

# Dimensional complement of the mathematical solution to the cosmological constant problem.

Stéphane Wojnow  
[wojnow.stephane@gmail.com](mailto:wojnow.stephane@gmail.com)

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Abstract

*We have proposed a mathematical solution to the cosmological constant problem with an attempted physical explanation. Here we propose a complement to this solution to validate the hypothetical energy density value of the cosmological constant in quantum field theory (QFT), showing that the dimensional method used can be applied to find the critical energy density of the  $\Lambda$ CDM model.*

**Keywords:** Cosmology, Quantum Field Theory, Cosmological constant problem, Vacuum catastrophe, Cosmological constant, zero point energy, critical energy density,  $\Lambda$ CDM model.

## Introction .

This document presents a complement to the proposed mathematical solution to the cosmological constant problem, with the aim of validating the hypothetical energy density value of the cosmological constant in quantum field theory. The proposed method utilizes dimensional analysis to find the critical energy density of the  $\Lambda$ CDM model. The document provides a reminder of the mathematical solution and defines relevant parameters, including Planck mass, Planck length, and Hubble constant. It then presents the proposed formula for the quantum critical energy density of the universe and demonstrates how it can be used to calculate the critical energy density of the  $\Lambda$ CDM model.

## Reminder of the result of the mathematical solution to the problem of the cosmological constant.

Here we define parameters with  $m_p$  as Planck mass,  $l_p$  as Planck length,  $\hbar$  reduced Planck constant,  $c$  speed of light in vacuum,  $G$  as Newton's constant,  $\Lambda$  as cosmological constant,  $A$  as zero-point energy density in quantum field theory [1],  $B$  as vacuum energy density assumed for the cosmological constant in the QFT,  $H_0$  as Hubble constant, and  $\rho_c$  as critical energy density of the  $\Lambda$ CDM model.

The energy density of the quantum vacuum in Planck units, i.e. that of the zero point of the QFT is :

$$A = \frac{m_p c^2}{l_p^3} = \hbar (l_p^{-2})^2 c \quad (1)$$

$$A = \frac{c^7}{G^2 \hbar} \quad (2)$$

By dimensional analysis, we can propose this hypothetical quantum energy density of the cosmological constant in the QFT [2] :

$$B = \frac{1}{(8\pi)^2} \hbar (\Lambda_{m-2})^2 c \quad (3)$$

to demonstrate that the cosmological constant  $C$  in  $J/m^3$  is [2] :

$$C = \sqrt{\hbar (l_p^{-2})^2 c} \sqrt{\frac{1}{(8\pi)^2} \hbar (\Lambda_{m-2})^2 c} \quad (4)$$

$$C = \sqrt{A} \sqrt{B} \quad (5)$$

### Dimensional complement of the mathematical solution to the cosmological constant problem.

Let us consider  $H_0$  the Hubble parameter (or Hubble constant) of dimension  $[T^{-1}]$ .

We want a dimension in  $[L^{-2}]$  to replace  $\Lambda_{m-2}$  in Eq(3),

As  $c^2$  is used to convert  $\Lambda_{s-2}$  to  $\Lambda_{m-2}$  by writing

$$\Lambda_{m-2} = \frac{\Lambda_{s-2}}{c^2} \quad (6)$$

we will write

$$\frac{H_0^2}{c^2} \quad (7)$$

to write a formula  $B'$  as "quantum critical energy density of the universe for  $H_0$ " assumed in the QFT with Eq(7) of dimension  $[L^{-2}]$  :

$$B'' = \frac{3^2}{(8\pi)^2} \hbar \left(\frac{H_0^2}{c^2}\right)^2 c \quad (8)$$

$$B'' = \frac{9\hbar}{(8\pi)^2} \frac{H_0^4 c}{c^4} \quad (9)$$

Finally, consider the critical energy density of the  $\Lambda$ CDM model for  $H_0$  :

$$\rho_c = \frac{3 c^2 H_0^2}{8\pi G} \quad (10)$$

We have:

$$\rho_c = \sqrt{A} \sqrt{B'} \quad (11)$$

This can be proved using Eq(2) and Eq(9) :

$$AB' = \frac{c^7}{G^2 \hbar} \frac{9 \hbar H_0^4 c}{(8\pi)^2 c^4} \quad (12)$$

$$AB' = \frac{c^7}{G^2} \frac{9 H_0^4 c}{(8\pi)^2 c^4} \quad (13)$$

$$AB' = \frac{9 c^4 H_0^4}{(8\pi)^2 G^2} \quad (14)$$

$$\sqrt{A}\sqrt{B'} = \frac{3 c^2 H_0^2}{8\pi G} = \rho_c \quad (15)$$

Eq(15) is the definition of the critical energy density of the  $\Lambda$ CDM model for a flat universe, i.e. Eq(10).

## CONCLUSION

The same dimensional methodology, to assume on the one hand the hypothetical quantum energy density of the cosmological constant QFT, on the other hand the hypothetical quantum critical energy density of the QFT, allows to find their equations in the  $\Lambda$ CDM model via their geometric mean with the zero-point energy density. In addition to attempting to make physical sense of the square roots of the energy density as a Hidebrandt solubility parameter [2], the reproducibility of the method reciprocally strengthens both results obtained.

## REFERENCES

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