ON TRYING TO MODIFY EINSTEIN FIELD EQUATIONS HYPOTHESIS

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ABSTRACT. I showed that there is a way of quantizing Einstein field equations by using complex space-time vector field turning into real fields.

CONTENTS

1. Complex space-time

Conflict between General Relativity and quantum physics is one of most important unsolved problems in theoretical physics. In this hypothesis I will try to show how to quantize it by starting with simple assumption, I start from complex space-time [1] tensor that I will denote $(\psi\psi^*)^\rho_\lambda$ that is mixed tensor field that takes two vectors of complex space-time , one normal complex vector and second it's complex conjugate and creates out of it a real mixed tensor. First I need to impose that field is symmetric tensor field:

$$(\psi\psi^*)^\rho_\sigma = (\psi\psi^*)^\sigma_\rho \tag{1.1}$$

From fact it's a complex field that uses $SU(4)$ matrix as rotation times it's complex conjugate I can rotate that field by this matrix and still get a valid solutions:

$$U^\rho_\kappa \left(U^\dagger\right)^\phi_\sigma (\psi\psi^*)^\rho_\sigma = (\psi\psi^*)^\sigma_\rho \tag{1.2}$$

Next there is a need for those equations to transform for flat space-time by Lorentz transformations [2], to preserve space-time interval for any observer:

$$\Lambda^{\rho'}_\rho \Lambda^\sigma_{\sigma'} (\psi\psi^*)^\rho_\sigma = (\psi\psi^*)^{\sigma'}_{\rho'} \tag{1.3}$$

In flat space-time I can calculate probability wave function of this field by taking how it changes in all directions and setting probability over whole space-time to one:

$$\psi^2(x) = \eta^{\sigma\kappa}\partial_\kappa\partial_\rho (\psi\psi^*)^\rho_\sigma \tag{1.4}$$

$$\int_{\mathcal{V}} \psi^2(x)\, d^4x = \int_{\mathcal{V}} \eta^{\sigma\kappa}\partial_\kappa\partial_\rho (\psi\psi^*)^\rho_\sigma\, d^4x = 1 \tag{1.5}$$

Combining Lorentz Transformation and $SU(4)$ matrix transformation [3][4][5] I will get complex picture of that field for flat space-time case, where I define space-time interval [6][7] as probability of each path distance in that space-time:

$$d\psi^2(x)\, ds^2(x) = d\psi^2(x)\, \eta_{\mu\nu} dx^\mu dx^\nu = \eta^{\sigma\kappa}\partial_\kappa\partial_\rho d (\psi\psi^*)^\rho_\sigma \eta_{\mu\nu} dx^\mu dx^\nu \tag{1.6}$$

It means that at each point of space-time I have probability of finding particle, and it's corresponding space-time interval. I can use equation for proper time:

$$\psi^2(x)\, \tau^2 = \frac{1}{c^2}\int_P d\psi^2(x)\, \eta_{\mu\nu} dx^\mu dx^\nu \tag{1.7}$$

Where P is some some path in space-time.

2. Curvature of complex space-time

I can extend this idea to curved scape-time, first I write relation between curvature tensor [8] [9][10][11] and covariant derivative acting on real mixed tensor field:

$$R^{\sigma}_{\lambda\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\sigma}(\psi\psi^{*})^{\kappa}_{\phi} - R^{\alpha}_{\rho\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi}$$
$$= \nabla_{\mu}\nabla_{\nu}U^{\alpha}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\rho}(\psi\psi^{*})^{\kappa}_{\phi} - \nabla_{\nu}\nabla_{\mu}U^{\alpha}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\rho}(\psi\psi^{*})^{\kappa}_{\phi} = 0 \tag{2.1}$$

It comes from fact that field is symmetric so I can write that both parts are equal:

$$R^{\sigma}_{\lambda\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\sigma}(\psi\psi^{*})^{\kappa}_{\phi} = R^{\alpha}_{\rho\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} \tag{2.2}$$

Now I can do Einstein field equations of that mixed tensor field, I will arrive at two equations that are equal to themself one is changing mixed field into covariant vector field in final equation and another one to contravariant vector field:

$$R^{\alpha}_{\rho} = g^{\mu\nu}R^{\alpha}_{\rho\mu\nu} \tag{2.3}$$

$$R^{\sigma}_{\lambda} = g^{\mu\nu}R^{\sigma}_{\lambda\mu\nu} \tag{2.4}$$

$$R^{\alpha}_{\rho\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} - \frac{1}{2}R^{\alpha}_{\rho}g_{\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} = \kappa T^{\alpha}_{\rho\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} \tag{2.5}$$

$$R^{\alpha}_{\alpha\mu\nu}U^{\alpha}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} - \frac{1}{2}R^{\alpha}_{\alpha}g_{\mu\nu}U^{\alpha}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} = \kappa T^{\alpha}_{\alpha\mu\nu}U^{\alpha}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} \tag{2.6}$$

$$R_{\mu\nu}(\psi\psi^{*})_{\lambda} - \frac{1}{2}Rg_{\mu\nu}(\psi\psi^{*})_{\lambda} = \kappa T_{\mu\nu}(\psi\psi^{*})_{\lambda} \tag{2.7}$$

$$R^{\sigma}_{\lambda\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} - \frac{1}{2}R^{\sigma}_{\lambda}g_{\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} = \kappa T^{\sigma}_{\lambda\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\lambda}(\psi\psi^{*})^{\kappa}_{\phi} \tag{2.8}$$

$$R^{\sigma}_{\sigma\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\sigma}(\psi\psi^{*})^{\kappa}_{\phi} - \frac{1}{2}R^{\sigma}_{\sigma}g_{\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\sigma}(\psi\psi^{*})^{\kappa}_{\phi} = \kappa T^{\sigma}_{\sigma\mu\nu}U^{\rho}_{\kappa}\left(U^{\dagger}\right)^{\phi}_{\sigma}(\psi\psi^{*})^{\kappa}_{\phi} \tag{2.9}$$

$$R_{\mu\nu}(\psi\psi^{*})^{\rho} - \frac{1}{2}Rg_{\mu\nu}(\psi\psi^{*})^{\rho} = \kappa T_{\mu\nu}(\psi\psi^{*})^{\rho} \tag{2.10}$$

Those two equations that are final product so equation 2.7 and 2.10 are field equations for mixed real tensor field that comes from complex vector fields.

3. Energy tensor and probability for curved space-time

Energy tensor has to obey few simple rules, first it has to be symmetric with respect to indexes:

$$T^{\sigma}_{\lambda\mu\nu} = T^{\lambda}_{\sigma\mu\nu} = T^{\lambda}_{\sigma\nu\mu} = T^{\sigma}_{\lambda\nu\mu} \tag{3.1}$$

$$T^{\alpha}_{\rho\mu\nu} = T^{\rho}_{\alpha\mu\nu} = T^{\rho}_{\alpha\nu\mu} = T^{\alpha}_{\rho\nu\mu} \tag{3.2}$$

Then contraction of that tensor will give energy-stress tensor:

$$T^{\sigma}_{\sigma\mu\nu} = T_{\mu\nu} \tag{3.3}$$

From it follows that there is conservation of energy of that that tensor:

$$g^{\mu\kappa} g^{\nu\lambda} T^{\sigma}_{\sigma\kappa\lambda} = T^{\mu\nu} \tag{3.4}$$

$$\nabla_{\nu} g^{\mu\kappa} g^{\nu\lambda} T^{\sigma}_{\sigma\kappa\lambda} = \nabla_{\nu} T^{\mu\nu} = 0 \tag{3.5}$$

I created probability function for flat space-time now I can move to curved one, first I start with geodesic equation that gives each point of space-time so each geodesic a probability:

$$g^{\lambda\kappa} \nabla_{\kappa} \nabla_{\rho} \left(\psi\psi^{*}\right)^{\rho}_{\lambda} \left(\frac{d^2 x^{\mu}}{ds^2} + \Gamma^{\mu}_{\alpha\beta} \frac{dx^{\alpha}}{ds} \frac{dx^{\beta}}{ds} \right) = 0 \tag{3.6}$$

This probability has to be normalized but I change from normal derivative to covariant derivative, otherwise it's same equation as before:

$$\int_{\mathcal{V}} g^{\lambda\kappa} \nabla_{\kappa} \nabla_{\rho} \left(\psi\psi^{*}\right)^{\rho}_{\lambda} d^4 x = 1 \tag{3.7}$$

And finally I can move to space-time interval with probability for curved space-time. I write probability function:

$$\psi^2 \left(x\right) = g^{\lambda\kappa} \nabla_{\kappa} \nabla_{\rho} \left(\psi\psi^{*}\right)^{\rho}_{\lambda} \tag{3.8}$$

Then I use it in space-time interval and proper time equation:

$$d\psi^2 \left(x\right) ds^2 \left(x\right) = g^{\lambda\kappa} \nabla_{\kappa} \nabla_{\rho} d \left(\psi\psi^{*}\right)^{\rho}_{\lambda} g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{3.9}$$

$$\psi^2 \left(x\right) \tau^2 = \frac{1}{c^2} \int g^{\lambda\kappa} \nabla_{\kappa} \nabla_{\rho} d \left(\psi\psi^{*}\right)^{\rho}_{\lambda} g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{3.10}$$

References

[1] https://mathworld.wolfram.com/ComplexVectorSpace.html

[2] https://mathworld.wolfram.com/LorentzTransformation.html

[3] https://mathworld.wolfram.com/SpecialUnitaryGroup.html

[4] https://mathworld.wolfram.com/UnitaryMatrix.html

[5] https://mathworld.wolfram.com/SpecialUnitaryMatrix.html

[6] https://mathworld.wolfram.com/MinkowskiMetric.html

[7] https://mathworld.wolfram.com/MinkowskiSpace.html

[8] https://mathworld.wolfram.com/RiemannTensor.html

[9] https://mathworld.wolfram.com/RicciCurvatureTensor.html

[10] https://mathworld.wolfram.com/CovariantDerivative.html

[11] https://mathworld.wolfram.com/EinsteinFieldEquations.html