A Theory of Multiple Interacting Realities

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Abstract.

We consider how two (or more) distinct physical realities can coexist within a common spacetime. As an example we will utilize quantum electrodynamics since this is a familiar and well-understood theory. We will designate one world the 'red' one and the other the 'green' one. We will illustrate how they can interact in a physically plausible way. The result is, in fact, a rather strange kind of Kaluza-Klein theory. If there are other such realities they could provide a possible explanation for dark matter.

Keywords: Multiple Realities; Kaluza-Klein Theories; Dark Matter; Dark Matter Halos.

Introduction.

Were there to exist a sector, or sectors, of particles that gravitated normally but could seldom or never be observed by us we might have an excellent candidate for dark matter. Indeed, Foot (1, 2, 3, 4) has suggested that 'Mirror Matter' – originally proposed by Lee and Yang – could be such a candidate. We will examine a different theory of multiple realities, mostly using quantum electrodynamics (QED) as a simple and familiar example (although we intend it to apply to the full Standard Model). We can imagine one reality – call it the 'red' one – populated with 'red' electrons and 'red' photons. We will suppose that there is another, 'green,' reality populated with 'green' electrons and photons. They share the same spacetime. If we are observers living in the 'red' reality we will imagine that the 'green' reality exists all around us and is defined over whatever spacetime coordinate system we decide to use. Ordinarily, we just cannot see this 'green' reality because its particles do not interact with our 'red' ones (except gravitationally). We will introduce a new function, $c(\boldsymbol{x}, t)$, which reflects the degree to which the 'red' and 'green' realities interact with one another. It is considered to be a real, dimensionless, scalar field. We will assume that the laws of physics are the same in both realities and that the two kinds of electrons have the same mass and charge in their respective realities.

QED in Two Realities.

We start out by writing the Lagrangian as it would look if these realities were always completely independent:

1) $$L_{\text{em}} = \overline{\psi_R}\,[\gamma^\mu[i\,\partial_\mu - e\,A_{R\,\mu}] - m]\psi_R - \tfrac{1}{4}F_R{}^{\mu\nu}F_{R\,\mu\nu} + \overline{\psi_G}\,[\gamma^\mu[i\,\partial_\mu - e\,A_{G\,\mu}] - m]\psi_G - \tfrac{1}{4}F_G{}^{\mu\nu}F_{G\,\mu\nu}.$$

The objects ψ_R, A_R, are understood to pertain to the 'red' reality. The 'G' subscript means they belong to the 'green' reality. $F_{R\mu\nu}$ is the electromagnetic field strength tensor appropriate to the 'red' world ($\partial_\mu\, A_{R\,\nu} - \partial_\nu\, A_{R\,\mu}$). $F_{G_{\mu\nu}}$ pertains to the 'green' one. Now an interaction between these realities could occur if there were to take place a *mixing* of A_R and A_G in their interaction with the electron fields according to:

2) $$A_{\mathrm{R}\mu} \longrightarrow \left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2}[A_{\mathrm{R}\mu} + c(\boldsymbol{x}, t)\; A_{\mathrm{G}\mu}] \text{ and}$$

$$A_{\mathrm{G}\mu} \longrightarrow \left(1 + c(\boldsymbol{x},\, t)^2\right)^{-1/2} \left[A_{\mathrm{G}\mu} + \, c(\boldsymbol{x},\, t)\;\, A_{\mathrm{R}\mu}\right].$$

Note that this mixing of quantum fields is confined to the photon fields. It is not applied to the electron fields. Nor is it applied within the $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ terms. When $c(\boldsymbol{x},\, t) = 0$ there is no interaction. As $c(\boldsymbol{x},\, t)$ becomes larger 'red' observers begin to experience some of the 'green' reality and vice-versa. $\left(1 + \, c(\boldsymbol{x},\, t)^2\right)^{-1/2}$functions as a kind of normalization factor. Under the influence of this transformation the Lagrangian becomes:

3) $$L_{\mathrm{em}} = \overline{\psi_R}\,[\gamma^\mu[\dot{\imath}\,\partial_\mu - e\left(1 + \, c(\boldsymbol{x},\, t)^2\right)^{-1/2}\left[A_{\mathrm{R}\mu} + \, c(\boldsymbol{x},\, t)\;\, A_{\mathrm{G}\mu}\right]] - m]\psi_R - \tfrac{1}{4}F_R{}^{\mu\nu}F_{R\,\mu\nu}$$
$$+\, \overline{\psi_G}\,[\gamma^\mu[\dot{\imath}\,\partial_\mu - e\left(1 + c(\boldsymbol{x},\, t)^2\right)^{-1/2}\left[A_{\mathrm{G}\mu} + \, c(\boldsymbol{x},\, t)\;\, A_{\mathrm{R}\mu}\right]] - m]\psi_G - \tfrac{1}{4}F_G{}^{\mu\nu}F_{G\,\mu\nu}.$$

We will assume, for the moment, that $c(\boldsymbol{x},\, t)$ is roughly constant over the spacetime volume of interest. While this new Lagrangian maintains local gauge invariance only under circumstances where $c(\boldsymbol{x},\, t)$ is constant it has the advantage of resulting, under these circumstances, in simple Feynman rules and a physics which, in many respects, corresponds with that we would like to see for a theory that doesn't grossly violate observed reality. In situations where $c(\boldsymbol{x},\, t)$ varies things become more complicated.

These new Feynman rules are similar to the familiar ones but with two important differences: Firstly, the vertices connecting an incoming and outgoing 'red' electron (or positron) line with a 'red' photon contribute with a coupling constant $e\left(1 + \, c(\boldsymbol{x},\, t)^2\right)^{-1/2}$. It is likewise for the 'green' particles. Secondly, new vertices appear which connect incoming and outgoing 'red' electron (or positron) lines with a 'green' photon and incoming and outgoing 'green' electron (or positron) lines with a 'red' photon (*fig.1*). (In the first two cases we omit drawing the graphs with the outgoing electrons exchanged. But we know they are there.) These contribute with a coupling constant which is $e\, c(\boldsymbol{x},\, t)\left(1 + \, c(\boldsymbol{x},\, t)^2\right)^{-1/2}$.

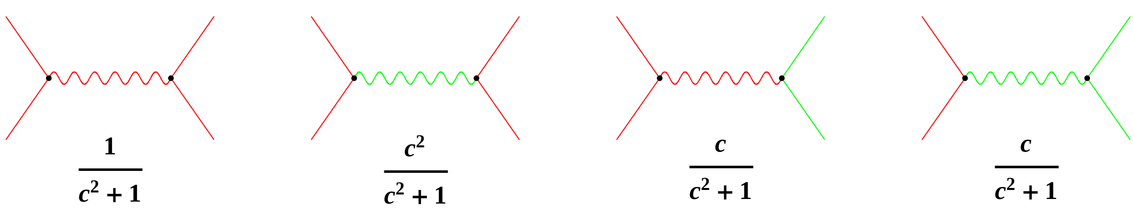

fig.1

Consider the scattering of one 'red' electron off another in the presence of an interaction. To find the probability amplitude for this process (to second order in the coupling constant) we will sum the amplitudes corresponding to the usual Feynman diagrams and new diagrams in which it is a 'green' virtual photon that is being exchanged. Straightforward arithmetic shows that the overall coupling constant is still e. Thus the resulting amplitude is unchanged by the presence of the interaction. The contribution from the 'green' virtual photon compensates exactly for the reduction in the coupling strength of the normal interaction. This is encouraging – as long as we are dealing with interactions between 'red' electrons and other 'red' electrons, electromagnetism should continue to work normally in the 'red' world even if $c(\boldsymbol{x},\, t)$ were different from zero. The same situation would obtain in the 'green' world. Suppose, instead, that we try to scatter 'red' electrons off of 'green' electrons. Now things are different. In each of the two relevant Feynman diagrams would be a vertex connecting either 'green' fermions with a virtual 'red' photon or 'red' fermions with a 'green' virtual photon. Arithmetic again yields a simple result. If we are 'red' observers looking at the behavior of 'red' electrons, we would have to conclude that the 'green' electrons had a charge that was only $2\,c(\boldsymbol{x},\,t)\left(1+\,c(\boldsymbol{x},\,t)^2\right)^{-1} e$. We would always assume that our 'red' electrons have charge e. If the 'green' electrons scatter abnormally it must be because they have a reduced charge. Also, since there are no vertices connecting an incoming 'red' electron with an outgoing 'green' electron, the scattering would be the same as that produced by two non-identical particles; this makes sense as we would not want to say that 'green' and 'red' particles are indistinguishable. There would be other consequences as well. If, for instance, we consider the Compton scattering of a 'red' photon off a 'red' electron in a high-$c(\boldsymbol{x},\,t)$ region there will be some chance of seeing a 'green' photon emerge.

What we have done with our A_R and A_G is reminiscent of what Foot (4) has done with his 'photons' and 'mirror photons' although the mathematics is not quite the same. And, for us, the realities interact through $c(\boldsymbol{x},\,t)$, which we regard as a function of spacetime. For Foot their interaction is mediated through what he calls 'ϵ' and considers a small physical constant. Also, we do not suppose that our 'red' and 'green' worlds differ as to their parity. We can have as many 'colored' worlds as we might want – we are not limited to two (*vide infra*). Foot (3) has, however, broadened his theory to encompass 'dissipative matter' which can also come in multiple forms.

These ideas can be easily generalized to the Standard Model. 'Red' vector bosons and gluons would mix as above with their 'green' counterparts. And this is, of course, what we *really are* proposing. We are not interested in simply producing a strange new version of QED. As has been mentioned, QED is utilized here only as an illustrative example in order to keep the math to a minimum. It is, however, a good example in that it allows us to easily investigate the large-scale phenomena potentially associated with our theory.

The Classical Limit.

We want to know what the physics resulting from this would look like to an ordinary, macroscopic, observer. And it is not clear how much more we can do in a quantum mechanical way. There, if we do not regard $c(\boldsymbol{x},\,t)$ as a constant, we have no easy way of doing the math. Let us look at Equation 3) from a semi-classical point of view. We recall that, according to Dirac theory, the 4-current density in the 'red' world is given by $e\;\overline{\psi_R}\;\gamma^\mu \psi_R$, and by $e\;\overline{\psi_G}\;\gamma^\mu \psi_G$ in the 'green' one. Varying Equation 3) by $A_{\mathrm{R}\mu}$ we find:

$$4)\qquad F_R{}^{\mu\nu}{}_{,\nu} = J^\mu \Big/ \left(1 + c(\boldsymbol{x},\,t)^2\right)^{1/2} + \tilde{J}^\mu\, c(\boldsymbol{x},\,t) \Big/ \left(1 + c(\boldsymbol{x},\,t)^2\right)^{1/2}$$

where J^μ denotes the 4-current density in the 'red' world and $\tilde{J}^\mu$ that in the 'green' world. Varying by $A_{\mathrm{G}\mu}$, we find a corresponding equation for things in the 'green' world. Let us now vary Equation 3) by $\overline{\psi_R}$ so as to get the Dirac equation for the behavior of 'red' electrons. We find:

$$5)\qquad [\gamma^\mu[\mathrm{i}\,\partial_\mu - e\left(1+\,c(\boldsymbol{x},\,t)^2\right)^{-1/2}\left[A_{\mathrm{R}\mu} +\; c(\boldsymbol{x},\,t)\;A_{\mathrm{G}\mu}\right]] - m]\psi_R = 0.$$

This tells us what effective "4-potential" the 'red' electron is responding to. We can perform the same exercise for the 'green' Dirac equation. We obtain, as a practical matter, a Lorentz force law for a 'red' electron which reads:

6) $$m\,\ddot{x}_R{}^{\mu} = e\Big(F_R{}^{\mu}{}_{\nu}\Big/\left(1+c(\boldsymbol{x},t)^2\right)^{1/2} + F_G{}^{\mu}{}_{\nu}\,c(\boldsymbol{x},t)\Big/\left(1+c(\boldsymbol{x},t)^2\right)^{1/2} - \left(1+c(\boldsymbol{x},t)^2\right)^{-3/2}\Big[c(\boldsymbol{x},t)\left(c(\boldsymbol{x},t)^{,\mu}A_{R\nu} - c(\boldsymbol{x},t)_{,\nu}A_R{}^{\mu}\right) - \left(c(\boldsymbol{x},t)^{,\mu}A_{G\nu} - c(\boldsymbol{x},t)_{,\nu}A_G{}^{\mu}\right)\Big]\Big)\,\dot{x}_R{}^{\nu}.$$

And we will obtain a reversed version for a 'green' electron, having the '*R*'s and '*G*'s interchanged.

Equation 6) is actually rather remarkable as it shows that we can deduce useful things by *not* trying to use the quantized theory. Equation 6) follows from 5) in the most simple way. We *know* that Dirac's equation – the one with A_μ as we are used to seeing it – gives us the familiar Lorentz force law when translated into the classical world. (It is actually rather hard to deduce this mathematically. But it is certainly true.) Thus by treating the strange term that appears in Equation 5) *exactly as if* it were A_μ (i.e. constructing an $F_{\mu\nu}$ from it) we arrive at Equation 6). And it must be true.

It will be observed that this equation of motion does not respect local gauge invariance, nor should it. Gauge invariance requires the constancy of $c(\boldsymbol{x}, t)$. And simply specifying a gauge will not help us here. We could require, for example, $\partial_\mu A_{R,G}{}^{\mu} = 0$. But this, alone, is insufficient. We could imagine adding a 4-vector, $\Lambda^{,\mu}$, to either A^μ and this would not disturb the gauge condition so long as $\Lambda^{,\mu}{}_{\mu} = 0$. It would, however, change Equation 6). The $A_{R,G}{}^{\mu}$ in this theory must be definite, unambiguous, and not subject to the addition of any factors. We would be better off endowing both of our photons with a vanishingly small mass. In effect we add terms $\epsilon^2 A_{R,G}{}^{\mu} A_{R,G_\mu}$ to the Lagrangians for our two photons (understanding that ϵ is so small that it can be taken to zero at the end of any practical calculation). The dynamical equations for the two A fields become Proca equations. This is invaluable both because it automatically ensures $\partial_\mu A_{R,G}{}^{\mu} = 0$ *and* also rules out the addition of any intrusive gradients to our A fields. We assume A_R and A_G go to zero in areas very far from any currents.

No assumptions regarding the constancy of $c(\boldsymbol{x}, t)$ have been made in deriving Equations 4) and 6) (and their two 'green' counterparts). We suspect that, under many circumstances, $c(\boldsymbol{x}, t)$ can be treated as, more-or-less, a constant. This allows us to make some simplifications to the mathematics. Since all we are interested in is the effective field that 'red' or 'green' electrons respond to, let us simplify matters by writing:

7) $$F^{\mu\nu} = F_R{}^{\mu\nu}\Big/\left(1+c(\boldsymbol{x},t)^2\right)^{1/2} + F_G{}^{\mu\nu}\,c(\boldsymbol{x},t)\Big/\left(1+c(\boldsymbol{x},t)^2\right)^{1/2} \text{ and}$$

8) $$\tilde{F}^{\mu\nu} = F_G{}^{\mu\nu}\Big/\left(1+c(\boldsymbol{x},t)^2\right)^{1/2} + F_R{}^{\mu\nu}\,c(\boldsymbol{x},t)\Big/\left(1+c(\boldsymbol{x},t)^2\right)^{1/2}.$$

It now becomes possible to write Maxwell's equations and the Lorentz force law, in the presence of an interaction, in a more compact form:

9) $$F^{\mu\nu}{}_{,\nu} = J^\mu + 2\,\tilde{J}^{\mu}\,c(\boldsymbol{x},t)\Big/\left(1+c(\boldsymbol{x},t)^2\right)$$

10) $$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0$$

11) $\tilde{F}^{\mu\nu}{}_{,\nu} = \tilde{J}^{\mu} + 2\, J^{\mu}\, c(\boldsymbol{x},\, t) \big/ \left(1 + c(\boldsymbol{x},\, t)^2\right)$

12) $\tilde{F}_{\alpha\beta,\gamma} + \tilde{F}_{\beta\gamma,\alpha} + \tilde{F}_{\gamma\alpha,\beta} = 0$

13) $m\, \ddot{x}_R{}^{\mu} = e\, F^{\mu}{}_{\nu}\, \dot{x}_R{}^{\nu}$

14) $m\, \ddot{x}_G{}^{\mu} = e\, \tilde{F}^{\mu}{}_{\nu}\, \dot{x}_G{}^{\nu}$.

where $F^{\mu\nu}$denotes the classical electromagnetic field strength tensor measured by the 'red' physicist and $\tilde{F}^{\mu\nu}$ that measured similarly by the 'green' one.

General Relativity and the Physics of $c(\boldsymbol{x}, t)$.

From Equation 4) and its 'green' counterpart we can deduce that $c(\boldsymbol{x},\ t)_{,\mu}\, J^{\mu} = 0$ and $c(\boldsymbol{x},\ t)_{,\mu}\, \tilde{J}^{\mu} = 0$. Otherwise, our theory can tell us nothing about the behavior of $c(\boldsymbol{x}, t)$. This problem can be addressed if we recognize that we are, in fact, dealing with a (rather peculiar) sort of Kaluza-Klein theory. Following Kerner's convention (5) we will say i, j, ... run from 1 to 4 (where 4 denotes the time coordinate). Our ansatz differs from that of Kerner, however. Consider a 6X6 metric having the form:

15) $g^{\mathrm{ij}} = \mathfrak{g}^{\mathrm{ij}}$

16) $g_{\mathrm{ij}} = \mathfrak{g}_{\mathrm{ij}} - (A_{R\, i}\, A_{R\, j} + A_{G\, i}\, A_{G\, j}) / \gamma$

17) $g^{5\, i} = - \left(1 + c(\boldsymbol{x},\, t)^2\right)^{-1/2} [A_R{}^{i} + c(\boldsymbol{x},\, t)\ A_G{}^{i}] / \gamma$

18) $g^{6\, i} = - \left(1 + c(\boldsymbol{x},\, t)^2\right)^{-1/2} [A_G{}^{i} + c(\boldsymbol{x},\, t)\ A_R{}^{i}] / \gamma$

!9) $g_{5\, i} = - \left(1 + c(\boldsymbol{x},\, t)^2\right)^{1/2} (A_{R\, i} - c(\boldsymbol{x}, t)\, A_{G\, i}) / (1 - c(\boldsymbol{x},\, t)^2)$

20) $g_{6\, i} = - \left(1 + c(\boldsymbol{x},\, t)^2\right)^{1/2} (A_{G\, i} - c(\boldsymbol{x}, t)\, A_{R\, i}) / (1 - c(\boldsymbol{x},\, t)^2)$

21) $g_{55} = g_{66} = - \gamma \left(1 + c(\boldsymbol{x},\, t)^2\right)^2 / \left(1 - c(\boldsymbol{x},\, t)^2\right)^2$

22) $g_{56} = g_{65} = 2\, \gamma \left(c(\boldsymbol{x},\, t) + c(\boldsymbol{x},\, t)^3\right) / \left(1 - c(\boldsymbol{x},\, t)^2\right)^2$.

$\gamma = \frac{1}{16\, \pi\, G\, \epsilon_0}$ and is introduced to keep the units correct and to ensure our results come out in a familiar form. This ansatz works only if $c(\boldsymbol{x}, t)$ = constant. If such is not the case we can still write down our metric although the mathematics becomes far more complex (see Supplementary Material).We note that $\sqrt{-\det[g]}$ =

$\gamma\sqrt{-\det[\mathfrak{g}]}\ \left|\left(1+\ c(\boldsymbol{x},\,t)^{2}\right)/\left(1-\ c(\boldsymbol{x},\,t)^{2}\right)\right|$. This result is true even if *c(**x**, t)* varies. We assume that the physically real quantities ($\mathfrak{g}_{\rm ij}$, A_R, A_G, and *c(**x**, t)*) depend only on x^i, not on x^5or x^6.

$\mathfrak{g}_{\rm ij}$ is the 4-metric that would be measured by 'red' and 'green' physicists using rulers and clocks. $\mathfrak{g}^{\rm ij}$ is its contravariant counterpart (its 4X4 matrix inverse). Gothic letters will be used to designate quantities belonging only to the 4-dimensional base space. We see that the above metric gives us exactly the theory we have outlined above. From -*R* we obtain the $-\frac{1}{4}F_R{}^{\rm ij}F_{R\,{\rm ij}\nu}$ and $-\frac{1}{4}\,F_G{}^{\rm ij}\,F_{G\,{\rm ij}}$ terms present in Equation 3). (The indices in $A_{R,G}{}^{i}$ and $F_{R,G}{}^{\rm ij}$ are lowered using $\mathfrak{g}_{\rm ij}$.) If *c(**x**, t)* is constant, and supposing a 'red' test particle having 'red' charge Q_R to have a 6-momentum with $p_5 = Q_R$ and $p_6 = 0$, the geodesic equation gives us Equation 13). For an analogous 'green' test particle we obtain Equation 14). (p_5 and p_6 are constants if the particle moves along a geodesic.) The Lagrangian density we set to ($L_{\rm em}$ - R/16 π G) $\sqrt{-\det[g]}\,/\,\gamma$. We find that - R = - $\mathfrak{R}$ - $\frac{1}{4\gamma}F_R{}^{\rm ij}F_{R\,{\rm ij}\nu}-\frac{1}{4\gamma}\,F_G{}^{\rm ij}\,F_{G\,{\rm ij}}$. This result is *always* true regardless whether *c(**x**, t)* changes or not.

To find the Einstein's equations appropriate to our system we follow the example of Kerner (5). Varying the Lagrangian density by $g^{\rm ij}$ we obtain a result proportional to:

23) $R_{\rm ij}$ - $\frac{1}{2}$ $\mathfrak{g}_{\rm ij}$ R + $[g_{5\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{6\,i}]\;R_{5\,j}/\gamma+[g_{\rm j5}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{\rm j6}]\;R_{i\,5}/\gamma+[g_{6\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{5\,i}]\;R_{6\,j}/\gamma+[g_{\rm j6}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{\rm j5}]\;R_{i\,6}/\gamma+\;[g_{5\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{6\,i}]\;[g_{5\,j}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{6\,j}]\;R_{55}/\gamma^{2}+\;[g_{6\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{5\,i}]\;[g_{6\,j}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{5\,j}]\;R_{66}/\gamma^{2}+\;[g_{5\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{6\,i}][\big(g_{6\,j}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{5\,j}]\;R_{56}/\gamma^{2}+\;[g_{\rm j5}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{\rm j6}][g_{\rm i6}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{\rm i5}]\;R_{65}/\gamma^{2}$.

Varying by $g^{5\,i}$ and $g^{6\,i}$ we find:

24) $R_{5\,i}+[g_{5\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{6\,i}]\;R_{55}/\gamma+\;[g_{6\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{5\,i}]\;R_{56}/\gamma$

25) $R_{6\,i}+[g_{6\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{5\,i}]\;R_{66}/\gamma+\;[g_{5\,i}+\frac{2\,c(\boldsymbol{x},t)}{1+c(\boldsymbol{x},t)^{2}}g_{6\,i}]\;R_{56}/\gamma$.

Expression 23) is valid only if *c(**x**, t)* is constant. Otherwise it becomes more complicated. When currents exist expressions 24) and 25) are set to $\frac{J_i}{2\,\epsilon_0}$ and $\frac{\tilde{J}_i}{2\,\epsilon_0}$ respectively. (We can consider the simple example of a 'red' Reissner-Nordstrom metric having *c(**x**, t)* = A_G = 0 and $A_{\rm R4}=\frac{1}{4\,\pi\,\epsilon_0\,r}$. We see that all of the above three expressions vanish. This is as it should be and provides a bit of a "reality-check" on our mathematics.)

The complicated-looking expression 23) can be rewritten as:

23') $\mathfrak{R}_{\rm ij}$ - $\frac{1}{2}$ $\mathfrak{g}_{\rm ij}$ $\mathfrak{R}$ - 8 π G $\mathfrak{O}_{\rm ij}$ where the latter term represents all the contributions from A_R, A_G, and *c(**x**, t)*. We interpret this as the stress-energy tensor of the EM fields(s) and *c(**x**, t)* in the base space. We can then write:

26) $\mathfrak{G}_{\rm ij}$ = 8 π G ($\mathfrak{O}_{\rm ij}$ + $\mathfrak{T}_{\rm ij}$) where $\mathfrak{T}_{\rm ij}$ is the stress-energy tensor corresponding to any matter fields that may be present.

The Lagrangian density must also be varied by $c(\mathbf{x}, t)$. Setting the result to zero provides an equation of motion for $c(\mathbf{x}, t)$. Since R is (formally) independent of $c(\mathbf{x}, t)$ our work becomes somewhat easier. Suppose $A_R = A_G = 0$ and that we are in Minkowski space. $R = 0$ and there is no restriction on the behavior of $c(\mathbf{x}, t)$. Of course, our actual base space has a complicated geometry; $\mathcal{R} \neq 0$ in many places. Where this condition exists, and remembering that the Lagrangian density contains the term $\left(1 + c(\mathbf{x}, t)^2\right) / \left(1 - c(\mathbf{x}, t)^2\right)$, we find:

27) $c(\mathbf{x}, t) = 0$ or $\pm\, \infty$

The latter solutions are no cause for concern. They simply represent a situation in which we have exchanged the names of A_R and A_G. Our solutions are not very interesting, however. But we must not forget Equation 3). From it comes 'source terms' for $c(\mathbf{x}, t)$. The mathematics quickly becomes difficult. There are, however, a few very simple cases that yield results. Suppose there were a small sphere of uniform 'red' charge density, ρ_R, and a similar green one with ρ_G, both centered at the origin ($\rho_R \neq \pm\rho_G$). Suppose the spheres are so small that $c(\mathbf{x}, t)$ within them can be treated as a constant. The quantity to be varied by $c(\mathbf{x}, t)$ is proportional to $\left(\rho_R^2 + \rho_G^2\right) + 4\, \rho_R\, \rho_G\, c(0, t) \Big/ \left(1 + c(0, t)^2\right)$ X $\sqrt{-\det[g]}$ since the electric fields depend only on the value of $c(\mathbf{x}, t)$ inside the spheres. We find $c(\mathbf{x}, t) = -\rho_R/\rho_G$ or $-\rho_G/\rho_R$ inside the spheres. The latter is, actually, the same solution with the names of A_R and A_G interchanged. Outside we, again, find Equation 27). If $\rho_R = \rho_G$ and $c(\mathbf{x}, t) = -1$ inside the spheres there are no electric fields anywhere. Outside, $c(\mathbf{x}, t)$ is unrestricted.

Dark Matter?

Let us assume, for simplicity, that $A_R = A_G = 0$ and $c(\mathbf{x}, t)$ is negligible. Suppose we can write the matter Lagrangian as $L_R + L_G$. The physics would derive equally from both the 'red' and 'green' worlds according to $\mathcal{G}_{ij} = 8\, \pi\, G\, (\mathcal{T}_{Rij} + \mathcal{T}_{Gij})$. Now the amount of dark matter that seems to be present exceeds the obvious matter by at least an order of magnitude. We could explain this by saying that the 'green' universe contained quite a bit of matter. We could, equally well, suppose that there are multiple other universes, each similar to our own. We can readily incorporate other 'colored' worlds into our theory (although the algebra becomes more tedious). (See Supplementary Material.) If there were 'red', 'green', and 'blue' realities we would require three $c(\mathbf{x}, t)$s, and more if there were additional ones. These differently colored particles would share many of the attributes of WIMPS as far as we were concerned.

The distribution of these types of matter would depend on conditions existing at the Big Bang. If we suppose that the 'red' matter originally existed as localized concentrations an interesting situation might arise. These concentrations would rapidly condense into 'red' galaxies. Suppose the other kind(s) of matter began very uniformly distributed. This matter would be drawn towards any 'red' galaxies in its vicinity. It might well remain too uniform and diaphanous to support star formation. But, as it collapsed, its pressure would increase. A stable state would result when it satisfied the Lane-Emden equation. We would end up with a dark matter halo. Indeed, galaxy rotation curves have been interpreted as suggesting that something like this may, actually, be the case (7, 8, 9, 10). Since particles can feel pressure only from others of their own color, a "multicolored" halo would be smaller and denser than an otherwise similar one consisting of a single color. Given the average mass of a 'red' galaxy, and its average distance from its nearest neighbors, we can (very roughly) estimate that the process of coalescence would require no more than about a billion years, probably considerably less. This is not cosmologically unreasonable.

If a roughly stellar-mass (or somewhat larger) gas cloud consisting of variously colored particles were to collapse fusion would be an inefficient process – nuclei can only fuse with others of their own color. The result might be an anomalously hot and dense star or large Jupiter-like planet. If fusion were unable to arrest the collapse, and the cloud sufficiently massive, the result would be a black hole. Such objects would not be easy to detect and could, in fact, be fairly common within galaxies (11). We also note that black holes are expected to evaporate through Hawking radiation. The process would be the same for the production of 'red' Hawking radiation, 'green' radiation, etc. If there were N other realities black holes would evaporate N + 1 times faster than Hawking predicts (ignoring the contribution from any gravitons that may be radiated since these come in only one color).

We note that Foot has already come a long way toward demonstrating that his theory is able to explain the cosmological data. This encourages the hope that the present idea, which (depending on how it is construed) would afford a similar cosmological phenomenology, might also accommodate the data.

Conclusion.

This theory is not unique in proposing the existence of other realities. It is, however, rather unusual in that it provides a mechanism whereby two or more realities could actually interact in such a manner that none would see any fatal disruption to its own physics but might, on occasion, encounter intrusions from the other(s). Now $c(\boldsymbol{x}, t)$ does not seem to get very large in very many places very often. But it could do so, here and there occasionally, and go pretty-much unnoticed. And it has never been looked for at all.

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Supplementary Material.

1) Below we construct the ansatz for a 6 X 6 metric where c (x, t) varies. CRG is what we have called c (x, t) and we will allow it to vary with x1. We could vary it otherwise but this is just a "simple" example. We omit the spatial components of $A_{R,G}$ to keep the arithmetic to a minimum. We really should include curvature of the base space as well. We will, however, not do this. We start with the contravariant metric where CRG is zero :

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\mathrm{AR}[x1]/\gamma & -\mathrm{AG}[x1]/\gamma \\ 0 & 0 & 0 & -\mathrm{AR}[x1]/\gamma & -1/\gamma + \mathrm{AR}[x1]\text{^}2/\gamma\text{^}2 & \mathrm{AR}[x1]\,\mathrm{AG}[x1]/\gamma\text{^}2 \\ 0 & 0 & 0 & -\mathrm{AG}[x1]/\gamma & \mathrm{AR}[x1]\,\mathrm{AG}[x1]/\gamma\text{^}2 & -1/\gamma + \mathrm{AG}[x1]\text{^}2/\gamma\text{^}2 \end{pmatrix}$$

$$\Big\{\{-1, 0, 0, 0, 0, 0\}, \{0, -1, 0, 0, 0, 0\}, \{0, 0, -1, 0, 0, 0\}, \Big\{0, 0, 0, 1, -\frac{\mathrm{AR}[x1]}{\gamma}, -\frac{\mathrm{AG}[x1]}{\gamma}\Big\}, \Big\{0, 0, 0, -\frac{\mathrm{AR}[x1]}{\gamma}, -\frac{1}{\gamma} + \frac{\mathrm{AR}[x1]^2}{\gamma^2}, \frac{\mathrm{AG}[x1]\,\mathrm{AR}[x1]}{\gamma^2}\Big\}, \Big\{0, 0, 0, -\frac{\mathrm{AG}[x1]}{\gamma}, \frac{\mathrm{AG}[x1]\,\mathrm{AR}[x1]}{\gamma^2}, -\frac{1}{\gamma} + \frac{\mathrm{AG}[x1]^2}{\gamma^2}\Big\}\Big\}$$

Here is its covariant counterpart.

```
Inverse[%] // Simplify
```

$$\Big\{\{-1, 0, 0, 0, 0, 0\}, \{0, -1, 0, 0, 0, 0\}, \{0, 0, -1, 0, 0, 0\}, \Big\{0, 0, 0, -\frac{-\gamma + \mathrm{AG}[x1]^2 + \mathrm{AR}[x1]^2}{\gamma}, -\mathrm{AR}[x1], -\mathrm{AG}[x1]\Big\}, \{0, 0, 0, -\mathrm{AR}[x1], -\gamma, 0\}, \{0, 0, 0, -\mathrm{AG}[x1], 0, -\gamma\}\Big\}$$

We recognize that Equation 2) actually represents a rotation in the internal space :

```
Xo = {x1, x2, x3, x4, x5, x6}
```

$$Xn = \left\{x1,\ x2,\ x3,\ x4,\ \frac{x5 + x6\,CRG[x1]}{\sqrt{1 + CRG[x1]^2}},\ \frac{x6 + x5\,CRG[x1]}{\sqrt{1 + CRG[x1]^2}}\right\}$$

Below is the new contravariant metric.

$$Table\left[\sum_{m=1}^{6}\sum_{n=1}^{6}(\partial_{Xo[[m]]}\,Xn[[i]])\,(\partial_{Xo[[n]]}\,Xn[[j]])\,\%1[[m,n]],\ \{i,1,6\},\ \{j,1,6\}\right]\ //\ FullSimplify$$

$$\Big\{\Big\{-1,\ 0,\ 0,\ 0,\ \frac{(-x6 + x5\,CRG[x1])\,CRG'[x1]}{\left(1 + CRG[x1]^2\right)^{3/2}},\ \frac{(-x5 + x6\,CRG[x1])\,CRG'[x1]}{\left(1 + CRG[x1]^2\right)^{3/2}}\Big\},\ \{0,\ -1,\ 0,\ 0,\ 0,\ 0\},$$

$$\{0,\ 0,\ -1,\ 0,\ 0,\ 0\},\ \Big\{0,\ 0,\ 0,\ 1,\ -\frac{AR[x1] + AG[x1]\,CRG[x1]}{\gamma\sqrt{1 + CRG[x1]^2}},\ -\frac{AG[x1] + AR[x1]\,CRG[x1]}{\gamma\sqrt{1 + CRG[x1]^2}}\Big\},$$

$$\Big\{\frac{(-x6 + x5\,CRG[x1])\,CRG'[x1]}{\left(1 + CRG[x1]^2\right)^{3/2}},\ 0,\ 0,\ -\frac{AR[x1] + AG[x1]\,CRG[x1]}{\gamma\sqrt{1 + CRG[x1]^2}},\ -\frac{1}{\gamma^2\left(1 + CRG[x1]^2\right)^3}$$

$$\Big(\left(1 + CRG[x1]^2\right)^2\left(\gamma - AR[x1]^2 - 2\,AG[x1]\,AR[x1]\,CRG[x1] + \left(\gamma - AG[x1]^2\right)CRG[x1]^2\right) +$$

$$\gamma^2\,(x6 - x5\,CRG[x1])^2\,CRG'[x1]^2\Big),\ \frac{1}{\gamma^2\left(1 + CRG[x1]^2\right)^3}$$

$$\Big(\left(1 + CRG[x1]^2\right)^2\,(-2\,\gamma\,CRG[x1] + (AR[x1] + AG[x1]\,CRG[x1])\,(AG[x1] + AR[x1]\,CRG[x1])) +$$

$$\gamma^2\,(x6 - x5\,CRG[x1])\,(-x5 + x6\,CRG[x1])\,CRG'[x1]^2\Big)\Big\},$$

$$\Big\{\frac{(-x5 + x6\,CRG[x1])\,CRG'[x1]}{\left(1 + CRG[x1]^2\right)^{3/2}},\ 0,\ 0,\ -\frac{AG[x1] + AR[x1]\,CRG[x1]}{\gamma\sqrt{1 + CRG[x1]^2}},\ \frac{1}{\gamma^2\left(1 + CRG[x1]^2\right)^3}$$

$$\Big(\left(1 + CRG[x1]^2\right)^2\,(-2\,\gamma\,CRG[x1] + (AR[x1] + AG[x1]\,CRG[x1])\,(AG[x1] + AR[x1]\,CRG[x1])) +$$

$$\gamma^2\,(x6 - x5\,CRG[x1])\,(-x5 + x6\,CRG[x1])\,CRG'[x1]^2\Big),\ -\frac{1}{\gamma^2\left(1 + CRG[x1]^2\right)^3}$$

$$\Big(\left(1 + CRG[x1]^2\right)^2\left(\gamma - AG[x1]^2 - 2\,AG[x1]\,AR[x1]\,CRG[x1] + \left(\gamma - AR[x1]^2\right)CRG[x1]^2\right) +$$

$$\gamma^2\,(x5 - x6\,CRG[x1])^2\,CRG'[x1]^2\Big)\Big\}\Big\}$$

The reader may be alarmed to see x5 and x6 appear. This is, however, perfectly normal. They will drop out of our physical calculations later.

Inverse[%] // FullSimplify;

We have to re – express x5 and x6 in their rotated form

$$Solve\left[\left\{\frac{x5 + x6\,CRG[x1]}{\sqrt{1 + CRG[x1]^2}} == xn5,\ \frac{x6 + x5\,CRG[x1]}{\sqrt{1 + CRG[x1]^2}} == xn6\right\},\ \{x5,\ x6\}\right]$$

$$\Big\{\Big\{x5 \to -\frac{xn5\sqrt{1 + CRG[x1]^2} - xn6\,CRG[x1]\sqrt{1 + CRG[x1]^2}}{-1 + CRG[x1]^2},$$

$$x6 \to -\frac{xn6\sqrt{1 + CRG[x1]^2} - xn5\,CRG[x1]\sqrt{1 + CRG[x1]^2}}{-1 + CRG[x1]^2}\Big\}\Big\}$$

```
%% /. %[[1]] // FullSimplify;
```

Below we write $g_{\mu\nu}$:

```
%8 /. xn5 → x5 /. xn6 → x6
```

$$\Big\{\Big\{-1-\frac{1}{\left(-1+\mathrm{CRG}[x1]^2\right)^4\left(1+\mathrm{CRG}[x1]^2\right)}\gamma\left(x5^2+x6^2-12\,x5\,x6\,\mathrm{CRG}[x1]+15\left(x5^2+x6^2\right)\mathrm{CRG}[x1]^2-\right.$$

$$\left.40\,x5\,x6\,\mathrm{CRG}[x1]^3+15\left(x5^2+x6^2\right)\mathrm{CRG}[x1]^4-12\,x5\,x6\,\mathrm{CRG}[x1]^5+\left(x5^2+x6^2\right)\mathrm{CRG}[x1]^6\right)\mathrm{CRG}'[x1]^2,$$

$$0,\ 0,\ \Big(\Big(\mathrm{AR}[x1]\left(x6-3\,x5\,\mathrm{CRG}[x1]+3\,x6\,\mathrm{CRG}[x1]^2-x5\,\mathrm{CRG}[x1]^3\right)+$$

$$\mathrm{AG}[x1]\left(x5-3\,x6\,\mathrm{CRG}[x1]+3\,x5\,\mathrm{CRG}[x1]^2-x6\,\mathrm{CRG}[x1]^3\right)\Big)$$

$$\mathrm{CRG}'[x1]\Big)\Big/\left(\left(-1+\mathrm{CRG}[x1]^2\right)^2\sqrt{1+\mathrm{CRG}[x1]^2}\right),$$

$$-\frac{\gamma\left(x6-4\,x5\,\mathrm{CRG}[x1]+6\,x6\,\mathrm{CRG}[x1]^2-4\,x5\,\mathrm{CRG}[x1]^3+x6\,\mathrm{CRG}[x1]^4\right)\mathrm{CRG}'[x1]}{\left(-1+\mathrm{CRG}[x1]^2\right)^3},$$

$$-\frac{\gamma\left(x5-4\,x6\,\mathrm{CRG}[x1]+6\,x5\,\mathrm{CRG}[x1]^2-4\,x6\,\mathrm{CRG}[x1]^3+x5\,\mathrm{CRG}[x1]^4\right)\mathrm{CRG}'[x1]}{\left(-1+\mathrm{CRG}[x1]^2\right)^3}\Big\},$$

$$\{0,-1,0,0,0,0\},\ \{0,0,-1,0,0,0\},$$

$$\Big\{\Big(\Big(\mathrm{AR}[x1]\left(x6-3\,x5\,\mathrm{CRG}[x1]+3\,x6\,\mathrm{CRG}[x1]^2-x5\,\mathrm{CRG}[x1]^3\right)+$$

$$\mathrm{AG}[x1]\left(x5-3\,x6\,\mathrm{CRG}[x1]+3\,x5\,\mathrm{CRG}[x1]^2-x6\,\mathrm{CRG}[x1]^3\right)\Big)\mathrm{CRG}'[x1]\Big)\Big/$$

$$\left(\left(-1+\mathrm{CRG}[x1]^2\right)^2\sqrt{1+\mathrm{CRG}[x1]^2}\right),\ 0,\ 0,\ -\frac{-\gamma+\mathrm{AG}[x1]^2+\mathrm{AR}[x1]^2}{\gamma},$$

$$\frac{\left(\mathrm{AR}[x1]-\mathrm{AG}[x1]\,\mathrm{CRG}[x1]\right)\sqrt{1+\mathrm{CRG}[x1]^2}}{-1+\mathrm{CRG}[x1]^2},\ \frac{\left(\mathrm{AG}[x1]-\mathrm{AR}[x1]\,\mathrm{CRG}[x1]\right)\sqrt{1+\mathrm{CRG}[x1]^2}}{-1+\mathrm{CRG}[x1]^2}\Big\},$$

$$\Big\{-\frac{\gamma\left(x6-4\,x5\,\mathrm{CRG}[x1]+6\,x6\,\mathrm{CRG}[x1]^2-4\,x5\,\mathrm{CRG}[x1]^3+x6\,\mathrm{CRG}[x1]^4\right)\mathrm{CRG}'[x1]}{\left(-1+\mathrm{CRG}[x1]^2\right)^3},$$

$$0,\ 0,\ \frac{\left(\mathrm{AR}[x1]-\mathrm{AG}[x1]\,\mathrm{CRG}[x1]\right)\sqrt{1+\mathrm{CRG}[x1]^2}}{-1+\mathrm{CRG}[x1]^2},$$

$$-\frac{\gamma\left(1+\mathrm{CRG}[x1]^2\right)^2}{\left(-1+\mathrm{CRG}[x1]^2\right)^2},\ \frac{2\,\gamma\,\mathrm{CRG}[x1]\left(1+\mathrm{CRG}[x1]^2\right)}{\left(-1+\mathrm{CRG}[x1]^2\right)^2}\Big\},$$

$$\Big\{-\frac{\gamma\left(x5-4\,x6\,\mathrm{CRG}[x1]+6\,x5\,\mathrm{CRG}[x1]^2-4\,x6\,\mathrm{CRG}[x1]^3+x5\,\mathrm{CRG}[x1]^4\right)\mathrm{CRG}'[x1]}{\left(-1+\mathrm{CRG}[x1]^2\right)^3},$$

$$0,\ 0,\ \frac{\left(\mathrm{AG}[x1]-\mathrm{AR}[x1]\,\mathrm{CRG}[x1]\right)\sqrt{1+\mathrm{CRG}[x1]^2}}{-1+\mathrm{CRG}[x1]^2},$$

$$\frac{2\,\gamma\,\mathrm{CRG}[x1]\left(1+\mathrm{CRG}[x1]^2\right)}{\left(-1+\mathrm{CRG}[x1]^2\right)^2},\ -\frac{\gamma\left(1+\mathrm{CRG}[x1]^2\right)^2}{\left(-1+\mathrm{CRG}[x1]^2\right)^2}\Big\}\Big\}$$

We can verify that our metric gives us what we desire :

```
g = %;
X = {x1, x2, x3, x4, x5, x6};
e = Simplify[Inverse[g]];
```

$$c = \frac{1}{2}\text{Table}\left[\partial_{X[[j]]} g[[i, k]] + \partial_{X[[i]]} g[[j, k]] - \partial_{X[[k]]} g[[i, j]], \{i, 1, 6\}, \{j, 1, 6\}, \{k, 1, 6\}\right];$$

$$d = \text{Table}\left[\sum_{l=1}^{6} c[[i, j, l]]\, e[[k, l]], \{i, 1, 6\}, \{j, 1, 6\}, \{k, 1, 6\}\right];$$

$$\text{Ricci} = \text{Table}\Big[\sum_{a=1}^{6}\left(\partial_{X[[a]]} d[[i, j, a]] - \partial_{X[[j]]} d[[i, a, a]]\right) + \sum_{a=1}^{6}\sum_{b=1}^{6}(d[[b, a, a]]\, d[[i, j, b]] - d[[b, j, a]]\, d[[i, a, b]]), \{i, 1, 6\}, \{j, 1, 6\}\Big];$$

$$R = \text{Simplify}\left[\sum_{i=1}^{6}\sum_{j=1}^{6} \text{Ricci}[[i, j]]\, e[[i, j]]\right];$$

We see that R is exactly what we want. x5 and x6 have disappeared.

```
R
```

$$-\frac{AG'[x1]^2 + AR'[x1]^2}{2\gamma}$$

```
Det[g] // Simplify
```

$$-\frac{\gamma^2\left(1 + CRG[x1]^2\right)^2}{\left(-1 + CRG[x1]^2\right)^2}$$

2) Below we write the Lagrangian where we have three colors (' red, ' ' green, ' and ' blue'). CRG, CRB, and CGB mix the three vector potentials.

$$L_{em} = \overline{\psi_R}\left[\gamma^\mu\left[i\,\partial_\mu - e\left(1 + \text{CRG}^2 + \text{CRB}^2\right)^{-1/2}\left[A_{R\mu} + \text{CRG}\,A_{G\mu} + \text{CRB}\,A_{B\mu}\right]\right] - m\right]\psi_R - \frac{1}{4}F_R{}^{\mu\nu}F_{R\,\mu\nu}$$
$$+ \overline{\psi_G}\left[\gamma^\mu\left[i\,\partial_\mu - e\left(1 + \text{CRG}^2 + \text{CGB}^2\right)^{-1/2}\left[A_{G\mu} + \text{CRG}\,A_{R\mu} + \text{CGB}\,A_{B\mu}\right]\right] - m\right]\psi_G - \frac{1}{4}F_G{}^{\mu\nu}F_{G\,\mu\nu}$$
$$+ \overline{\psi_B}\left[\gamma^\mu\left[i\,\partial_\mu - e\left(1 + \text{CGB}^2 + \text{CRB}^2\right)^{-1/2}\left[A_{B\mu} + \text{CRB}\,A_{R\mu} + \text{CGB}\,A_{G\mu}\right]\right] - m\right]\psi_B - \frac{1}{4}F_B{}^{\mu\nu}F_{B\,\mu\nu}.$$

Below we write the 7 X 7 $g_{\mu\nu}$.

$$\Big\{\{-1, 0, 0, 0, 0, 0, 0\}, \{0, -1, 0, 0, 0, 0, 0\}, \{0, 0, -1, 0, 0, 0, 0\}, \Big\{0, 0, 0,$$
$$-\frac{AB^2 + AG^2 + AR^2 - \gamma}{\gamma}, \frac{\sqrt{1 + CRB^2 + CRG^2}\,\left(AR - AR\,CGB^2 + AG\,CGB\,CRB - AG\,CRG + AB\,(-CRB + CGB\,CRG)\right)}{-1 + CGB^2 + CRB^2 - 2\,CGB\,CRB\,CRG + CRG^2},$$
$$\frac{\sqrt{1 + CGB^2 + CRG^2}\,\left(AG + AR\,CGB\,CRB - AG\,CRB^2 - AR\,CRG + AB\,(-CGB + CRB\,CRG)\right)}{-1 + CGB^2 + CRB^2 - 2\,CGB\,CRB\,CRG + CRG^2},$$
$$\frac{\sqrt{1 + CGB^2 + CRB^2}\,\left(AB - AG\,CGB - AR\,CRB + AR\,CGB\,CRG + AG\,CRB\,CRG - AB\,CRG^2\right)}{-1 + CGB^2 + CRB^2 - 2\,CGB\,CRB\,CRG + CRG^2}\Big\},$$

$$\left\{0,\,0,\,0,\,\frac{\sqrt{1+CRB^2+CRG^2}\,\left(AR-AR\,CGB^2+AG\,CGB\,CRB-AG\,CRG+AB\,(-CRB+CGB\,CRG)\right)}{-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2},\right.$$
$$-\frac{\left(1+CRB^2+CRG^2\right)\left(1+CGB^4+CRB^2-4\,CGB\,CRB\,CRG+CRG^2+CGB^2\left(-2+CRB^2+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2},$$
$$\frac{\sqrt{1+CGB^2+CRG^2}\,\sqrt{1+CRB^2+CRG^2}\,\left(CGB^3\,CRB+2\,CRG+CGB\,CRB\left(-3+CRB^2-CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2},$$
$$\left.\frac{\sqrt{1+CGB^2+CRB^2}\,\sqrt{1+CRB^2+CRG^2}\,\left(2\,CRB-CGB\,CRB^2\,CRG+CGB\,CRG\left(-3+CGB^2+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2}\right\},$$
$$\left\{0,\,0,\,0,\,\frac{\sqrt{1+CGB^2+CRG^2}\,\left(AG+AR\,CGB\,CRB-AG\,CRB^2-AR\,CRG+AB\,(-CGB+CRB\,CRG)\right)}{-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2},\right.$$
$$\frac{\sqrt{1+CGB^2+CRG^2}\,\sqrt{1+CRB^2+CRG^2}\,\left(CGB^3\,CRB+2\,CRG+CGB\,CRB\left(-3+CRB^2-CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2},$$
$$-\frac{\left(1+CGB^2+CRG^2\right)\left(1+CRB^4+CGB^2\left(1+CRB^2\right)-4\,CGB\,CRB\,CRG+CRG^2+CRB^2\left(-2+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2},$$
$$\left.-\frac{\sqrt{1+CGB^2+CRB^2}\,\sqrt{1+CGB^2+CRG^2}\,\left(-2\,CGB+CGB^2\,CRB\,CRG-CRB\,CRG\left(-3+CRB^2+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2}\right\},$$
$$\left\{0,\,0,\,0,\,\frac{\sqrt{1+CGB^2+CRB^2}\,\left(AB-AG\,CGB-AR\,CRB+AR\,CGB\,CRG+AG\,CRB\,CRG-AB\,CRG^2\right)}{-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2},\right.$$
$$\frac{\sqrt{1+CGB^2+CRB^2}\,\sqrt{1+CRB^2+CRG^2}\,\left(2\,CRB-CGB\,CRB^2\,CRG+CGB\,CRG\left(-3+CGB^2+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2},$$
$$-\frac{\sqrt{1+CGB^2+CRB^2}\,\sqrt{1+CGB^2+CRG^2}\,\left(-2\,CGB+CGB^2\,CRB\,CRG-CRB\,CRG\left(-3+CRB^2+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2},$$
$$\left.\left.-\frac{\left(1+CGB^2+CRB^2\right)\left(-4\,CGB\,CRB\,CRG+\left(-1+CRG^2\right)^2+CGB^2\left(1+CRG^2\right)+CRB^2\left(1+CRG^2\right)\right)\gamma}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2}\right\}\right\}$$

```
Det[%] // Simplify
```

$$\frac{\left(1+CGB^2+CRB^2\right)\left(1+CGB^2+CRG^2\right)\left(1+CRB^2+CRG^2\right)\gamma^3}{\left(-1+CGB^2+CRB^2-2\,CGB\,CRB\,CRG+CRG^2\right)^2}$$