Proof of Dubner's Conjecture

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Abstract: Dubner's conjecture was proposed by American electrical engineer and mathematical researcher Harvey Dubner (1928 – 2019) in 1996, that is, all sufficiently large even numbers (greater than 4208) are the sum of some two twin prime numbers.

Since there is no mathematical model for prime numbers to be completely accurate, prime numbers are randomly distributed on the number axis. The distribution of prime numbers in the latest research results is deterministic random distribution, so the problems related to prime numbers can be studied, analyzed and proved by using probability statistics methods.

In this paper, the Dubner's conjecture is proved by the method of probability and statistics.

Keywords: Dubner's conjecture, deterministic random distribution, probability statistics, twin Goldbach prime

Dubner's conjecture was proposed by American electrical engineer and mathematical researcher Harvey Dubner (1928 – 2019) in 1996, that is, all sufficiently large even numbers (greater than 4208) are the sum of some two twin prime numbers [1].

In this paper, the probability statistics method is used to prove the conjecture, and the method is introduced as follows:

1.The Proof Method of Dubner's Conjecture

Since there is no mathematical model for prime numbers to be fully accurate, prime numbers are randomly distributed on the number axis [2] [3]. The latest research results show that the distribution of prime numbers is deterministic random distribution, so the problems related to prime numbers can be studied, analyzed and proved by using probability statistics methods [4] [5] [6].

Let x be any sufficiently large even number. We define that when the even number x is expressed as the sum of two prime numbers, which are called the Goldbach prime number of the even number x. From this definition, we can see that Dubner prime number is both Goldbach prime number and twin prime number.

Because any even number can be decomposed into the sum of a larger prime number and a smaller odd number, where the larger prime number is greater than one half of the even number and the smaller odd number is less than one half of the even number.

Also, since the mantissa of any prime number greater than 5 cannot be the number 5, the mantissa of the corresponding smaller odd number can only be one of 1, 3, 7 and 9. According to the prime number theorem, the probability that the smaller odd number is a prime number is:

$$\frac{5}{2}\frac{1}{ln(x)}$$

And according to the prime number theorem, the number of the integers, which are greater than x/2 and less than x and are prime number, is

$$\frac{x}{ln(x)}-\frac{\frac{x}{2}}{ln(\frac{x}{2})}$$

Therefore, the number that the smaller odd number is prime number, that is, the sufficiently large even number x is expressed as the sum of two prime numbers, is

$$\frac{5}{2}\frac{1}{ln(x)}(\frac{x}{ln(x)}-\frac{\frac{x}{2}}{ln(\frac{x}{2})})$$

Simplified as:

$$\frac{5}{4}\frac{x}{ln(x)^2}(\frac{ln(x)-2ln(2)}{ln(x)-ln(2)})$$

When x increases, ln (x) - 2ln (2) and ln (x) - ln (2) approach the value of ln (x), which becomes

$$\frac{5}{4}\frac{x}{ln(x)^2}$$

Therefore, the probability that the integer less than an even number x is a Goldbach prime is

$$\frac{5}{4}\frac{1}{ln(x)^2}$$

Similarly, the probability that the integer less than an even number x is a twin prime number is [5]

$$\frac{2}{ln(x)^2}$$

Thus, it can be concluded that the probability that an integer less than the even number x is both a Goldbach prime number and a twin prime number is

$$\frac{5}{4}\frac{1}{ln(x)^2}\frac{2}{ln(x)^2}$$

Furthermore, the integer less than the even number x is both a pair of Goldbach primes and a pair of twin primes, that is, the number of Dubner's prime pairs is

$$\frac{x}{2}\frac{5}{4}\frac{1}{ln(x)^2}\frac{2}{ln(x)^2}$$

A discriminant function is obtained after simplification

$$\frac{5}{4}\frac{x}{ln(x)^4} \qquad (1)$$

Because the discriminant function (1) is similar to a curve with a slight radian, showing a divergent form, as shown in Figure 1 (when x is greater than 4208, the value of y is greater than 1.08), the calculated value gradually increases with the increase of even number x.

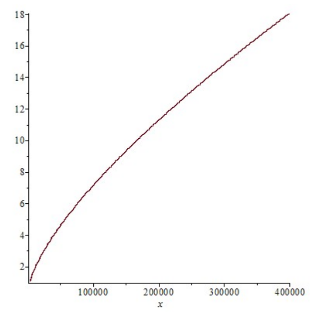

Fig. 1 Discriminant function.

The first derivative of discriminant function (1) is

$$\frac{5(\ln(x) - 4)}{4ln(x)^5} \tag{2}$$

It is always positive, and the x-axis is the asymptote, as shown in Figure 2. It can be seen that the discriminant function increases monotonously, taking infinity as the extreme point.
Similarly, the second derivative of discriminant function (1) is

$$-\frac{5(\ln(x) - 5)}{x\,ln(x)^6} \tag{3}$$

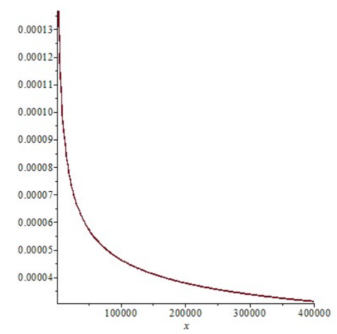

Fig. 2 First derivative of discriminant function.

It can be seen from Figure 3 that the second derivative is always negative, taking the X axis as the asymptote; The maximum value of the discriminant function is infinity.

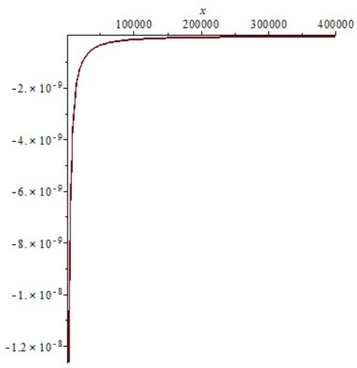

Fig. 3 Second derivative of discriminant function.

As the even value x increases, the discriminant function (1) increases at the same time, and is far greater than the value 1; Because the value calculated by the discriminant function (1) is not only the number of twin prime pairs, but also the number of even numbers x expressed as the sum of two prime numbers, it shows that a sufficiently large even number can be written as the sum of two twin prime numbers, which proves the Dubner's conjecture.

2.Conclusion

Any sufficiently large even number can be written as the sum of two twin prime numbers, which proves the Dubner's conjecture.

For a sufficiently large even number x, the number of Dubner prime pairs is about

$$\frac{5}{4}\frac{x}{ln(x)^4}$$

3.Corollaries

(1). The twin Goldbach prime theorem

All sufficiently large even numbers x (about more than 13,900,000) can be the sum of some two twin Goldbach prime numbers p, p+2, where p is Goldbach prime; the number of twin Goldbach prime pairs is about

$$\frac{25}{16}\frac{x}{ln(x)^6}$$

(2).Dubner's prime pair theorem

For all sufficiently large even numbers x (about greater than 124,000,000,000), and any natural number k, 1<k<<x/2, there exist Dubner prime pairs (p, p+2k), where p is Dubner prime; the number of Dubner prime pairs is about

$$\frac{25}{16}\frac{x}{ln(x)^8}\left(\frac{x}{2}-k\right)$$

(3). Goldbach's prime number theorem for quadruplets

For all sufficiently large even numbers x (about more than 29,000,000,000,000,000,000) there exists a quadruplet of Goldbach prime p, p+2, p+6, p+8, where p is the Goldbach prime;the number of groups of Goldbach prime quadruplets is about

$$\frac{625}{256}\frac{x}{ln(x)^{12}}$$

4.References

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