Dimensional complement of the mathematical solution to the cosmological constant problem.

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Abstract/Introduction

We have proposed a mathematical solution to the cosmological constant problem with an attempted physical explanation. Here we propose a complement to this solution to validate the hypothetical energy density value of the cosmological constant in quantum field theory (QFT), showing that the dimensional method used can be applied to find the critical energy density of the ΛCDM model.

Keywords: Cosmology, Quantum Field Theory, Cosmological constant problem, Vacuum catastrophe, Cosmological constant, zero point energy, critical energy density, ΛCDM model.

Reminder of the result of the mathematical solution to the problem of the cosmological constant.

Here we define parameters with m_p as Planck mass, l_p as Planck length, $\hbar$ reduced Planck constant, c speed of light in vacuum, G as Newton's constant, $\boldsymbol{\Lambda}$ as cosmological constant, $\boldsymbol{A}$ as zero-point energy density in quantum field theory [1], $\boldsymbol{B}$ cas vacuum energy density assumed for the cosmological constant in the QFT, H_0 as Hubble contant, and ρ_c as critical energy density of the ΛCDM model.

The energy density of the quantum vacuum in Planck units, i.e. that of the zero point of the QFT is :

$$A = \frac{m_p c^2}{l_p^3} = \hbar (l_p^{-2})^2 c \tag{1}$$

$$A = \frac{c^7}{G^2 \hbar} \tag{2}$$

By dimensional analysis, we can propose this hypothetical quantum energy density of the cosmological constant in the QFT [2] :

$$B = \frac{1}{(8\pi)^2} \hbar (\Lambda_{m^{-2}})^2 c \tag{3}$$

to demonstrate that the cosmological constant C in J/m³ is [2] :

$$C = \sqrt{\hbar (l_p^{-2})^2 c} \sqrt{\frac{1}{(8\pi)^2} \hbar (\Lambda_{m^{-2}})^2 c} \tag{4}$$

$$C = \sqrt{A}\sqrt{B} \tag{5}$$

Reinforcement of the mathematical solution to the cosmological constant problem.

Let us consider H_0 the Hubble parameter (or Hubble constant) of dimension [T^{-1}].

We want a dimension in [L^{-2}] to replace $\Lambda_{m^{-2}}$ in Eq(3),

As c^2 is used to convert $\Lambda_{s^{-2}}$ to $\Lambda_{m^{-2}}$ by writing

$$\Lambda_{m^{-2}} = \frac{\Lambda_{s^{-2}}}{c^2} \tag{6}$$

we will write

$$\frac{H_0^2}{c^2} \tag{7}$$

to write a formula ***B'*** as "quantum critical energy density of the universe for H_0" assumed in the QCT with Eq(7) of dimension [L^{-2}] :

$$B" = \frac{3^2}{(8\pi)^2}\hbar(\frac{H_0^2}{c^2})^2 c \tag{8}$$

Finally, consider the critical energy density of the ΛCDM model for H_0 :

$$\rho_c = \frac{3\ c^2 H_0^2}{8\pi G} \tag{9}$$

We have:

$$\rho_c = \sqrt{A}\sqrt{B'} \tag{10}$$

This can be proved using Eq(2) and Eq(8) :

$$AB' = \frac{c^7}{G^2\hbar}\frac{9\ \hbar H_0^4 c}{(8\pi)^2 c^4} \tag{11}$$

$$AB' = \frac{c^7}{G^2}\frac{9\ H_0^4 c}{(8\pi)^2 c^4} \tag{12}$$

$$AB' = \frac{9\ c^4 H_0^4}{(8\pi)^2 G^2} \tag{13}$$

$$\sqrt{A}\sqrt{B'} = \frac{3\ c^2 H_0^2}{8\pi G} = \rho_c \tag{14}$$

Eq(14) is the definition of the critical energy density of the ΛCDM model for a flat universe, i.e. Eq(9).

CONCLUSION

The same dimensional methodology, to assume on the one hand the hypothetical quantum energy density of the cosmological constant QFT, on the other hand the hypothetical quantum critical energy density of the QFT, allows to find their equations in the ΛCDM model via their geometric mean with the zero-point energy density. In addition to attempting to make physical sense of the square roots of the energy density as a Hidebrandt solubility parameter [2], the reproducibility of the method reciprocally strengthens both results obtained.

REFERENCES

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