyz}\hat{z}\hat{y}\hat{x} \quad (5)$$

$$R = r_0 + x\hat{x} + y\hat{y} + z\hat{z} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + r_{xyz}\hat{z}\hat{y}\hat{x} \quad (6)$$

Now let's go to the bivectors. In Fig.2 you can see that there is a bivector b^c that represents the orientation of a preferred plane in the particle/rigid body. This plane is in general the plane where the rotation will take effect (the plane perpendicular to the rotation axis n) when it happens.

As we are still in a position multivector there is no rotation at this stage, so let's say that this bivector tells us the orientation of the particle/rigid body at a certain moment. If you select a preferred plane solidary to the particle/rigid body, it tells us the orientation of this plane at a certain time. So:

$$b \wedge c = r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} \quad (7)$$

Introducing in R:

$$R = r_0 + a + b \wedge c + r_{xyz}\hat{z}\hat{y}\hat{x} \quad (8)$$

$$R = r_0 + x\hat{x} + y\hat{y} + z\hat{z} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + r_{xyz}\hat{z}\hat{y}\hat{x} \quad (6)$$

Regarding the $\hat{z}\hat{y}\hat{x}$, as we commented in chapter 8 of [6] and [7] this corresponds to the $\hat{t}$ vector (being instead the $\hat{x}\hat{y}\hat{z}$ its inverse, the $\hat{t}^{-1}$ vector). So, the element r_{xyz} corresponds to the coordinate of time t. This is:

$$\hat{z}\hat{y}\hat{x} = \hat{t} \quad (9)$$

$$r_{xyz} = t \quad (10)$$

$$R = r_0 + a + b \wedge c + r_{xyz}\hat{z}\hat{y}\hat{x} \quad (8)$$

$$R = r_0 + a + b \wedge c + t\hat{t} \quad (11)$$

$$R = r_0 + x\hat{x} + y\hat{y} + z\hat{z} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + t\hat{t} \quad (12)$$

$$R = r_0 + x\hat{x} + y\hat{y} + z\hat{z} + t\hat{t} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} (13)$$

So, we see that we have the four traditional dimensions (three of space and one of time) included in the position multivector (apart form other elements). This happens even if we have considered only the three space dimensions to start. The time has appeared naturally as the trivector $\hat{z}\hat{y}\hat{x}$.

The only pending element is r_0. The meaning of this element is more obscure. As I have commented in [5[6] the scalars in the multivectors are a kind of scalation factor that affects all the magnitudes that are multiplied by it. So, it could be related to a kind of scalation in the metric appearing in non-Euclidean metrics (kind of local Ricci scalar or trace of the metric tensor).

Another simpler interpretation for r_0, is that the scalars appear when we multiply or divide vectors by themselves. So, it is a necessary degree of freedom to accommodate these results when they appear. For example, we will divide later the time trivector by itself leading to a scalar. But the origin of the value would still remain related to time.

Coming back to the equation of R, we will use the following form (we will use $\hat{z}\hat{y}\hat{x}$ instead of $\hat{t}$) to facilitate the operations:

$$R = r_0 + x\hat{x} + y\hat{y} + z\hat{z} + r_{xy}\hat{x}\hat{y} + r_{yz}\hat{y}\hat{z} + r_{zx}\hat{z}\hat{x} + t\hat{z}\hat{y}\hat{x} \quad (14)$$

3. The Del operator

In [13] we made the following definition for the Del (∇) operator. All the partial derivatives make reference to the coordinates defined in previous chapter.

I will keep in bold the ones are not normally standard in physics so we can follow them separately.

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} + \boldsymbol{\frac{\partial}{\partial r_0}\hat{x}\hat{y}\hat{z}} - \boldsymbol{\frac{\partial}{\partial r_{xy}}\hat{x}\hat{y}} - \boldsymbol{\frac{\partial}{\partial r_{yz}}\hat{y}\hat{z}} - \boldsymbol{\frac{\partial}{\partial r_{zx}}\hat{z}\hat{x}} + \frac{\partial}{\partial t} \quad (15)$$

In paper [6] we obtained one of the representations of the Dirac Equation in Geometric Algebra $Cl_{3,0}$ as follows:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)\psi=0 \qquad (16)$$

As you can see, the Del operator, seems quite different in (16) than in (15). As the equation is equal to zero, we could pre or post multiply with no effect in the results. ´

So, if we premultiply the definition of Del in (15) by the trivector, we obtain:

$$\hat{x}\hat{y}\hat{z}\nabla=\frac{\partial}{\partial x}\hat{y}\hat{z}+\frac{\partial}{\partial y}\hat{z}\hat{x}+\frac{\partial}{\partial z}\hat{x}\hat{y}-\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}\hat{\boldsymbol{z}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}\hat{\boldsymbol{x}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}\hat{\boldsymbol{y}}+\frac{\partial}{\partial t}\hat{x}\hat{y}\hat{z} \ (17)$$

And reordering terms, we have:

$$\hat{x}\hat{y}\hat{z}\nabla=\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}+\hat{y}\hat{z}\frac{\partial}{\partial x}+\hat{z}\hat{x}\frac{\partial}{\partial y}+\hat{x}\hat{y}\frac{\partial}{\partial z}-\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}\hat{\boldsymbol{z}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}\hat{\boldsymbol{x}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}\hat{\boldsymbol{y}} \ (18)$$

Which is almost the operator we needed (apart from the new elements). Only one sign in the trivector is different. If you have seen the papers [6][7][13] that signs are a bit of a nightmare and they are the result of not having a clear formulism, convention for this algebra. The reason for this problem in the sign has to be studied and solved in the future.

Our goal in the paper is to expand as much as possible the Dirac equation with all the parameters that could be taken into account in Geometric Algebra $Cl_{3,0}$. That normally are considered to be zero, but the idea is to have the expanded equation with all of them, so no information is lost if it is discovered that some are not zero, or are oscillatory with a zero mean value for example.

So, as we know that equation (16) works (in fact another representation of that equation, wee [6]), what we can do is just to expand the Del operator in a way that we know will work for the Dirac Equation and at the same time add all the elements we want. So, we will use (18) but with of change of sign in the bivectors, like this:

$$\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}\hat{\boldsymbol{z}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}\hat{\boldsymbol{x}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}\hat{\boldsymbol{y}} \ (19)$$

This way, we know that using the expanded expression of the Del operator like (19) we will be able to reproduce an equation as (16) already proved in previous paper [6].

4. Generalization of the Dirac Equation

Now, we will perform the calculations (16) done in [6]:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)\psi=0 \qquad (16)$$

But using (19) as the operator to the wavefunction ψ.

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m+\hat{\boldsymbol{x}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}+\hat{\boldsymbol{y}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}+\hat{\boldsymbol{z}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}\right)\psi=0 \quad (20)$$

We have included the m with negative sign in the operator as it was done with the previous operator. The wavefunction was defined in [6] as:

$$\psi = \psi_0 + \hat{x}\psi_x + \hat{y}\psi_y + \hat{z}\psi_z + \hat{x}\hat{y}\psi_{xy} + \hat{y}\hat{z}\psi_{yz} + \hat{z}\hat{x}\psi_{zx} + \hat{x}\hat{y}\hat{z}\psi_{xyz} \quad (21)$$

So, the operation is:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t} - \hat{y}\hat{z}\frac{\partial}{\partial x} - \hat{z}\hat{x}\frac{\partial}{\partial y} - \hat{x}\hat{y}\frac{\partial}{\partial z} - m + \boldsymbol{\hat{x}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}} + \boldsymbol{\hat{y}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}} + \boldsymbol{\hat{z}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}\right)$$
$$\left(\psi_0 + \hat{x}\psi_x + \hat{y}\psi_y + \hat{z}\psi_z + \hat{x}\hat{y}\psi_{xy} + \hat{y}\hat{z}\psi_{yz} + \hat{z}\hat{x}\psi_{zx} + \hat{x}\hat{y}\hat{z}\psi_{xyz}\right) = 0 \quad (22)$$

Operating, we get:

$$\begin{aligned}
&\hat{x}\hat{y}\hat{z}\frac{\partial\psi_0}{\partial t} + \hat{y}\hat{z}\frac{\partial\psi_x}{\partial t} + \hat{z}\hat{x}\frac{\partial\psi_y}{\partial t} + \hat{x}\hat{y}\frac{\partial\psi_z}{\partial t} - \hat{z}\frac{\partial\psi_{xy}}{\partial t} - \hat{x}\frac{\partial\psi_{yz}}{\partial t} - \hat{y}\frac{\partial\psi_{zx}}{\partial t} - \frac{\partial\psi_{xyz}}{\partial t} -\\
&-\hat{y}\hat{z}\frac{\partial\psi_0}{\partial x} - \hat{x}\hat{y}\hat{z}\frac{\partial\psi_x}{\partial x} + \hat{z}\frac{\partial\psi_y}{\partial x} - \hat{y}\frac{\partial\psi_z}{\partial x} - \hat{z}\hat{x}\frac{\partial\psi_{xy}}{\partial x} + \frac{\partial\psi_{yz}}{\partial x} + \hat{x}\hat{y}\frac{\partial\psi_{zx}}{\partial x} + \hat{x}\frac{\partial\psi_{xyz}}{\partial x} -\\
&-\hat{z}\hat{x}\frac{\partial\psi_0}{\partial y} - \hat{z}\frac{\partial\psi_x}{\partial y} - \hat{x}\hat{y}\hat{z}\frac{\partial\psi_y}{\partial y} + \hat{x}\frac{\partial\psi_z}{\partial y} + \hat{y}\hat{z}\frac{\partial\psi_{xy}}{\partial y} - \hat{x}\hat{y}\frac{\partial\psi_{yz}}{\partial y} + \frac{\partial\psi_{zx}}{\partial y} + \hat{y}\frac{\partial\psi_{xyz}}{\partial y} -\\
&-\hat{x}\hat{y}\frac{\partial\psi_0}{\partial z} + \hat{y}\frac{\partial\psi_x}{\partial z} - \hat{x}\frac{\partial\psi_y}{\partial z} - \hat{x}\hat{y}\hat{z}\frac{\partial\psi_z}{\partial z} + \frac{\partial\psi_{xy}}{\partial z} + \hat{z}\hat{x}\frac{\partial\psi_{yz}}{\partial z} - \hat{y}\hat{z}\frac{\partial\psi_{zx}}{\partial z} + \hat{z}\frac{\partial\psi_{xyz}}{\partial z} -\\
&-m\psi_0 - \hat{x}m\psi_x - \hat{y}m\psi_y - \hat{z}m\psi_z - \hat{x}\hat{y}m\psi_{xy} - \hat{y}\hat{z}m\psi_{yz} - \hat{z}\hat{x}m\psi_{zx} - \hat{x}\hat{y}\hat{z}m\psi_{xyz} +\\
&\boldsymbol{\hat{x}}\frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_{yz}}} + \boldsymbol{\hat{x}\hat{y}}\frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_{yz}}} - \boldsymbol{\hat{z}\hat{x}}\frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_{yz}}} + \boldsymbol{\hat{y}}\frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_{yz}}} + \boldsymbol{\hat{x}\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_{yz}}} - \boldsymbol{\hat{z}}\frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_{yz}}} + \boldsymbol{\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_{yz}}} +\\
&\boldsymbol{\hat{y}}\frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_{zx}}} - \boldsymbol{\hat{x}\hat{y}}\frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_{zx}}} + \boldsymbol{\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_{zx}}} - \boldsymbol{\hat{x}}\frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_{zx}}} + \boldsymbol{\hat{z}}\frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_{zx}}} + \boldsymbol{\hat{x}\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_{zx}}} + \boldsymbol{\hat{z}\hat{x}}\frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_{zx}}} +\\
&\boldsymbol{\hat{z}}\frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_{xy}}} + \boldsymbol{\hat{z}\hat{x}}\frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_{xy}}} - \boldsymbol{\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_{xy}}} + \boldsymbol{\hat{x}\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_{xy}}} - \boldsymbol{\hat{y}}\frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_{xy}}} + \boldsymbol{\hat{x}}\frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_{xy}}} + \boldsymbol{\hat{x}\hat{y}}\frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_{xy}}} +\\
&\frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{x}}\frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{y}}\frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{z}}\frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{x}\hat{y}}\frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{z}\hat{x}}\frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_0}} + \boldsymbol{\hat{x}\hat{y}\hat{z}}\frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_0}}\\
&= 0 \quad (23)
\end{aligned}$$

Separating in equations, we get:

$$\frac{\partial\psi_0}{\partial t} - \frac{\partial\psi_x}{\partial x} - \frac{\partial\psi_y}{\partial y} - \frac{\partial\psi_z}{\partial z} - m\psi_{xyz} + \frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_0}} = 0 \quad (24)$$

$$\frac{\partial\psi_x}{\partial t} - \frac{\partial\psi_0}{\partial x} + \frac{\partial\psi_{xy}}{\partial y} - \frac{\partial\psi_{zx}}{\partial z} - m\psi_{yz} + \frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_{zx}}} - \frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_0}} = 0 \quad (25)$$

$$\frac{\partial\psi_y}{\partial t} - \frac{\partial\psi_{xy}}{\partial x} - \frac{\partial\psi_0}{\partial y} + \frac{\partial\psi_{yz}}{\partial z} - m\psi_{yz} - \frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_0}} = 0 \quad (26)$$

$$\frac{\partial\psi_z}{\partial t} + \frac{\partial\psi_{zx}}{\partial x} - \frac{\partial\psi_{yz}}{\partial y} - \frac{\partial\psi_0}{\partial z} - m\psi_{xy} + \frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_{yz}}} - \frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_{xyz}}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_0}} = 0 \quad (27)$$

$$-\frac{\partial\psi_{xy}}{\partial t} + \frac{\partial\psi_y}{\partial x} - \frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_{xyz}}{\partial z} - m\psi_z - \frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_0}} = 0 \quad (28)$$

$$-\frac{\partial\psi_{yz}}{\partial t} + \frac{\partial\psi_{xyz}}{\partial x} + \frac{\partial\psi_z}{\partial y} - \frac{\partial\psi_y}{\partial z} - m\psi_x + \frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_{yz}}} - \frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_{zx}}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_0}} = 0 \quad (29)$$

$$-\frac{\partial\psi_{zx}}{\partial t} - \frac{\partial\psi_z}{\partial x} + \frac{\partial\psi_{xyz}}{\partial y} + \frac{\partial\psi_x}{\partial z} - m\psi_y + \frac{\boldsymbol{\partial\psi_{xy}}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_{zx}}} - \frac{\boldsymbol{\partial\psi_{yz}}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_0}} = 0 \quad (30)$$

$$-\frac{\partial\psi_{xyz}}{\partial t} + \frac{\partial\psi_{yz}}{\partial x} + \frac{\partial\psi_{zx}}{\partial y} + \frac{\partial\psi_{yx}}{\partial z} - m\psi_0 + \frac{\boldsymbol{\partial\psi_x}}{\boldsymbol{\partial r_{yz}}} + \frac{\boldsymbol{\partial\psi_y}}{\boldsymbol{\partial r_{zx}}} + \frac{\boldsymbol{\partial\psi_z}}{\boldsymbol{\partial r_{xy}}} + \frac{\boldsymbol{\partial\psi_0}}{\boldsymbol{\partial r_0}} = 0 \quad (31)$$

In these equations, we see that apart from the partial equations regarding linear position and time, we gave also partial derivatives with respect to angular momentum/orientation as defined in chapter 2 with the multivector position R.

These derivatives should be zero or oscillating with an average value of zero, so they are not important to calculate the movement, trajectory or wavefunction (and that is the reason they are not necessary for that) but could affect in oscillatory movements, zitterbewegung, erratic position of it. Somehow, if known probably a more accurate definition of the position of the particle could be done. Even, if the meaning of these parameters is not properly known or defined, we know we have them as free parameters that can be used to define more properly the status of the particle in case it is necessary or that they can affect or be a variable affecting measurements even if it is not known.

Moving on, in [6] we did not use equation [16] to get a one-to-one map with standard matrix calculations of Dirac Algebra.

Instead, we had to use the equation:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}\right)\psi-m\psi_{even}\hat{z}+m\psi_{odd}\hat{z}=0 \quad (32)$$

That is another representation of the same equation but projecting in the $\hat{z}$ axis as a preferred direction. Something as changing the internal values Dirac Matrices but keeping their properties (the value of their products). This type of projection is done typically in Geometric Algebra as you can check in [3] (Chapters 8.1 and 8.3 for example).

Ψ_{even} and ψ_{odd} are defined as following, so its sum is the complete wavefunction ψ:

$$\psi_{even}=\psi_0+\hat{x}\hat{y}\psi_{xy}+\hat{y}\hat{z}\psi_{yz}+\hat{z}\hat{x}\psi_{zx} \quad (33)$$
$$\psi_{odd}=\hat{x}\psi_x+\hat{y}\psi_y+\hat{z}\psi_z+\hat{x}\hat{y}\hat{z}\psi_{xyz} \quad (34)$$
$$\psi=\psi_{even}+\psi_{odd}=\psi_0+\hat{x}\psi_x+\hat{y}\psi_y+\hat{z}\psi_z+\hat{x}\hat{y}\psi_{xy}+\hat{y}\hat{z}\psi_{yz}+\hat{z}\hat{x}\psi_{zx}+\hat{x}\hat{y}\hat{z}\psi_{xyz} \quad (35)$$

Using (32) but expanding with (19) we have:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}\hat{\boldsymbol{z}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}\hat{\boldsymbol{x}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}\hat{\boldsymbol{y}}\right)\psi-m\psi_{even}\hat{z}+m\psi_{odd}\hat{z}=0 \quad (36)$$

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_0}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{xy}}}\hat{\boldsymbol{z}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{yz}}}\hat{\boldsymbol{x}}+\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r_{zx}}}\hat{\boldsymbol{y}}\right)\left(\psi_0+\hat{x}\psi_x+\hat{y}\psi_y+\hat{z}\psi_z+\hat{x}\hat{y}\psi_{xy}+\hat{y}\hat{z}\psi_{yz}+\hat{z}\hat{x}\psi_{zx}+\hat{x}\hat{y}\hat{z}\psi_{xyz}\right)$$
$$-m\left(\psi_0+\hat{x}\hat{y}\psi_{xy}+\hat{y}\hat{z}\psi_{yz}+\hat{z}\hat{x}\psi_{zx}\right)\hat{z}$$
$$+m\left(\hat{x}\psi_x+\hat{y}\psi_y+\hat{z}\psi_z+\hat{x}\hat{y}\hat{z}\psi_{xyz}\right)\hat{z}=0 \quad (37)$$

Operating:

$$\hat{x}\hat{y}\hat{z}\frac{\partial\psi_0}{\partial t}+\hat{y}\hat{z}\frac{\partial\psi_x}{\partial t}+\hat{z}\hat{x}\frac{\partial\psi_y}{\partial t}+\hat{x}\hat{y}\frac{\partial\psi_z}{\partial t}-\hat{z}\frac{\partial\psi_{xy}}{\partial t}-\hat{x}\frac{\partial\psi_{yz}}{\partial t}-\hat{y}\frac{\partial\psi_{zx}}{\partial t}-\frac{\partial\psi_{xyz}}{\partial t}-$$
$$-\hat{y}\hat{z}\frac{\partial\psi_0}{\partial x}-\hat{x}\hat{y}\hat{z}\frac{\partial\psi_x}{\partial x}+\hat{z}\frac{\partial\psi_y}{\partial x}-\hat{y}\frac{\partial\psi_z}{\partial x}-\hat{z}\hat{x}\frac{\partial\psi_{xy}}{\partial x}+\frac{\partial\psi_{yz}}{\partial x}+\hat{x}\hat{y}\frac{\partial\psi_{zx}}{\partial x}+\hat{x}\frac{\partial\psi_{xyz}}{\partial x}-$$

$$-\hat{z}\hat{x}\frac{\partial \psi_0}{\partial y} - \hat{z}\frac{\partial \psi_x}{\partial y} - \hat{x}\hat{y}\hat{z}\frac{\partial \psi_y}{\partial y} + \hat{x}\frac{\partial \psi_z}{\partial y} + \hat{y}\hat{z}\frac{\partial \psi_{xy}}{\partial y} - \hat{x}\hat{y}\frac{\partial \psi_{yz}}{\partial y} + \frac{\partial \psi_{zx}}{\partial y} + \hat{y}\frac{\partial \psi_{xyz}}{\partial y} -$$
$$-\hat{x}\hat{y}\frac{\partial \psi_0}{\partial z} + \hat{y}\frac{\partial \psi_x}{\partial z} - \hat{x}\frac{\partial \psi_y}{\partial z} - \hat{x}\hat{y}\hat{z}\frac{\partial \psi_z}{\partial z} + \frac{\partial \psi_{xy}}{\partial z} + \hat{z}\hat{x}\frac{\partial \psi_{yz}}{\partial z} - \hat{y}\hat{z}\frac{\partial \psi_{zx}}{\partial z} + \hat{z}\frac{\partial \psi_{xyz}}{\partial z} -$$
$$\boldsymbol{\hat{x}\frac{\partial \psi_0}{\partial r_{yz}} + \frac{\partial \psi_x}{\partial r_{yz}} + \hat{x}\hat{y}\frac{\partial \psi_y}{\partial r_{yz}} - \hat{z}\hat{x}\frac{\partial \psi_z}{\partial r_{yz}} + \hat{y}\frac{\partial \psi_{xy}}{\partial r_{yz}} + \hat{x}\hat{y}\hat{z}\frac{\partial \psi_{yz}}{\partial r_{yz}} - \hat{z}\frac{\partial \psi_{zx}}{\partial r_{yz}} + \hat{y}\hat{z}\frac{\partial \psi_{xyz}}{\partial r_{yz}}} +$$
$$\boldsymbol{\hat{y}\frac{\partial \psi_0}{\partial r_{zx}} - \hat{x}\hat{y}\frac{\partial \psi_x}{\partial r_{zx}} + \frac{\partial \psi_y}{\partial r_{zx}} + \hat{y}\hat{z}\frac{\partial \psi_z}{\partial r_{zx}} - \hat{x}\frac{\partial \psi_{xy}}{\partial r_{zx}} + \hat{z}\frac{\partial \psi_{yz}}{\partial r_{zx}} + \hat{x}\hat{y}\hat{z}\frac{\partial \psi_{zx}}{\partial r_{zx}} + \hat{z}\hat{x}\frac{\partial \psi_{xyz}}{\partial r_{zx}}} +$$
$$\boldsymbol{\hat{z}\frac{\partial \psi_0}{\partial r_{xy}} + \hat{z}\hat{x}\frac{\partial \psi_x}{\partial r_{xy}} - \hat{y}\hat{z}\frac{\partial \psi_y}{\partial r_{xy}} + \frac{\partial \psi_z}{\partial r_{xy}} + \hat{x}\hat{y}\hat{z}\frac{\partial \psi_{xy}}{\partial r_{xy}} - \hat{y}\frac{\partial \psi_{yz}}{\partial r_{xy}} + \hat{x}\frac{\partial \psi_{zx}}{\partial r_{xy}} + \hat{x}\hat{y}\frac{\partial \psi_{xyz}}{\partial r_{xy}}} +$$
$$\boldsymbol{\frac{\partial \psi_0}{\partial r_0} + \hat{x}\frac{\partial \psi_x}{\partial r_0} + \hat{y}\frac{\partial \psi_y}{\partial r_0} + \hat{z}\frac{\partial \psi_z}{\partial r_0} + \hat{x}\hat{y}\frac{\partial \psi_{xy}}{\partial r_0} + \hat{y}\hat{z}\frac{\partial \psi_{yz}}{\partial r_0} + \hat{z}\hat{x}\frac{\partial \psi_{zx}}{\partial r_0} + \hat{x}\hat{y}\hat{z}\frac{\partial \psi_{xyz}}{\partial r_0}}$$
$$-\hat{z}m\psi_0 - \hat{z}\hat{x}m\psi_x + \hat{y}\hat{z}m\psi_y + m\psi_z - \hat{x}\hat{y}\hat{z}m\psi_{xy} - \hat{y}m\psi_{yz} + \hat{x}m\psi_{zx} + \hat{x}\hat{y}m\psi_{xyz}$$
$$= 0 \quad (38)$$

And dividing in equations according to the element they are multiplying (scalar, vector, bivector or trivector):

$$\frac{\partial \psi_0}{\partial t} - \frac{\partial \psi_x}{\partial x} - \frac{\partial \psi_y}{\partial y} - \frac{\partial \psi_z}{\partial z} - m\psi_{xy} + \boldsymbol{\frac{\partial \psi_{yz}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial \psi_{zx}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial \psi_{xy}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_{xyz}}{\partial r_0}} = 0 \quad (39)$$
$$\frac{\partial \psi_x}{\partial t} - \frac{\partial \psi_0}{\partial x} + \frac{\partial \psi_{xy}}{\partial y} - \frac{\partial \psi_{zx}}{\partial z} + m\psi_y + \boldsymbol{\frac{\partial \psi_{xyz}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial \psi_z}{\partial r_{zx}}} - \boldsymbol{\frac{\partial \psi_y}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_{yz}}{\partial r_0}} = 0 \quad (40)$$
$$\frac{\partial \psi_y}{\partial t} - \frac{\partial \psi_{xy}}{\partial x} - \frac{\partial \psi_0}{\partial y} + \frac{\partial \psi_{yz}}{\partial z} - m\psi_x - \boldsymbol{\frac{\partial \psi_z}{\partial r_{yz}}} + \boldsymbol{\frac{\partial \psi_{xyz}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial \psi_x}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_{zx}}{\partial r_0}} = 0 \quad (41)$$
$$\frac{\partial \psi_z}{\partial t} + \frac{\partial \psi_{zx}}{\partial x} - \frac{\partial \psi_{yz}}{\partial y} - \frac{\partial \psi_0}{\partial z} + m\psi_{xyz} + \boldsymbol{\frac{\partial \psi_y}{\partial r_{yz}}} - \boldsymbol{\frac{\partial \psi_x}{\partial r_{zx}}} + \boldsymbol{\frac{\partial \psi_{xyz}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_{xy}}{\partial r_0}} = 0 \quad (42)$$
$$-\frac{\partial \psi_{xy}}{\partial t} + \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_{xyz}}{\partial z} - m\psi_0 - \boldsymbol{\frac{\partial \psi_{zx}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial \psi_{yz}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial \psi_0}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_z}{\partial r_0}} = 0 \quad (43)$$
$$-\frac{\partial \psi_{yz}}{\partial t} + \frac{\partial \psi_{xyz}}{\partial x} + \frac{\partial \psi_z}{\partial y} - \frac{\partial \psi_y}{\partial z} + m\psi_{zx} + \boldsymbol{\frac{\partial \psi_0}{\partial r_{yz}}} - \boldsymbol{\frac{\partial \psi_{xy}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial \psi_{zx}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_x}{\partial r_0}} = 0 \quad (44)$$
$$-\frac{\partial \psi_{zx}}{\partial t} - \frac{\partial \psi_z}{\partial x} + \frac{\partial \psi_{xyz}}{\partial y} + \frac{\partial \psi_x}{\partial z} - m\psi_{yz} + \boldsymbol{\frac{\partial \psi_{xy}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial \psi_0}{\partial r_{zx}}} - \boldsymbol{\frac{\partial \psi_{yz}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_y}{\partial r_0}} = 0 \quad (45)$$
$$-\frac{\partial \psi_{xyz}}{\partial t} + \frac{\partial \psi_{yz}}{\partial x} + \frac{\partial \psi_{zx}}{\partial y} + \frac{\partial \psi_{yx}}{\partial z} + m\psi_z + \boldsymbol{\frac{\partial \psi_x}{\partial r_{yz}}} + \boldsymbol{\frac{\partial \psi_y}{\partial r_{zx}}} + \boldsymbol{\frac{\partial \psi_z}{\partial r_{xy}}} + \boldsymbol{\frac{\partial \psi_0}{\partial r_0}} = 0 \quad (46)$$

When again, regarding the partial derivatives with respect to $\mathbf{r_{ij}}$ or $\mathbf{r_0}$ it applies the same as commented for equations (24) to (31) above.
The difference between equations (24) to (31) and (39) to (46) is that for the latter there is a one-to-one map to the standard Dirac Equation.

Considering the standard representation of the wavefunction in standard Algebra of Dirac Equation:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \psi_{1r} + i\psi_{1i} \\ \psi_{2r} + i\psi_{2i} \\ \psi_{3r} + i\psi_{3i} \\ \psi_{4r} + i\psi_{4i} \end{pmatrix} \qquad (47)$$

And for the Geometric Algebra $Cl_{3,0}$ defining ψ as commented:

$$\psi = \psi_0 + \hat{x}\psi_x + \hat{y}\psi_y + \hat{z}\psi_z + \hat{x}\hat{y}\psi_{xy} + \hat{y}\hat{z}\psi_{yz} + \hat{z}\hat{x}\psi_{zx} + \hat{x}\hat{y}\hat{z}\psi_{xyz}$$

In [6] it was shown that the following one-to-one map existed:

$$\psi_{1r} = -\psi_y \quad (48)$$

$$\psi_{1i} = -\psi_x \quad (49)$$
$$\psi_{2r} = \psi_{xyz} \quad (50)$$
$$\psi_{2i} = \psi_z \quad (51)$$
$$\psi_{3r} = -\psi_{yz} \quad (52)$$
$$\psi_{3i} = \psi_{zx} \quad (53)$$
$$\psi_{4r} = \psi_{xy} \quad (54)$$
$$\psi_{4i} = \psi_0 \quad (55)$$

You can check [6] for more information.
But the idea of this paper was to show the expanded version of the Dirac Equation. This, you can see both in equations (24) to (31) and (39) to (46) in the bold elements that did not exist originally in the Dirac Equation and which possible meaning has been explained along the chapter.

5. Conclusions

In this paper, we have calculated a generalization of the Dirac Equation using Geometric Algebra $Cl_{3,0}$. Apart from the partial derivatives with respect to position and time, also partial derivatives regarding orientation (or angular momentum) have appeared.

The reason that they have not been considered is probably because their value is very small or directly zero or because they represent an oscillatory movement or value which mean value is zero. Meaning they can influence in local effects (helicoidal movement, rotations etc.) but not in the mean trajectory of the particles.

Two representations of the equations have been shown, being the following one, the one that has a one-to-one map to Standard Matrix Dirac Algebra. In bold the new elements appearing:

$$\frac{\partial\psi_0}{\partial t} - \frac{\partial\psi_x}{\partial x} - \frac{\partial\psi_y}{\partial y} - \frac{\partial\psi_z}{\partial z} - m\psi_{xy} + \boldsymbol{\frac{\partial\psi_{yz}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial\psi_{zx}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial\psi_{xy}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_0}} = 0 \quad (39)$$
$$\frac{\partial\psi_x}{\partial t} - \frac{\partial\psi_0}{\partial x} + \frac{\partial\psi_{xy}}{\partial y} - \frac{\partial\psi_{zx}}{\partial z} + m\psi_y + \boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial\psi_z}{\partial r_{zx}}} - \boldsymbol{\frac{\partial\psi_y}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_{yz}}{\partial r_0}} = 0 \quad (40)$$
$$\frac{\partial\psi_y}{\partial t} - \frac{\partial\psi_{xy}}{\partial x} - \frac{\partial\psi_0}{\partial y} + \frac{\partial\psi_{yz}}{\partial z} - m\psi_x - \boldsymbol{\frac{\partial\psi_z}{\partial r_{yz}}} + \boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial\psi_x}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_{zx}}{\partial r_0}} = 0 \quad (41)$$
$$\frac{\partial\psi_z}{\partial t} + \frac{\partial\psi_{zx}}{\partial x} - \frac{\partial\psi_{yz}}{\partial y} - \frac{\partial\psi_0}{\partial z} + m\psi_{xyz} + \boldsymbol{\frac{\partial\psi_y}{\partial r_{yz}}} - \boldsymbol{\frac{\partial\psi_x}{\partial r_{zx}}} + \boldsymbol{\frac{\partial\psi_{xyz}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_{xy}}{\partial r_0}} = 0 \quad (42)$$
$$-\frac{\partial\psi_{xy}}{\partial t} + \frac{\partial\psi_y}{\partial x} - \frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_{xyz}}{\partial z} - m\psi_0 - \boldsymbol{\frac{\partial\psi_{zx}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial\psi_{yz}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial\psi_0}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_z}{\partial r_0}} = 0 \quad (43)$$
$$-\frac{\partial\psi_{yz}}{\partial t} + \frac{\partial\psi_{xyz}}{\partial x} + \frac{\partial\psi_z}{\partial y} - \frac{\partial\psi_y}{\partial z} + m\psi_{zx} + \boldsymbol{\frac{\partial\psi_0}{\partial r_{yz}}} - \boldsymbol{\frac{\partial\psi_{xy}}{\partial r_{zx}}} + \boldsymbol{\frac{\partial\psi_{zx}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_x}{\partial r_0}} = 0 \quad (44)$$
$$-\frac{\partial\psi_{zx}}{\partial t} - \frac{\partial\psi_z}{\partial x} + \frac{\partial\psi_{xyz}}{\partial y} + \frac{\partial\psi_x}{\partial z} - m\psi_{yz} + \boldsymbol{\frac{\partial\psi_{xy}}{\partial r_{yz}}} + \boldsymbol{\frac{\partial\psi_0}{\partial r_{zx}}} - \boldsymbol{\frac{\partial\psi_{yz}}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_y}{\partial r_0}} = 0 \quad (45)$$
$$-\frac{\partial\psi_{xyz}}{\partial t} + \frac{\partial\psi_{yz}}{\partial x} + \frac{\partial\psi_{zx}}{\partial y} + \frac{\partial\psi_{yx}}{\partial z} + m\psi_z + \boldsymbol{\frac{\partial\psi_x}{\partial r_{yz}}} + \boldsymbol{\frac{\partial\psi_y}{\partial r_{zx}}} + \boldsymbol{\frac{\partial\psi_z}{\partial r_{xy}}} + \boldsymbol{\frac{\partial\psi_0}{\partial r_0}} = 0 \quad (46)$$

Being the wavefunction ψ defined as:

$$\psi = \psi_0 + \hat{x}\psi_x + \hat{y}\psi_y + \hat{z}\psi_z + \hat{x}\hat{y}\psi_{xy} + \hat{y}\hat{z}\psi_{yz} + \hat{z}\hat{x}\psi_{zx} + \hat{x}\hat{y}\hat{z}\psi_{xyz} \quad (21)$$

Bilbao, 5th November 2022 (viXra-v1).
Bilbao, 6th November 2022 (viXra-v2).
Bilbao, 12th November 2022 (viXra-v3).

6. Acknowledgements

To my family and friends. To Paco Menéndez and Juan Delcán. To the current gods of Geometric Algebra: David Hestenes, Chris Doran, Anthony Lasenby, Garret Sobczyk, Joy Christian and many others.

AAAAÁBCCCDEEIIILLLLLMMMOOOPSTU

If you consider this helpful, do not hesitate to drop your BTC here:

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Bueno, se acabó por hoy, vamos a ver el lol.

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