Precision measurement of the frequency drift of the binary star system HM Cancri

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The binary star system HMC (J0806.3+1527) emits gravitational waves near 6220.5 μHz, which can be detected with superconducting gravimeters. A newly developed method improves the signal-to-noise ratio of the signal so much that the second derivative of the orbital frequency can now also be precisely measured. This frequency evolution is needed to understand the transport of matter in this mysterious star system.

1 Introduction

Our galaxy hosts the double star system RX J0806.3+1527 with orbital period of 5.4 minutes [1]. The emitted continuous gravitational wave (GW) excites the Earth to forced oscillations which are clearly different from the natural resonances of the Earth: The frequency is very stable, has a much smaller half-width than the natural resonances of the Earth [2] and the average amplitude remains constant for years. As the Earth rotates around the sun and around its own axis, all GWs are phase modulated with the corresponding frequencies. A GW is also amplitude modulated because each antenna has some directivity. These different signatures are important criteria to identify GW signals in the records of gravimeters.

Although GWs were detected many years ago [3], only recently has it been possible to measure continuous GW and show that gravimeters are useful antennas in a broad range around 2 mHz [5]. They measure neither the diameter of the Earth nor its change, but the change in acceleration at the surface. The signals are evaluated using proven standard methods of communications engineering.

During the initial investigation of HMC [4], the GW could be successfully detected in the records of many individual gravimeters. Because of the poor S/N, the important parameter *frequency drift* could only be determined as a linear approximation. This is physically unsatisfactory because the enormous energy emission from the star system HMC suggests that the drift will change measurably over a ten-year observation period.

Now, it has been possible to use a new method to improve the S/N to such an extent that the change in drift can be measured. The key point is the phase-coherent addition of the available long-term records to improve the S/N. The procedure is based on the following consideration: Four low-noise gravimeters (BF1, BF2, MB and ST) are located – closely adjacent – in Central Europe and one on the opposite side of the Earth in Canberra/Australia. Since all gravimeters change their distance from the center of the earth synchronously, the amplitude of the GW signal must increase when adding the records of all accelerometers. Because the noise is asynchronous, the S/N improves.

2 Construction of the database

One problem has to be overcome: gravimeters are geophysical instruments and are calibrated at $f \approx 30$ μHz. Geophysicists are interested in the frequency range $f < 4$ mHz and dampen the higher frequencies differently: If you look at a narrow range around 6 mHz, the spectra of the records from MB and CB show amplitudes about ten times higher than the corresponding spectra from BF and ST . This also applies to the well-known natural resonances of the earth. Apparently, the data from the gravimeters are not filtered with standardized low-pass filters.

At this point it must be emphasized: All previous experiences confirm the impression that the inherent noise of the gravimeters is not an instrument error, but a consequence of the numerous GW that modulate the gravitational acceleration [6].

In order to level the different output amplitudes of the five low-noise gravimeters, the phase coherent sum $\Sigma = CB + MB + 5 \cdot (BF1 + BF2 + ST)$ is chosen as a reference. The application of the MSH method [4] confirms that this trick significantly improves the signal-to-noise ratio (S/N $\approx$ 20). To enable an estimation of the measurement errors, a database is constructed using a procedure that corresponds to the jackknife method of statistics [8]:

- A record, for example MB, is subtracted from Σ ("leave-one-out" estimate).
- For the remaining subsample, the parameters of the equation (2) are calculated using the MSH method. (This is the subject of the following section.)
- In order to broaden the database, you can also work with "half" records or add a record instead of subtracting it. There is plenty of room for experimentation.
- According to the rules of the Jackknife method, the individual results are combined to form an overall result.

The basis of the following investigation are the records of superconducting gravimeters from the period 2009-01-01 to 2018-12-31, which are stored at IGETS [9]. The conversion of the raw data from the gravimeters to long-term records is described in [4].

3 The compensation of the phase modulation

The records of the gravimeters are summed and run through an IQ bandpass filter which reduces the center frequency from 6220 μHz to 90 μHz. The bandwidth of 100 μHz is sufficient to pass the essential sideband frequencies of all suspected PMs. The subsequent decimation of the file length by a factor of 48 accelerates the data processing. The usual approach for a phase modulated oscillation with constant frequency is

$$y = sin(2\pi t \cdot f + \phi_{modulation}) \tag{1}$$

The two parameters f and $\phi_{modulation}$ in equation (1) must be adapted to the problem:

- The frequency of HMC is not constant, it increases over time. Because of the high S/N, it quickly becomes apparent that the drift does not increase linearly. The corresponding approach is therefore $f = f_{GW} + \dot{f}t + 0.5\ddot{f}t^2 + 0.167\dddot{f}t^3$. f_{GW} is the frequency of HMC at the beginning of the analysis period (2009-01-01).

- The phase ϕ_{year} indicates the position of the Earth on its way around the sun. We assume a circular orbit $\phi_{year} = \eta_{year} \cdot sin(2\pi t \cdot f_{year} + p_{year})$.

- The PM in 24-hour or 12-hour cycles is

  $\phi_{day} = \eta_{day} \cdot sin(2\pi t \cdot f_{day} + p_{day}) + \eta_{day/2} \cdot sin(4\pi t \cdot f_{day} + p_{day/2})$.

Translated into mathematical language, the output from the auxiliary oscillator is:

$$y = sin(2\pi t(f_{GW} + \dot{f}t + \frac{1}{2}\ddot{f}t^2 + \frac{1}{6}\dddot{f}t^3) + \phi_{year} + \phi_{day}) \tag{2}$$

The equation (2) contains 10 parameters to describe the receivable GW. It is iterated with the aim of maximizing the amplitude of the spectral line of f_{GW}. The initial values for HMC are:
$f_{GW} = 6220.31\ \mu$Hz (date=2009-01-01, from [4], figure 2)
$\eta_{year} = 19.5$, result from [4]
$\dot{f} = 760 \times 10^{-18}\ s^{-2}$, result from [4]
$\ddot{f} = \dddot{f} = 0$ due to uncertain estimates [11, 12]
Technical remark: Both phase modulation and amplitude modulation with a fixed frequency f_{mod} produce spectral lines at a distance f_{mod} above and below the carrier frequency (sidebands). These differ in their phases. The compensation method MSH [4] used in this section reacts *only* to the sidebands caused by PM. Once these are removed, the remaining sidebands may be analyzed using AM demodulation methods (see section 7).

4 Results

After eliminating drift and PM, the parameters of equation (2) have the following values:
$\eta_{day} = 3.95 \pm 0.08$, $p_{day} = 3.03 \pm 0.02$
$\eta_{day/2} = 0.46 \pm 0.06$, $p_{day/2} = 2.50 \pm 0.13$
$\eta_{year} = 19.82 \pm 0.03$, $p_{year} = 5.566 \pm 0.002$
$f_{GW\ (2009)} = 6220.362939\ \mu\text{Hz} \pm 0.487$ nHz
$\dot{f}_{GW\ (2009)} = (7.587 \pm 0.022) \times 10^{-16}\ \text{s}^{-2}$;
$\ddot{f}_{GW\ (2009)} = (94.3 \pm 4.6) \times 10^{-27}\ \text{s}^{-3}$;
$\dddot{f}_{GW\ (2009)} = (5.16 \pm 0.26) \times 10^{-34}\ \text{s}^{-4}$;

The values in the last four lines apply from the start of this analysis (2009-01-01) and may depend on the year.

Earlier measurements in the X-ray range [7, 11] are based on short data sets with a low frequency resolution of $\Delta f \approx 200$ nHz. The data sets of the present measurement cover ten years and reduce Δf to around 2 nHz. At the beginning of the iteration, the signal processing bandwidth is large so that the signal from the HMC does not leave the filter range. As the results converge, the bandwidth is reduced to improve the S/N.

5 Comparison of previous results

HMC differs from other binary star systems by its particularly short orbital period and is therefore a touchstone for the predictions of the general RT. The increase in rotation frequency $\dot{f}_{GW}$ is a measure of the energy radiated by GW. The value of $\ddot{f}_{GW}$ allows statements about the flow of matter between the two stars [12] and the future of HMC.

Year	f_{GW} μHz	$\dot{f}_{GW}$ $\times 10^{-18}$ s^{-2}	$\ddot{f}_{GW}$ $\times 10^{-27}$ s^{-3}	$\dddot{f}_{GW}$ $\times 10^{-34}$ s^{-4}	Author
2005	6220.2764	726 ± 12	$<$4000 ?	—	Strohmayer [7]
2006	—	—	0.25 ± 0.05	—	Deloye,Taam [12]
2009	6220.362939	758.7 ± 2.2	94.3 ± 4.64	5.16 ± 0.26	Weidner (GW)
2020	6220.5838	698 ± 6	—	—	Strohmayer-1 [11]
2020	6220.2759	711.4 ± 1	-17.9 ± 2.8	—	Strohmayer-2 [11]

Table 1): *Characteristics of the GW from HMC. For better comparison, the data measured by Strohmayer have been doubled. The value in the highlighted cell is obviously wrong.*

6 Checking the amplitude constancy of the GW

A spectrum contains no information whether and how the amplitude of a signal changes over time. A phase-sensitive integrator acts like an extremely narrow-band filter and answers this question. One possible method is based on the addition theorem of trigonometric functions and measures only a single frequency f_{check}. The SG data are digitized and consist of discrete values. Between successive readings z_n and z_{n+1} of the signal, a certain time interval passes (the sampling time T_s). With each step, the phase angle increases by the value $\alpha = 2\pi T_s \cdot f_{check}$. If one wants to know whether the received noise z_n, z_{n+1}, z_{n+2}.... contains a signal of frequency f_{check}, one sums the amplitudes by alternately calculating the two formulas:

$$x_{n+1} = z_n + cos(\alpha)x_n + sin(\alpha)y_n \tag{3}$$
$$y_{n+1} = cos(\alpha)y_n - sin(\alpha)x_n \tag{4}$$

The sequence of values x_n and y_n depends on the choice of parameters:

- Without an injected signal and with the initial values $x_1 = 1$, $y_1 = 0$, the formulas calculate a table of values for $x = sin(2\pi t f_{check})$ and $y = cos(2\pi t f_{check})$ with constant amplitude.
- Setting $x_1 = y_1 = 0$ and injecting a monochromatic signal z_n whose frequency matches f_{check}, the formulas calculate an oscillation whose amplitude increases in proportion to time.
- If the injected frequency differs from f_{check} or if the injected signal z_n varies in phase or amplitude, the output signal of the integrator is small and not linear.
- If noise is injected, the formulas calculate a low-bandwidth frequency mixture in the vicinity of f_{check} , whose amplitude fluctuates irregularly.

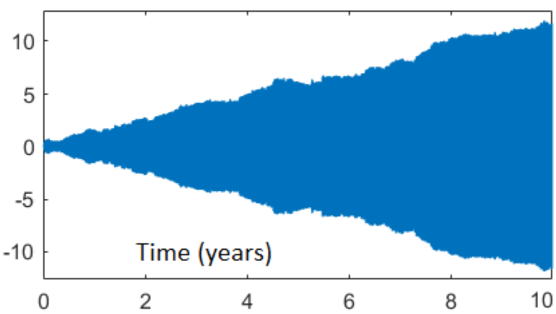

Figure 1): *After the signal of the GW is freed from all PM and the frequency drift, the frequency is constant and the integrated amplitude increases proportionally to the time. Phase fluctuations produce noticeable deviations from linearity. Since the gravimeters are connected to the ground without damping elements, earthquakes are disruptive.*

Choosing $f_{check} = 6220.363$ μHz and injecting the *unfiltered* signal recorded by the gravimeters, we obtain the result shown in Figure 1. Since the integrated amplitude increases in proportion to time, the injected signal mixture contains a signal whose frequency and amplitude are constant throughout the entire ten-year period – as expected from a continuous GW.

7 Amplitude modulation in half-day rhythm

The MSH procedure determines and compensates PM. Subsequently, there are no more sidebands that can be assigned to a PM. This does not exclude that further modulations are present. If the GW is additionally amplitude modulated (AM), sidebands with corresponding frequencies still exist. If the signal is modulated with a *single* frequency f_{AM}, the spectrum shows three spectral lines with the mutual spacing f_{AM}. With a period of 24 hours, the wide gap between f_{GW} and the 11.6 μHz distant sidebands is filled with noise. With low S/N, the weak spectral lines can hardly be found in the noise of the many other GWs. To isolate the neccessary AM sidebands, one needs a matched filter (BW = 2 nHz) and a *precise* value of f_{AM}. This enables the measurement of the AM.

Possible AM periods of 12 and 24 hours follow from the rotation period of the Earth; other rhythms were not studied. Figure 2 shows the results:

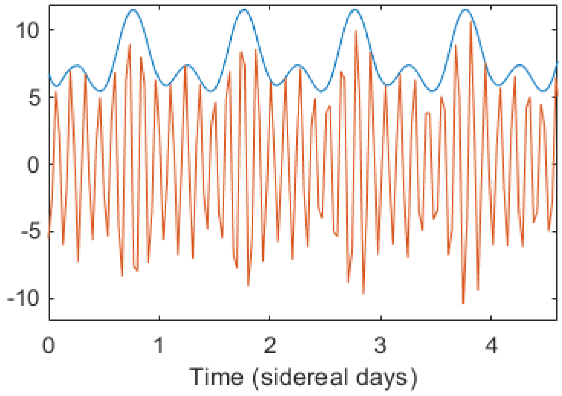

Figure 2): *Highly stretched section of the amplitude modulation of the GW. Red: Signal of the GW after frequency shift to* f_{ZF} *and after the matched filter. Blue: Envelope for better visualization. This fine structure remains constant during the ten-year total duration. The time counting of the t-axis starts on 2009-01-01 at 0 o'clock.*

- Rough structure: the mean amplitude is nearly constant during the whole time span of ten years (±3%). That causes the linear increase of the integrated amplitude in figure 1. The small deviations are a consequence of the not perfectly rectangular filter shapes. The extremely narrow-band matched filter eliminates all disturbances caused by earthquakes.
- Fine structure: The GW of HMC is amplitude modulated with two time constants: 11.9672 ± 0.0001 hours (modulation index = 26%) and 23.93447 ± 0.00002 hours (modulation index = 35%). The oscillation period of the AM deviates only 4×10^{-8} from the sidereal day length 23.93447192 hours.
- Daily, 18.7 ± 0.03 hours after midnight the amplitude of the GW reaches its maximum value (measured in the sidereal time frame starting on 2009-01-01 at 0 o'clock).
- A weaker maximum follows 10.7 ± 0.2 hours later. The phase shift between high and low maximum remains constant during the ten-year measurement period. These details are mean values over the entire measurement period of 3652 days.

The striking deviation from synodic daylength 24 hours suggests a GW source outside the solar system. A possible explanation: If one interprets the straight line between Europe and Canberra/Australia as a linear GW antenna, it rotates by 180^o per 11.9672 hours. As with all linear antennas, the reception amplitude depends on the angle between the orientation of this antenna and the propagation direction of the GW. Without knowledge of the directional pattern of this antenna, the direction to the source of the GW cannot be determined.

8 Summary and Discussion

In all records of the eight superconducting gravimeters (at different locations), one finds a spectral line whose frequency and mean amplitude are strikingly constant throughout the ten-year measurement period. All modulations of this signal indicate an excitation by a GW:

- In January 2009, HM Cancri generated a GW of frequency 6220.36294 μHz.
- The frequency drift of f_{GW} is exactly twice as large as the drift of f_{orbit} [7], as predicted by the general theory of relativity.
- The signal is phase modulated with f_{year} and the frequency deviation Δf_{year} almost exactly equals to the Doppler shift calculated from the motion of the Earth in orbit. This confirms that the ecliptic latitude if the GW source is very small. The actual ecliptic latitude of HM Cancri is -4.7^0.
- The signal is also phase modulated at a much higher frequency, which matches very well with the sidereal day duration. Therefore, it is very unlikely that the signal is generated in the Sun or in near-Earth space.
- The frequency deviation of this PM in a daily rhythm far exceeds the limit due to the low circumferential velocity at the equator. The cause of this strong PM is unclear.
- The spectral line f_{GW} does not correspond to any of the many natural resonances of the Earth [2] and differs by a much smaller half-width. Therefore we can exclude that f_{GW} has anything to do with the natural resonances of the Earth.
- Integration of the signal with a phase-sensitive method confirms an extremely small variation of the phase during the ten-year measurement period. This rules out excitation by non-synchronized Earthquakes.

9 Data availability

The recordings of all gravimeters may be downloaded from GFZ Potsdam [9]. A detailed description of the IGETS data base, the IGETS products and the registration procedure are described in [10]. The raw data are formatted as ASCII files and cover one month each.

References

[1] Israel, G. L. et al., The discovery of 321 s pulsations in the ROSAT HRI light curves of J080622.8+152732 = RX J0806.3+1527, Astron. Astrophys. 349, L1–L4 (1999)

[2] Harvard Seismology, Resources, Normal-Mode Observations, Normal-Mode Observations, http://www.seismology.harvard.edu/downloads/mode.html

[3] Abbott, B.P. et al., GW151226: Observation of gravitational gaves from a 22-solar-mass binary black hole coalescence., Phys. Rev. Lett. 2016, 116, 241103.

[4] Weidner, H., Detection of the Continuous Gravitational Wave of HM Cancri, (2022), https://vixra.org/pdf/2201.0172v2.pdf

[5] Weidner, H., Investigation of the Gravitational Wave of the Binary Star System J0651+2844, (2022), https://vixra.org/abs/2208.0125

[6] Weidner, H., A Mystery is Solved: Gravitational Waves Generate the Constant Hum of the Earth, (2022), https://vixra.org/abs/2208.0018

[7] Strohmayer, T., Precision X-ray timing of RX J0806.3+1527 with CHANDRA: Evidence for Gravitational Radiation from an Ultracompact Binary, (2005), Preprint at arxiv.org/pdf/astro-ph/0504150.pdf

[8] Quenouille, M. H., "Problems in Plane Sampling." Ann. Math. Statist. 20 (3) 355 - 375, September, 1949. https://doi.org/10.1214/aoms/1177729989

[9] IGETS Datenbank, GFZ Potsdam, https://isdc.gfz-potsdam.de/igets-data-base/

[10] Voigt, C. et.al., Report on the data base of the international geodynamics and Earth tide service (IGETS), (2016) https://gfzpublic.gfz-potsdam.de/rest/items/item_5003806_1/component/file_5003807/content

[11] Strohmayer, T., A Real-time View of Orbital Evolution in HM Cancri, The Astrophysical Journal Letters, 912:L8 (9pp), (2021)

[12] Deloye, C., Taam, R., THE TURN-ON OF MASS TRANSFER IN AM CVN BINARIES: IMPLICATIONS FOR RX J0806+1527 AND RX J1914+2456, ApJL, 649, L99, (2006), https://arxiv.org/abs/astro-ph/0608442v1