Fizeau Experiment revisited and physical meaning of the refraction index

Non-ad-hoc classic explanation of the Fizeau Experiment and association of refractive index with molecule’s distances

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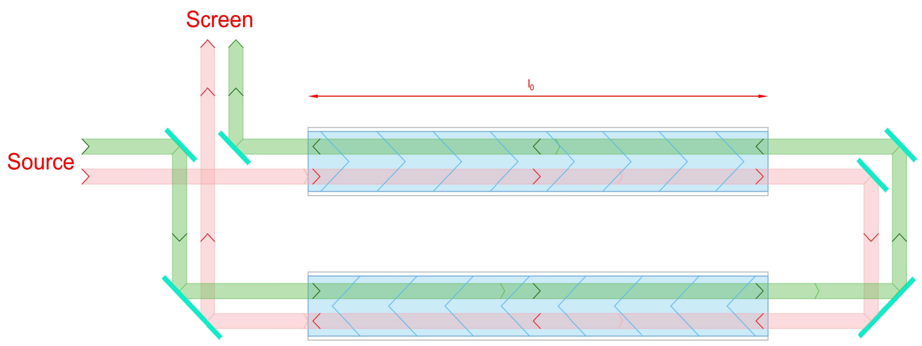
Abstract

The Fizeau experiment was one of the milestones on the long road to the discovery of relativity. In this paper, we will expose a fundamental flaw in Fizeau's own Fresnel-based interpretation, namely the exclusion of the different wavelengths of light in vacuum and in water from the calculation. We will show a physically tractable explanation of the effect using classical physics. As a side effect, the formulas found offer the possibility to clarify the physical meaning of the refractive index itself and the question why refractive indices usually, but not always, correlate with matter density. Furthermore, it will be possible to calculate the relationship between the molecular radius and the distance between the molecules of a medium from the refractive index.

1. Description and critic on the experiment

Fizeau [1] relied on Fresnel's work [2] on the reduction of the speed of light in refracting media to explain the result of his experiment. The latter is rightly called an ad hoc explanation, and mathematics cannot explain the problem in a physically derivable way, as we will show. The experimental setup is as follows:



Looking to Fizeau's formula which is concluding to Einstein's [3] formula when wavelength within the medium is being used:

$$\Delta l = 4 \cdot l_0 \cdot \left(\frac{v_{Medium}}{c_{Vacuum}} \right) \cdot \left(n^2 - 1 \right)$$

with:

Δl = change in length
l_0 = length of one tube of the lightpath at rest
v_{Medium} = movement speed of medium
c_{Vacuum} = light speed in vacuo
n = refractive index

After some rearrangement we obtain:

$$\Delta l = 4 \cdot l_0 \cdot \frac{v_{Medium}}{c_{Vacuum}} \left(n - \frac{1}{n} \right)$$

And finally, cleaning out n to obtain only the relevant figures for physical comprehension, i.e. velocities and distance:

$$\Delta l = 4 \cdot l_0 \cdot \left(\frac{v_{Vacuum}}{c_{Medium}} - \frac{v_{Medium}}{c_{Vacuum}} \right)$$

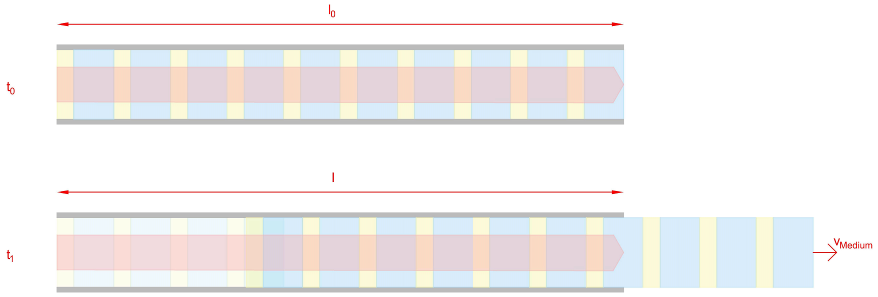
The attempt to find a form of meaningful velocity addition must fail, because velocities in the medium and vacuum are put in relation. Also the appearance of the speed of the vacuum in this formula (derived from $c_{Vakuum} = v_{Medium} \cdot n$) seems doubtful.

2. Reinterpretation from the scratch

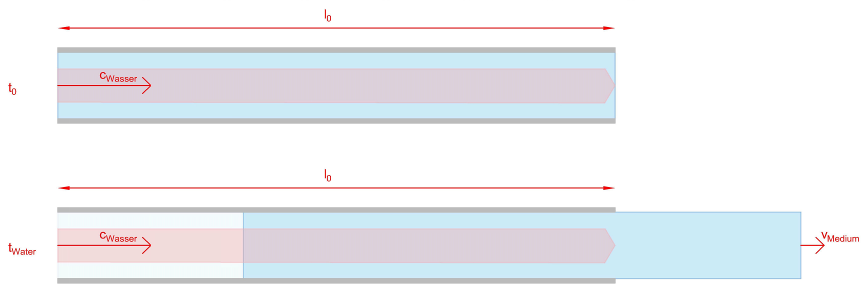
The assumption that the medium consists of molecules with a vacuum between them is the basis for our investigation. We assume that light travels between the molecules at vacuum velocity, and near the molecules at reduced velocity according to the refractive index. Although we cannot reliably determine the distances traveled in vacuum or in the medium, we will see later that this is surprisingly not relevant.

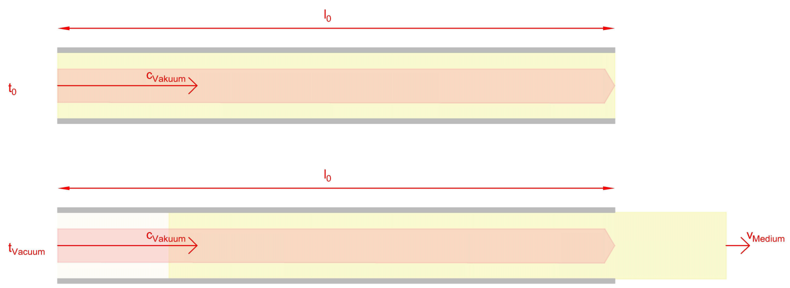
For simplicity, let us examine only one direction of the medium flow. Since the rays are moving counterclockwise, there is no second order effect to be expected from velocity addition. Therefore, it is convenient to consider only one length of pipe with one medium flow direction and simply multiply the results by four later. Furthermore, it is extremely important to differentiate between the respective wavelengths in the medium or vacuum.

To begin with, we will show how one can picture the principle outlined before:

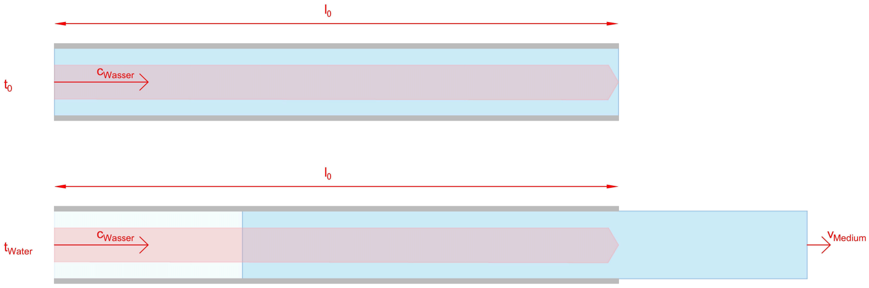


Light would now travel a fraction of the distance of l_0 within vacuum, another fraction within the medium, and both distances together must be l_0. For now we assume, that each travel lengths would be l_0, and later we will deal with the fraction.





We focus now on the light propagation within the medium, whereas calculating the time for the light ray with speed c_{Medium} to traverse l_0 whilst the medium is moving with v_{Medium}. Velocities must be added classically:



As a reference, first we find the time that light would need to cover the distance l_0 if the medium was not moving:

$$t_{0,Medium} = \frac{l_0}{c_{Medium}}$$

And now the time for the same distance, but assuming that the light ray must follow behind the moving medium (the movement of the source being irrelevant as per classic wave theory):

$$t_{Medium} = \frac{l_0}{c_{Medium} - v_{Medium}}$$

We have now the difference in time caused by the moving medium:

$$\Delta t_{Medium} = \frac{l_0}{c_{Medium} - v_{Medium}} - \frac{l_0}{c_{Medium}}$$

And herewith the difference in distance on basis of light speed in the medium:

$$\Delta s_{Medium} = \left(\frac{l_0}{c_{Medium} - v_{Medium}} - \frac{l_0}{c_{Medium}} \right) \cdot c_{Medium}$$

Finally calculating the fringe shift it is indispensable to use the wavelength in the medium:

$$\Delta fringe_{Medium} = \frac{\left(\frac{l_0}{c_{Medium} - v_{Medium}} - \frac{l_0}{c_{Medium}} \right) \cdot c_{Medium}}{\lambda_{Medium}} = l_0 \cdot \frac{c_{Medium}}{\lambda_{Medium}} \cdot \left(\frac{1}{c_{Medium} - v_{Medium}} - \frac{1}{c_{Medium}} \right)$$

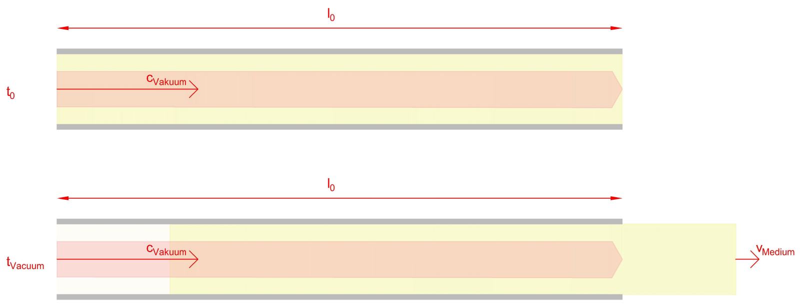
After some rearrangement we obtain:

$$\Delta fringe_{Medium} = \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{\left(c_{Medium} - v_{Medium} \right)} \quad (1)$$

And since v_{Medium} should be minor in the denominator at non-relativistic speeds, we have:

$$\Delta fringe_{Medium} \approx \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{c_{Medium}}$$

Now we come to the vacuum part and use the same approach. Also in this case we have to use the wavelength in vacuum, where the vacuum moves with the same speed as the medium before:



$$\Delta fringe_{Vacuum} = \frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{\left(c_{Vacuum} - v_{Medium}\right)} \quad (2)$$

$$\Delta fringe_{Vacuum} \approx \frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{c_{Vacuum}}$$

And finally we want to know the fringe shift difference of both medium and vacuum, deducting (2) from (1):

$$\Delta fringe = \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{\left(c_{Medium} - v_{Medium}\right)} - \frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{\left(c_{Vacuum} - v_{Medium}\right)} \quad (3)$$

And for low speeds v_{Medium} :

$$= l_0 \cdot v_{Medium} \cdot \left(\frac{1}{\lambda_{Medium} c_{Medium}} - \frac{1}{\lambda_{Vakuum} c_{Vakuum}} \right) = \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{c_{Medium}} \left(1 - \frac{1}{n^2}\right)$$

And since until now we have used only one tubelength:

$$4 \cdot \Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{c_{Medium}} \left(1 - \frac{1}{n^2}\right) \quad (4)$$

(In this case using the more convenient wavelength within medium)

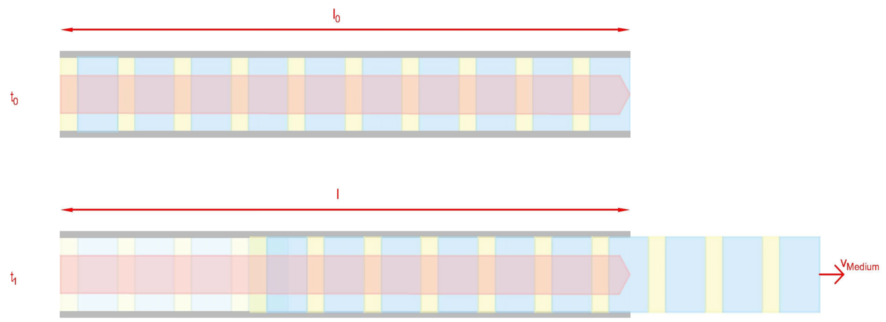
As we can see, the formula found leads to the same results as Fizeau's formula, but we have derived it from solid physical assumptions.

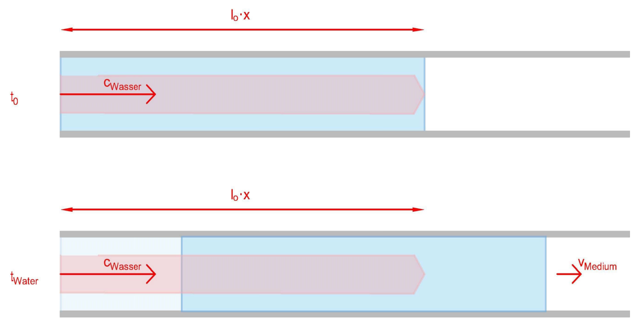
Nevertheless, we have not yet answered the most important question. We have tacitly assumed that a ray of light travels the full distance l_0 in a vacuum while simultaneously traveling the same distance in the medium. Consequently, we must now add any division of the distance l_0 for vacuum and the medium (x and y) as a fraction of together 1:

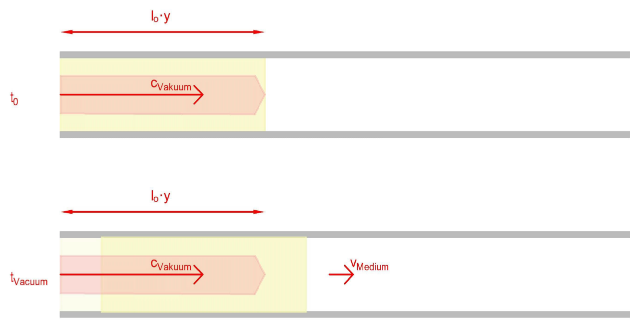
$$l_{Medium} = l_0 \cdot x$$

$$l_{Vacuum} = l_0 \cdot y$$

In other words, we add up all the little distances within the medium and the distances within vacuum over the whole length of l_0. The following picture shows how we have to imagine this:







And finally we obtain an altered formula on the same basis (this time also for relativistic velocities), taking (3):

$$\Delta fringe = \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{\left(c_{Medium} - v_{Medium}\right)} - \frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{\left(c_{Vacuum} - v_{Medium}\right)}$$

And extending with x and y

$$4 \cdot \Delta fringe = 4\left(x \cdot \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{\left(c_{Medium} - v_{Medium}\right)} - y\frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{\left(c_{Vacuum} - v_{Medium}\right)}\right)$$

$$\boxed{4 \cdot \Delta fringe = 4 \cdot l_0 \cdot v_{Medium}\left(\frac{x}{\lambda_{Medium} \cdot \left(c_{Medium} - v_{Medium}\right)} - \frac{y}{\lambda_{Vakuum} \cdot \left(c_{Vakuum} - v_{Medium}\right)}\right)} \quad (5)$$

And again simplified for lower speeds:

$$\boxed{4 \cdot \Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Medium}} \cdot \frac{v_{Medium}}{c_{Medium}}\left(x - \frac{y}{n^2}\right)} \quad (6)$$

At first sight it seems impossible to gain the same results as per Fizeau's formula, because depending on the fraction x and y the overall fringe shift must differ. We would have to find terms for x and y so that the result again becomes equal with Fizeaus's formula.

After some testing on an excel sheet it turns out that the following correlation of x and y with the refractive index n, summing up to 1, will always do the job as a pair:

$$\boxed{x = \frac{1}{\frac{1}{n^2} + 1}} \quad (7)$$

$$\boxed{y = \frac{1}{n^2 + 1}} \quad (8)$$

Where $x + y = 1$

Now what does this mean? We have found the relation of empty space and matter within a medium, depending on the refractive index. This means, we have found a relation of the distance between two molecules and the molecule's diameter, to be expressed by the equations (7) and (8)!

Since the above terms for x and y lead to the experimentally correct results, x and y obviously must be an inherent property of the material, derived from the refractive index. In other words: The refractive index is nothing but caused by these inherent properties!

Let's do a test: Calculating the proportion of the diameter of a water molecule against the distance between two molecules at say room temperature, derived from the known dimensions:

Given mass of molecule:

$$m_{H_2O} = 3{,}07 \cdot 10^{-26} kg$$

Number of molecules z in one liter:

$$z = 3{,}07 \cdot 10^{25}$$

Volume of one molecule including its distance to the next:

$$vol = 3{,}07 \cdot 10^{-29} m^3$$

Equivalent to diameter+distance of molecule d_{H_2O} + s_{H_2O} :

$$d_{H_2O} + s_{H_2O} = 313{,}189\, pm$$

Given d_{H_2O} :

$$d_{H_2O} \approx 200\, pm = 0{,}63859 \text{ of } d_{H_2O} + s_{H_2O}$$

Thus

$$s_{H_2O} = 113{,}189\, pm = 0{,}36140 \text{ of } d_{H_2O} + s_{H_2O}$$

According to new formula above and a refractive index of water of 1,333:

$$d_{H_2O} = 0{,}63988$$

and

$$s_{H_2O} = 0{,}36011$$

Obviously we have a match with a deviation of only 2‰!

3. Conclusion

We have found a way to a classical interpretation of the Fizeau experiment [1] based on purely physical assumptions. As a side effect, we have found a way to correlate molecular dimensions with the space between molecules and thus explain the true cause of the refractive index of materials. Since it is proving difficult to find unambiguously applicable given dimensions for alternative liquids with more complex molecules and different indices for cross-checking, I would like the community to continue research in this area.

References and Acknowledgments (in order of appearance):

[1] H. Fizeau, Sur les hypothèses relatives à l'éther lumineux., Comptes Rendus. 33, 1851, S. 349–355.

[2] Fresnel, Augustin, Lettre d'Augustin Fresnel à François Arago sur l'influence du mouvement terrestre dans quelques phénomènes d'optique, Bde. %1 von %2Oeuvres complètes d'Augustin Fresnel / publiées par MM. Henri de Senarmont, http://visualiseur.bnf.fr/CadresFenetre?O=NUMM-91937&I=633&M=tdm, Gallica, 1818.

[3] Einstein, Albert, Zur Elektrodynamik bewegter Körper, Annalen der Physik und Chemie. 17, 1905, S. 891–921, 1905.