The assumption of a continuous Lorentzian spacetime manifold in quantum gravity

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Spacetime is more and more often suspected of being at the origin of the problem of quantum gravity, and it is said that the concept of spacetime needs to be revised.

In this paper, we want to provide the concrete reason why the Lorentzian spacetime manifold is not compatible with quantum gravity, by showing that it is a man-made artefact: unlike the Euclidean metric, no Lorentzian pseudometric is able to span up a real-valued manifold. This is why - since its introduction with Minkowski's famous lecture "Space and time" and until today - Lorentzian manifolds require always the addition of a second metric in order to override the appearance of negative squares and of imaginary values.

This artificial "patchwork" of two opposite metrics is not only incompatible with quantum mechanics, it is even contradicting the very principles of general relativity.

1. Observation is indirect - the Euclidean metric of observational interfaces

What is the reason why the current theories of quantum gravity are failing? The answer is that they are all based on the same false assumption: the assumption of a Lorentzian spacetime manifold, as it was presented by Minkowski in his famous lecture in 1908.

Before Minkowski, there was the Newtonian absolute spacetime, a fourdimensional Euclidean manifold:

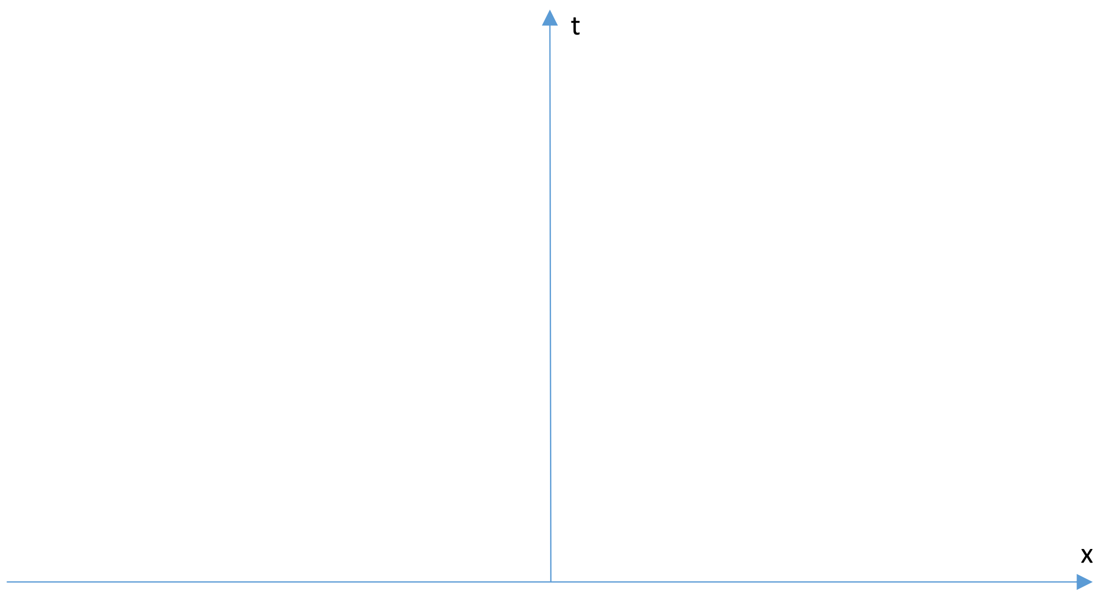

Fig. 1 - Newtonian spacetime

It had absolute, observer-independent space distances and time intervals, but no spacetime interval was defined, there were no mixed space-and-time intervals.

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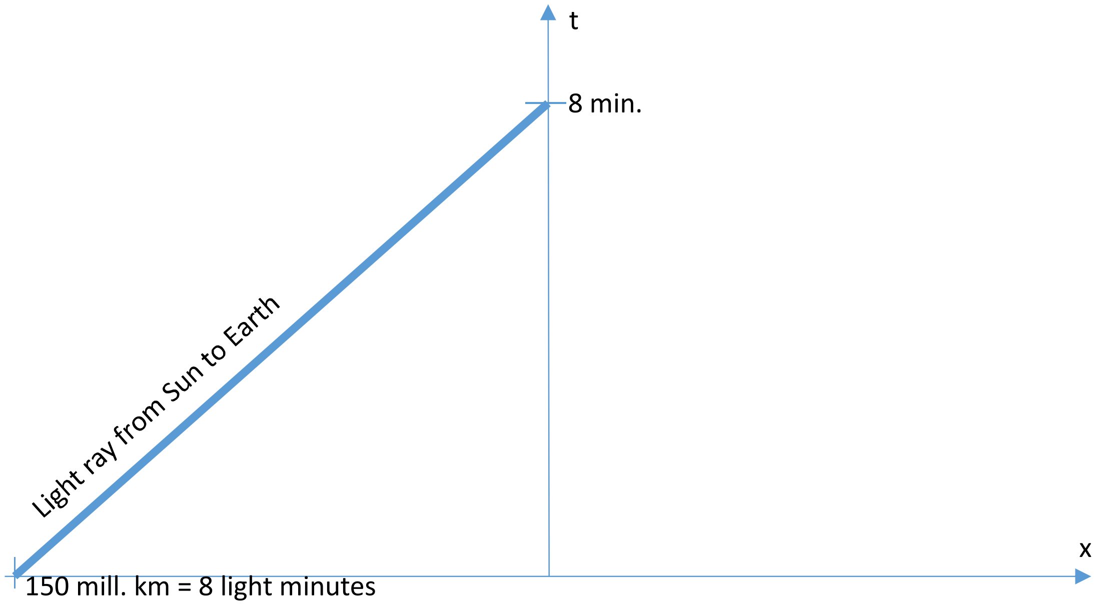

Fig. 2 - Newtonian spacetime with light ray

For instance, a light ray traveling from Sun to Earth takes 8 minutes for 150 million kilometers, but the length of the spacetime interval was not defined in Newtonian spacetime.

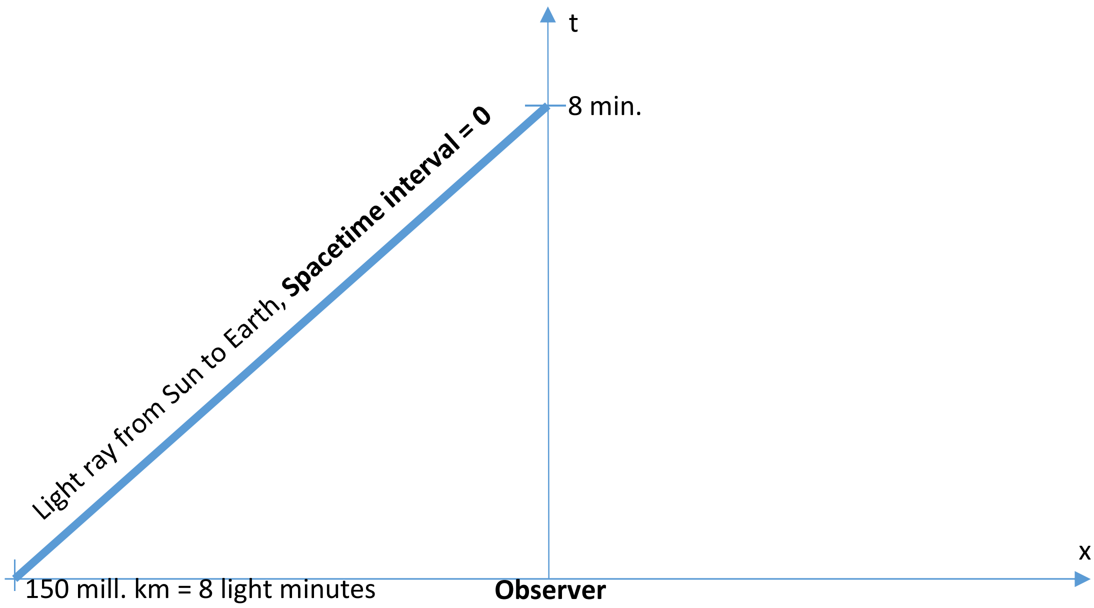

Fig. 3 - Spacetime according to Einstein and Minkowski:

Minkowski introduced the Lorentzian spacetime interval - for instance, in the diagram, the lightlike spacetime interval is zero. But as we can see, the spacetime diagram is completely identic with Newtonian spacetime, it is Euclidean, not Lorentzian, otherwise the lightlike worldline of the light ray would have the length zero and it would be reduced to a point. This Euclidean metric of the "sheet of paper" is well existing, even if this length of the light ray is completely meaningless. In contrast, the Lorentzian spacetime interval (such as the zero spacetime interval of the light ray) is not part of the spacetime diagram, it is an additional information which is not observable, it is an underlying reality ("the real age of the traveling twin" in the twin paradox) which is hidden and may be retrieved by calculation.

The distinction between the Euclidean metric of the sheet of paper and the Lorentzian metric shows that observation is indirect, and we must distinguish between the real universe and the way how it is observed within an observational interface:

Observational interface such as "a light ray sketched on a sheet of paper"	**Underlying reality** such as "the zero spacetime interval of light rays"
Spacetime manifold	Real universe
Observer-dependent	Lorentz-invariant
Euclidean metric	Lorentzian metric
Observable	Hidden, retrieved by calculation

Fig. 4 - Observation is indirect

The observational interface on the left is like a spacetime diagram with a light ray, sketched on a sheet of paper, it is a spacetime manifold, in contrast to the underlying reality on the right which represents the zero spacetime interval of the light ray. Observation is observer-dependent, while reality is Lorentz invariant. The first has a Euclidean metric and is observable, the second has a Lorentzian metric which is not observable, it is hidden and may be retrieved by calculation.

2. The twofold pseudometric of Lorentzian manifolds

Unfortunately, in his famous lecture "Space and Time", Minkowski did not see the role of the Euclidean metric, and he tried to use the Lorentz-invariant Lorentzian metric for the construction of an observer-dependent spacetime manifold. Doing this, he mixed up elements (colored) of the observational interface with those of the underlying reality:

Observational interface such as "a light ray sketched on a sheet of paper"	**Underlying reality** such as "the zero spacetime interval of light rays"
Spacetime manifold	Real universe
Observer-dependent	Lorentz-invariant
Euclidean metric	Lorentzian metric
Observable	Hidden, retrieved by calculation

Fig. 5 - Minkowski's Lorentzian manifold mixes up observational interface and underlying reality

But there was a crucial problem: When he introduced the Lorentzian squared metric he called "F"

$$"F = c^2t^2 - x^2 - y^2 - z^2" \ \textbf{[1]},$$

it was obvious that this pseudometric was not always real, it produced negative squares for spacelike intervals, corresponding to an imaginary metric, that means clearly that spacelike spacetime intervals are not defined. But Minkowski might have felt constrained to build up a Lorentzian manifold, so he simply skipped this problem. He introduced the distinction between timelike and spacelike intervals and forced the real result by inversing the sign for spacelike intervals, by introducing the opposite spacelike metric "- F",

$$" - F = x^2 + y^2 + z^2 - c^2t^2 = k^2 " \ \textbf{[1]},$$

without further explanation. This is how the Lorentzian manifold was born.

Today, the Lorentzian manifold is widely accepted, but the Lorentzian spacetime interval seems to be the most ill-defined, obscure and contradictory chapter of physics: There are many diverging "conventions" with different signatures, with either imaginary intervals or negative squares, but nobody cares. Moreover, whoever is the author, all concepts require a twofold pseudometric.

For instance, Misner Thorne Wheeler presented one metric for proper time $\Delta\tau$ and one metric for the spacetime interval Δs:

$$\Delta s^2 = -\Delta\tau^2 = g_{\mu\nu}(x^\alpha)\Delta x_\mu \Delta x_\nu \text{ } \mathbf{[2]},$$

implying the second metric of proper time

$$\Delta\tau^2 = -\Delta s^2 = -g_{\mu\nu}(x^\alpha)\Delta x_\mu \Delta x_\nu$$

A positive square equals a negative square, that means: on one side we have the square of a real number, on the other side the square of an imaginary number, in other words: real equals imaginary (!). The whole problem is there, and whatever is the way how we are trying to work around the problem, the result is always the same: The Lorentzian spacetime interval is only the appropriate metric for timelike and lightlike worldlines, it is not able to span up any manifold.

3. The problems of the Lorentzian manifold

The Lorentzian manifold is an artefact, and whatever is the approach, there are always problems instead of solutions. In the following, we enumerate not less than 7 different approaches:

- <u>In general relativity:</u> The spacetime interval with its multitude of contradicting definitions and signatures (as mentioned above) witnesses that the Lorentzian twofold "patchwork" pseudometric is not compatible with general relativity.

- <u>In quantum mechanics:</u> The Lorentzian spacetime manifold does not comply with quantum mechanics (the unresolved problem of quantum gravity). For quantum gravity, gravity may not only be described within curved spacetime but equivalently also in the form of gravitational time dilation within absolute, uncurved space (**see annex 1**).

- <u>Experiments:</u> There is no experimental evidence which would require the existence of such a fourdimensional Lorentzian manifold and in particular of spacelike spacetime intervals. The reason is that the spacelike spacetime metric corresponds to a forbidden displacement, exceeding the speed limit of general relativity which is speed of light.

- <u>Topology:</u> There is no straightforward "nice" topology of spacetime. The main theory makes even the 1+3 breakdown, with the separation between time and space dimensions. **[3]**

- <u>Vacuum points:</u> there is no clear notion of spacetime vacuum. General relativity, and in particular also the two postulates of special relativity

- Laws of physics are the same in all inertial frames of reference.
- As measured in any inertial reference frame, light is observed to travel with velocity c.

are referring to reference frames and to lightlike phenomena, but not to the spacetime vacuum between the worldlines.

- The discrepancy between space distances and spacelike spacetime intervals: If we apply Minkowski's signature (+,-,-,-), a real space distance of "5 meters" ($\Delta t = 0$) corresponds to an imaginary spacetime interval of "5i meters". This might have been the reason why the opposite signature (-,+,+,+) was introduced, such that 5 meters of space distance corresponds to 5 meters of spacetime interval. However, many authors are not following this signature convention.

- And finally, even the curved spacetime of gravity does not require any spacetime manifold (!) The propagation of gravity is lightlike, and its effects may be described with timelike and lightlike worldlines, there are no spacelike interactions in curved spacetime, and no continuity of spacetime in spacelike direction is required:

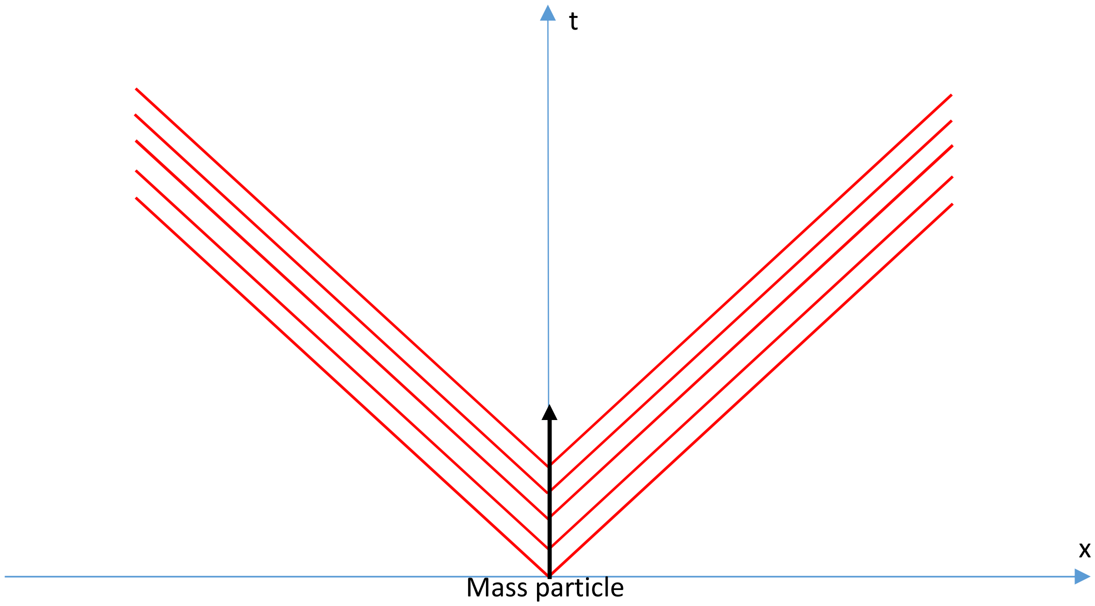

Fig. 6 - The propagation of gravity does not require any spacelike spacetime intervals

Annex 1: Gravity without curved spacetime

Based on Minkowski's twofold Lorentzian metric, Einstein and Grossmann developed the concept of curved spacetime for gravity. This concept corresponded perfectly to the experimental evidence in the field of general relativity, and this is why today the Lorentzian manifold is considered to be an integral part of general relativity, even if we are not able to put it in harmony with quantum mechanics.

However, it will be shown here at the example of the Schwarzschild metric that gravity may not only be described within curved spacetime but equivalently also in the form of gravitational time dilation within absolute, uncurved space.

The Schwarzschild metric is an exact solution to the Einstein field equations. One of its characteristics is its amazing simplicity. The equation[2]

[2] Following the current sign convention (- + + +)

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2(d\Theta + sin^2\Theta\, d\phi^2)$$

is simply a combination of the flat Minkowski metric with gravitational time dilation. In order to show this, we denote by **C** (upper case) the gravitational time dilation of the clock of a particle in a gravity field with reference to a potential-free far-away observer:

$$C = \frac{d\tau}{dt} = \sqrt{1 - \frac{r_s}{r}} = \sqrt{1 - \frac{2GM}{c^2 r}}$$

By inserting **C** into the equation above, we get a modified form of the Schwarzschild metric:

$$ds^2 = -c^2(\boldsymbol{Cdt})^2 + \left(\frac{\boldsymbol{dr}}{\boldsymbol{C}}\right)^2 + r^2(d\Theta + sin^2\Theta\, d\phi^2)$$

Now we compare this equation with the equation of flat Minkowski metric **[3]**:

$$ds^2 = -c^2\boldsymbol{dt}^2 + \boldsymbol{dr}^2 + r^2(d\Theta + sin^2\Theta\, d\phi^2)$$

We see that the Schwarzschild metric and the Minkowski metric are very similar, and the gravitational time dilation C is the only difference between curved and uncurved spacetime: Time dt is multiplied with gravitational time dilation, and distance dr is divided by it. By consequence, gravity may be entirely described without spacetime, by gravitational time dilation in absolute space, and there is no gravitational interaction whatsoever beyond gravitational time dilation.

4. References

[1] Hermann Minkowski: Raum und Zeit (1908), in: Space and Time, Minkowski's Papers on Relativity, Minkowski Institute Press 2012, p. 111

[2] Charles Misner, Kip Thorne, John Archibald Wheeler: Gravitation, 1973, p. 305

[3] See the overview of Renee Hoekzema: On the Topology of Lorentzian manifolds, 2011