Quantum Gravity Asymptotic Safety from 2D Universal Regime and Smooth Transition to Dual Superstrings

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Abstract:

In the context of our work on multi-fold universes, and the emergence of gravity from entanglement, we argued that quantum gravity is asymptotically safe in multi-fold universe, as well as in the real universe, where the arguments rely either on the belief that multi-fold mechanisms correctly model these aspects, or on arguments captured in the literature by asymptotic safe quantum gravity aficionados. We discussed the consequences of asymptotic safety: the incompatibility of supersymmetry, and superstrings, or M-theory, with the Standard Model (SM) or SM_G *(The Standard Model with non-negligible gravity effects at the SM scales).*

On the other hand, string afficionados, and others to be fair, view asymptotic safety claims for gravity with skepticism, to say the least. Arguments that gravity would be asymptotically safe, and incompatible with superstrings, have not reduced their claims of supremacy, or viability of the superstrings theory. Indeed, asymptotic safety is often treated as heretic, derided, or labeled by a few, as non-sensical, or fundamentally mistaken. It is despite several hints that asymptotic safety might indeed apply to General Relativity (GR)-based quantum gravity.

Having accumulated more and more arguments that position superstrings with respect to multi-fold universes, and that seem to explain some of their challenges, we wonder if we could settle the asymptotic safety once and for all, using conventional Physis, i.e. not multi-fold, or even better, stringy arguments, just as we recently did, in another paper, for de Sitter vacua, and the string swampland, and of course, taking such latter results into account.

In this paper, we present arguments that justify, non-perturbatively, the asymptotic safety of gravity in a conventional universe, and in a multi-fold universe. More surprisingly, we encounter a possible touch point between the resulting UV fixed point, and gravity dimensional reduction to 2D processes, where the cosmological coupling in the presence of matter may first be negative, and the UV fixed point might therefore potentially also be described by, or emerge from, superstrings. There are possible smooth transitions between AdS dual and spacetime in the conventional, superstrings and multi-fold cases. However, we do not know if that can be reconciled with the other incompatibilities between SM and superstrings, resulting from the asymptotic safety, which might still exist when considering, the trajectories of the couplings running away from the UV fixed point. In order to be non-perturbative, our approach provides arguments and microscopic justifications for the effects of running the gravitational couplings.

In multi-fold universes, a new phenomena has been identified: short range entanglement between concretized spacetime locations that contribute negatively to the cosmological constant at very small scales (in 2D regimes).

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1. Introduction

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The paper [1] proposes contributions to several open problems in physics like the reconciliation of General Relativity (GR) with Quantum Physics, explaining the origin of gravity, proposed as emerging from quantum (EPR-Einstein Podolsky Rosen) entanglement between particles, detailing contributions to dark matter, and dark energy, and explaining other Standard Model (SM) mysteries, without requiring New Physics beyond the Standard Model, other than the addition of gravity to the Standard Model Lagrangian, i.e. no new fundamental particles. All this is achieved in a multi-fold universe that may well model our real universe.

With the proposed model of [1], spacetime and Physics are modeled from Planck scales to quantum and macroscopic scales, and semi classical approaches appear valid till very small scales. In [1], it is argued that spacetime is discrete, with a random walk-based fractal structure, fractional and noncommutative at, and above, Planck scales (with a 2-D behavior and Lorentz invariance preserved by random walks till the early moments of the universe). Spacetime results from past random walks of (Higgs) particles. Concretized spacetime locations and particles can be modeled as microscopic black holes (Schwarzschild for photons and spacetime coordinates, and metrics between Reisner Nordstrom [2] and Kerr Newman [3] for massive and possibly charged particles – the latter being possibly (over) extremal). Although surprising, [1] recovers results consistent with others like [4], while also being able to justify the initial assumptions of black holes from the gravity or entanglement model in a multi-fold universe. The resulting gravity model recovers General Relativity (GR) at larger scale, as a 4-D process, with massless gravity, but also with massive gravity components at very small scale that make gravity significant at these scales. Semi-classical models also turn out to work well till way smaller scales that usually expected.

Follow-up work led us to conclude that more open issues of the Standard Model (SM) and Standard Cosmological model can be addressed with multi-fold mechanisms [1], and SM_G (see [5] to track all related latest results). Since, we also concluded that GR, and quantum gravity, appear to be asymptotically safe in multi-fold universes, as well as in the real universe [9], and that de Sitter spacetime can't emerge from superstrings [7]. The implications are immediately daunting: supersymmetry and superstrings, with super partners and any dimensions, appear incompatible with SM / SM_G in (de Sitter) 4D.

This paper starts from these results, asking if we can motivate the asymptotic safety considerations for gravity, from a stringier point of view, as well as conventional ones, which would hopefully be more acceptable to the string community. We first review our assertions that (super)strings actions contain, from the get-go, GR, the Hilbert Einstein action and therefore gravitons when quantized (even if gravitons may just be quasi or pseudo particle concepts) [1,6]. We review our analysis that de Sitter vacua are part of the string swampland, something actively discussed as conjectures within the string community, and motivated, without requiring multi-fold assumptions, in [7]. Then, we review our readings of the implications of asymptotic safety for superstrings and supersymmetry. Starting from the paper [8], that internalized de Sitter swampland as a swampland concern, and proposed a plausible stringy answer. In the present paper, we argue that the proposal of [8] can give interesting new insights. We note that, surprisingly considering the opposition of the community, conventional superstrings seem to in fact predict asymptotic safety of quantum gravity; at least at the level of the string actions, and strings in background fields. As far as we know, this is a key result, different from all the computations results and other arguments encountered so far in the literature.

As the string community raised many other objections to asymptotic safety (e.g. [9]), based on other considerations and results from string theory, we try to address some of the key concerns via non-perturbative arguments for both QFT approaches and multi-fold universes. We think that such an approach is original on its own.

Notes added on 8/22/22: We have since provided a different non-perturbative proof of asymptotic safety of GR-based gravity [53].

2. Asymptotic Safety and the Standard Model: No-Go for Many Theories

In [10], we examine [11,12] that convey the following observations:

- There are indications, that quantum gravity is asymptotically safe. This is achieved by truncating an effective action, above an IR cutoff, and modeling the evolutions of the gravity running couplings [9].
- The papers study these evolutions, under many models, involving different amounts of particles and dimensions, as expected from the Standard Model, Supersymmetry, Superstrings and GUTs.
- The main conclusion, of interest for us, is that, if quantum gravity is asymptotically safe, then it does not appear to be compatible with the numbers of particles, or dimensions introduced by supersymmetry, superstrings, and most related GUTs and TOEs. Dimension compactification does not change the results.

These were conventional considerations.

In [10], we also argued that multi-fold universes, especially at very small scales, with its discrete spacetime generated by random walks and Lorentz invariant as well as non-commutative, indicates asymptotic safety and gravity, and spacetime, is essentially a 2D process; something initially proposed in [21] *(Note added on 3/20/21: [21] is with the caveat discussed at [51]),* and encountered by most approaches to quantum gravity [22].

The conclusions of [10-12] potentially have tremendous impact for all the affected supersymmetric theories. Yet, the potential incompatibility with the standard model are mostly passed under silence in the literature. It is simply not discussed, maybe because asymptotic safety of gravity is so strongly discounted anyway. Indeed, one mostly encounters arguments against the idea of asymptotic safety for gravity including arguing against the concept of running the dimensionless Newton and cosmological couplings [10,13,14]. A priori, such arguments seem to make it difficult to confidently draw suitable conclusions from [10-12].

The purpose of this paper is to discuss other stringy, or conventional, ways, of course preferably not based on multi-fold mechanisms, to investigate, or even confirm, asymptotic safety of quantum gravity. Doing so, we hope to provide some new answers to criticisms raised against asymptotic safety of gravity.

3. Asymptotically Safe Gravity, A String Action Argument?

Considering the criticisms raised, by the string community, to the program of asymptotically safe quantum gravity, we wish to find new reasonings to explore. To that effect, we propose to repeat our analysis that strings actions (e.g., Nambu-Goto or Polyakov) contain, and are essentially equivalent to the first order of, the Hilbert Einstein action, for a given manifold emerging from superstrings [6]. We already based our proof rejecting the possibility of emergence of a positive cosmological constant on a similar reasoning [7].

Can we actually prove from a superstring point of view that, if superstrings generate a spacetime, then gravity in that spacetime is asymptotically safe? Furthermore, what would it take for this result to hold in spacetimes with de Sitter vacua, or in asymptotically de Sitter spacetimes?

3.1. Well-behaved Quantum Gravity Emerging from Superstrings

The Hilbert Einstein action extremization by a string action extremization implies that, in the presence of other fields, for each “position” considered to compute the string action (See [6], and references in that paper, like, in particular, [15]), we have a well-behaved outcome (resulting from 2-D CFT contributions, and therefore renormalizable) due to the sigmoid monotonic transformation performed by the string actions. The gravity action does not diverge, no matter how high energies are involved, or how small the scales are. The result can be seen as nonlocal to the extent that its value is the result of an integration along the extent of the superstring. The extremized metric is not a CFT. This good behavior comes from the stabilized number of degrees of freedom, because of the integration over the string length, which amounts to asymptotic safety.

The result is very important. With this reasoning, we have shown that, ignoring the problem of spacetime vacua, superstrings by themselves predict an asymptotically safe quantum gravity, at least in the absence of matter.

What about in the presence of matter (fermions, bosons, scalars) i.e. other fields? Conformality of all the other (Yang Mills) fields results from the consistency conditions discussed, for example, in [15-19] (see [6] for more references). As the number of matter fields increase, the cosmological coupling at the UV fixed point decreases towards negative territories. Everywhere matter or energy appears, curvature accentuates the effects, and so the world sheet expands: the number of degrees of freedom increase instead of remaining plateaued. This happens wherever the fields are not zero. If too many fields are also present, it means that that the reasoning above breaks apart: no UV fixed point can exist. Similarly, as the number of spacetime dimension increase[2], we expect increase of degrees of freedom that accelerates the effects of more fields, making the problem occur earlier. Note that such an analysis is especially unique and useful as it does not depend on complex numerical computations. Such an analysis has the beauty to be non-perturbative.

The result is very important. We have shown that, ignoring the problem of spacetime vacua, superstrings by themselves predict an asymptotically safe quantum gravity, at least in the absence of matter. Furthermore the resulting reasoning, based entirely on the superstring action, qualitatively recovers the results of [10-12], and the resulting incompatibilities of supersymmetry, superstrings and large number of dimensions with SM (inspired GUTs and ToEs are similarly rendered incompatible). It is entirely contained in the superstring actions.

3.2 Dealing with the AdS vs. de Sitter Vacua Questions

In [7], we used the same action-based reasoning to deduct that de Sitter vacua hence without other fields, and asymptotic de Sitter spacetime, are part of the string swampland.

On that basis, well-behaved, as in non-divergent and renormalizable, gravity fields can a priori only emerge in AdS spacetime. It was something that we already knew. The AdS/CFT conjecture, and its superstring derivation with supersymmetric New Physics, ensure already that non-perturbative gravity in AdS is well-behaved, and perturbative models for stringy gravity are renormalizable in AdS. It could seem in contradiction with [20], where it is argued that the same conjecture, and the holographic principles, refute renormalizability of gravity. We argue that it is not the case. The difference comes from the fact that, at very small scales, quantum gravity appears as a 2D process[3] instead of a higher dimensionality process. It was presented first in [21] *(Note added on 3/20/21: [21] is with the caveat discussed in [51])*, and encountered in most quantum gravity models [22] ([1] provides a physical interpretation in terms of random walks, as part of its discussion of multi-fold spacetime reconstruction).

Superstrings can also explain such a model. Indeed, above the Hagedorn energy, the possible microscopic states are contributed by every unit along the string, and hence the entropy grows linearly with the energy (or temperature). The formal derivation can be found at [24]. Therefore, when the Hagedorn phase transition takes

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[2] We will see later that we associate the UV fixed point to 2D process due to random walks, including with QFT reasonings [21,22], not just in the case of multi-fold universe. As more and more dimensions would be considered, the reduction to 2D becomes more tenuous to achieve. Indeed, stabilizing the number of degrees of freedom growth is more difficult, and also affected by the disrupting effects from other fields.

[3] It is a process aspect. We equate it as dimensions following [21] because at such scales it is quite acceptable to believe that the processes dictate how, and what, dimensions are perceived, i.e., the dynamics and kinematics of the system.

place, the entropy evolution describes a fixed point; exactly our result. It resolves the discrepancies about entropy evolutions pointed out in the literature [20,25]. The same argument applies for a flat spacetime.

In a(n asymptotic) de Sitter spacetime, per [7], no solution exists without the introduction of new sources that would not be not emerging from superstrings[4]. We do not know what such sources would be, as fields rather push towards AdS vacua, but for the sake of discussion they could be considered (examples are discussed in [7] and references therein). As long that such sources are well behaved, section 3.1 shows that the result will remain well behaved, and can be asymptotically safe, again due to an evolution to a 2D behavior.

We also note that 2D quantum gravity (QFT) is renormalizable with proven asymptotically safe for (2+ε)D (See e.g. [30,33] and references therein), which is another way to demonstrate that if we have dimensional reduction to 2D, then we demonstrate asymptotically safety for a QFT gravity.

4. A Running Cosmological Coupling from dS to AdS

So far, [8] is, to our knowledge. the only paper in the superstrings camp, which seems to accept and build on both asymptotic safety of gravity, and dS vacua as part of the swampland, and yet provides a way forward for superstrings. The proposal from [8] goes as follows:

1. The paper authors do not contest that de Sitter vacua may belong to the string swampland.
2. They accept that quantum gravity appears asymptotically safe.
3. They observe as a result that the fixed point for the cosmological coupling (i.e., a running cosmological constant) in presence of SM (matter) appears to be negative[5]. For this, they rely on a model of the trajectories of the cosmological coupling obtained from [11]: with SM fields, the UV fixed point has a negative cosmological coupling[6].
4. Therefore, they propose that, as energy scales increase, quantum gravity evolves from a de Sitter spacetime to a AdS spacetime that can then itself emerge from superstrings.

In our view, the proposal of [8] is a very ingenious hypothesis that forces us to address two aspects:

- Why would the cosmological run from positive to negative? We will come back to this in the context of multi-fold universes where we propose a dark energy model resulting into a positive dark energy effect [1,23]. Considering our model for dark energy [1]. There is a plausible explanation about an effect we had mentioned, but that we have not yet discussed so far in terms of the implications: the short scale, and temporary, entanglement of spacetime concretized (or encountered/visited) by random walks.
- Is the outcome physically plausible? Unfortunately, this is where the proposal may a-priori appear to fall apart. If one admits that validity of a trajectory for the cosmological coupling described with the

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[4] That should a priori also exclude D-branes, albeit proposals like KKLT argue, for example, for use $\overline{D}$3-branes (anti branes).

[5] This is immediately seen from Einstein GR field equation: matter as energy momentum tensor and cosmological constant have opposite effects.

[6] We did not discuss this detail in past papers like [7,52], which is a shame because we already had a physical explanation for such behavior at the level of the random walks, and resulting local entanglement between spacetime location concretized or visited by same particle. We will revisit in more details later. *Note added on 8/22/22: [54] provides another way to also see such entanglement and associated multi-folds (implementable as wormholes) in GR-based gravity at Planck scale: entanglement creates gravity attraction that amounts to a negative contribution to the cosmological constant.*

> equations of [8], one see that it is derived in [11]. [11,12] use these models to show that the asymptotic safety is not compatible with the amounts of particles that supersymmetry, or superstrings, would provide or its high dimensions. So [8] would require an explanation for modeling only the SM particles, i.e., disregarding the super partners, we can't justify doing so; invoking supersymmetry breaking does not work: the super partners would still exist even if at way higher energies, and / or handling ~~super~~strings (without supersymmetry) in a 4D spacetime, where the emergence of gravity, and Yang Mills fields, are not consistently supported because, as afore mentioned, consistency requires conformance conditions, that require 10D or 26 / 27D [6,15-19]. Therefore, while superstrings could indeed generate the AdS vacua as proposed, they can't do it consistently with only the standard model, nor in 4D only. The proposal of [8], as is, does not hold.

However , this leads us to possible tenable alternative, that honestly we did not really consider so far, until re-reading [8]. This alternative assumes that, near the UV fixed point, we would already be in already in the gravity phase[7], with dominant 2D processes. Now the incompatibility may be manageable: [11,12] do not exactly apply.

In such a case, do superstrings really bring anything new over, for example, random walk and discrete spacetime? In 2D, they just amount to small segments of the string length moving on a 1D axis. Maybe it helps, maybe it doesn't. So studying superstrings as source for our spacetime may arguably be moot if that's all that superstrings can contribute vs. random walk on a discrete, fractal lattice. Superstrings impact is not that distinguishable at such scales from multi-fold reconstruction, or any of the other quantum gravity alternatives with a 2D regime. One could even argue that superstrings in 2D are just a way to model random walk from the point of view of a larger scale in a continuous spacetime. Furthermore, it is not entirely clear if it would actually salvage the problem of incompatibility with SM in 4D [11,12], mentioned above. Even if it were to be a correct model at the UV fixed point, it can't be used to model gravity at lower energy because of these incompatibilities: they do not well complete gravity QFT below the UV fixed point.

A good summary at this stage is that superstrings actually predict asymptotically safe gravity, and could be compatible with modelling it when gravity reduces to 2D processes. However, we have not (yet) managed to find a way to physically support superstrings as soon that the scale increase. It is more like there is a very small scale a fuzzy region where processes are essentially 2D and physics in spacetime (from random walks, in multi-fold universe) and from the dual tangent AdS spacetime (especially as encountered, in multi-fold universes) seem to coexist (converge) at a scale ε [8]. There, QFT, 2D-CFTs or superstrings (and therefore M-Theory) above the Hagedorn energy/temperature may[9] all correctly model physics.

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[7] The situation is different from the arguments about the entropy present earlier where the processes matter to determine the evolution of the entropy. Here we are talking about the spacetime dimensions.

[8] In AdS(5), there is therefore a need for a 2D phase also. This happens per section 3.1.

[9] Equally well, or just approximatively well. The questions about supersymmetry and super partners (e.g., do they exist at these scales) remain unanswered for now. We would suggest that when it comes to supersymmetry, there is probably just a mathematical convergence to ensure consistency of the fields leaking from spacetime to its AdS dual tangent, and conversely. The theories continue each other well, but super partners do not appear to leak outside of AdS. In a multi-fold universe, w plausible explanation is that supersymmetry from AdS(5) is probably broken in that ε scale region (e.g., by quantum fluctuations, and by the fact that superstrings and fields only approximate the physical reality of a multi-fold noncommutative Lorentz invariant discrete and randomly fractal spacetime with random walk processes, which has nothing to do with supersymmetry). *Note on 8/22/22: [55] indeed shows also that multi-fold mechanisms do not bring supersymmetry to the embedding space, which within an ε neighborhood, can easily imply also no supersymmetry in the ε region, where AdS or Ricci flat may not be distinguishable [56]).*

5. Handling other criticisms of the Asymptotically Free Gravity

Many key objections have been made to the conventional asymptotic safety program for gravity [42]. In this section, we discuss the most relevant ones that we have encountered.

[25] argues that quantum gravity cannot be described by local fixed theory because:

- Gravity infinities cannot be tamed. It is also along the line of the challenges made in [32]. Yet, that is not aligned with truncation results. So, these papers challenge the validity or relevancy of these results.
- The discrepancies in scaling of entropy with the energy argued in [20,25]. Yet, this has been addressed in section 3.2, relying on 2D. 2D and Ultimate Unification [1,29] are also the answer in multi-fold universe.
- Information preservation in black holes would require non-local New Physics. Yet it has been allegedly addressed in the context of QFT and gravity [28]. A model for black holes in multi-fold universes aligned with [28] is discussed in [27]. *Note added on 8/22/22: [1,57] further shows how black hole interactions and particle interactions actually relate.*
- The trans-Planckian problem of black holes [26] would not be handled well in the presence of a UV fixed point that is scale invariant. However, and as discussed in section 4, it is addressed if the asymptotically safe fixed point coincide with the 2D phase of gravity and the associated discrete spacetime. We have shown that it must be the case for superstrings to have a chance to be involved, albeit hardly relevant, in order to have a chance of consistency with SM. Without involving superstrings, it can simply be similarly handled by the discrete 2D phase of spacetime. It is also the case in a multi-fold universe. These effects are also discussed in [1] when modeling black hole horizon to explain [43], and allegedly encountering strings as approximation of particles evaporating from blackholes because of the quantum fluctuations of the horizon; trans-Planckian effects are just like 2D effects at very smalls scales; they look like strings when seen from larger scales; but that does not make them superstrings. *Note added in 8/22/22: [58] provides a detailed view on the trans-Planckian censorship Conjecture, by explaining it without contradiction with asymptotic safety of gravity: essentially a minimum length, as for example provided by spacetime discreteness, is all what is required.*

[13] suggests that there would be no correct real-world justification for running the gravitational couplings (Newton constant and cosmological constant), and no universality, i.e., same applicability to any particle. We admit that there may not be an easy conventional QFT answer to that. However, in section 3.1 and below, we provide non-perturbative reasonings for the UV and IR Fixed points, and trajectories of the couplings, which match the results, that we rely on, here and in [52], and that are obtained by the asymptotic safety program. Also, in a multi-fold universe we can justify these trajectories of the cosmological coupling, even from a positive to a negative value, and a running of the gravitational coupling to support the UU behavior [1,29] (See section 8.2). All our arguments relate to the 2D phase transition discussed earlier, and, as such, they are somehow supported by most quantum gravity models. We believe that [13] misses fundamentally the consideration of this phase transition in its arguments, or misses the points made in next paragraph that the 2D phase of quantum gravity validates running the Newton and cosmological couplings.

For the rest, [31,32] provide discussions, also often referenced, that argue against the computations used by most asymptotic safe projects: they would be incorrect, because of problems with the underlying measure, due to non-invariance of IR fixed points and, or incomplete domains of path integration, which may have non-fully connected topologies, rendering some regions inaccessible to perturbative methods. It may be the case. However, we know already that, if we have dimensional collapse to 2D, then we have asymptotic safety of gravity (Abstractly because 2D and (2+ε)D gravity are asymptotically safe, and making the Newton and the cosmological couplings run is therefore proven defensible. It is another answer to the criticism in [13]) and it renders the arguments moot as all models have such a collapse. Of course, the claim of alleged incorrect computations remains troublesome if it could affect the equations characterizing the trajectories of the couplings (e.g. see for example [33] where in truth one does not know at the end if the trajectory for the Newton coupling go up or down in 4D, despite claims that there would be anti-screening), and therefore weaken the assertions of [11,12], and anything built on these results (e.g. [8,10]). Yet because the computation works for (2+ε)D, and because we can reproduce (without, as well as with, multi-fold considerations) the trajectories and results of [11,12,38], we assert that the analysis is probably consistent at the level of the estimates of the equations describing the runs of the couplings.

Furthermore, related works on gravity have also uncovered clear existence of IR fixed points [38-40] (without matter). We believe that these are very strong arguments in favor of the cutoff being correctly modeled contrary to the concerns of [31,32] about the cutoff selection, and suitable, or not, BSRT invariance, and path integral measures.

The next section provides a conventional nonperturbative motivation to [11,12,38]. We believe that it addresses most of the criticism: yes the analyses are correct and explainable, and many of the "problems" alleged by critics come from not factoring in the 2D dimension reduction that occurs at the fixed point.

Note added on 8/8/22: Anyway, by now, [53] has settled the issues for GR-based gravity: one can non-perturbatively prove that it is asymptotically safe, with a rather different approach.

6. Non-perturbative Analysis With Conventional Considerations.

In a conventional universe, we can predicts results for the space of gravity theories that show trajectories of couplings in a (λ,g_N) phase diagram, which match the results of Figure 4 of [38]. This curve is for a model without additional fields, i.e. no matter (loosely used to also include scalars, gauge bosons and fermions). λ is the cosmological coupling defined as the dimensionally corrected (dimensionless) cosmological coupling. G_N is the dimensionless Newton gravity coupling.

The analysis is qualitative and based on impact of effects on the couplings. This may surprise the reader but doing it at such a level is essential and the only way to achieve non-perturbative analyses. Anything else would imply arguable assumptions that may not please all. And we just saw that there are many skeptics.

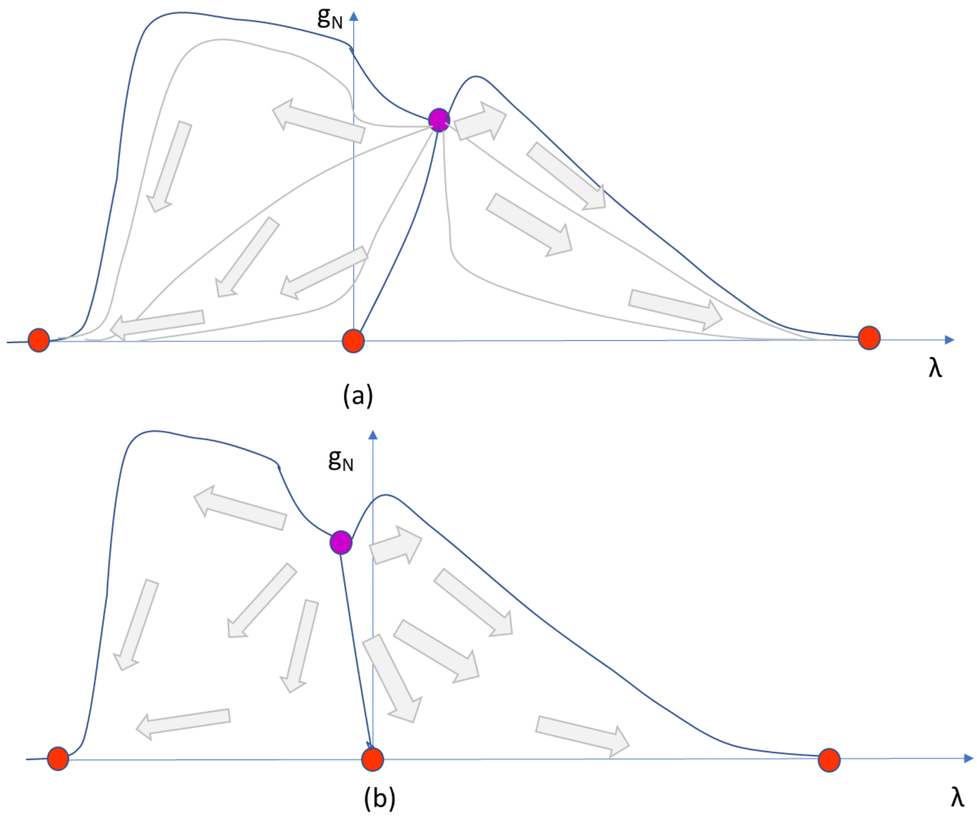

Figure 1: It illustrates the general forms of the UV critical curves as a (λ,g_N) phase diagram per the non-perturbative reasoning presented in this paper (conventional Physics). This is for a situation, where no matter or fields are present. (a) correspond to a positive λ at the UV fixed point while (b) corresponds to a negative one. Violet circle marks the UV fixed point. Red circles are IR fixed point. They are assumed to exist. A multi-fold reasoning renders (a) more probable while a superstrings origin would force a (b) diagram. In the presence of matter or fields, the diagrams are modified (typically and deformed to the left (UV point moving to the left until it (possibly) can't exist anymore). These diagrams are to be compared to the results of actual computations as done in Figure 4 of [38].

Accordingly:

- Let us assume that we start with a case where the 2D dimension reduction takes place for a positive cosmological constant (UV Fixed point). In such cases:
  - If the cosmological constant decreases, then it amounts to first g_N increasing (as decreasing cosmological coupling first amounts to stronger gravity effects) before decreasing, or decreasing all the time (as weaker gravity helps negative cosmological couplings). In that case, the effect can plateau (as effects change from opposite to aligned), when the cosmological couplings becomes negative, then finally dives down, when the cosmological constant become too large to even allow propagation of gravity interactions. It ends up in an IR fixed point.
  - If the cosmological constant increases, then one can expect that g_N consistently decreases as gravity effects are weakened by the cosmological constant increases. At some point, spacetime is growing so fast that gravity effects becomes negligible with g_N collapsing (it is an IR fixed point). Of course, there could be a region, near the UV fixed point, where increasing λ initially amounts to letting gravity interactions reach further, and so, some localized (by opposition to the increase/plateau associated to a decreasing of λ) increases of g_N near the UV fixed point could occur before the decrease to the IR fixed point.

- Conversely, if we let us assume that we start with a case where the 2D dimension reduction takes place for a negative cosmological coupling (UV Fixed point). In such cases:
  - The decrease of the cosmological constant will amount to a decrease of g_N towards an IR fixed point (some flatness then shared decrease can also occur)
  - The increase of the cosmological constant implies a flattened decrease of g_N while λ is negative then a steeper descent to a IR fixed point; with again a possible localized (by opposition to the increase/plateau associated to a decrease of λ) initial increase.
- In between these two sets of flows, (for increasing and decreasing λ), g_N can decrease also to another IR fixed point where λ is essentially null (it can come from positive or negative λ UV fixed point). This trajectory separates the increasing and decreasing λ trajectories discussed in the previous bullets.

Amazingly our reasoning is exactly reproducing Figure 4 of [38][10], where numerical estimates obtain a positive λ for the UV fixed point. Remember that this is without matter.

With matter, we can add the following reasoning that concretizes what we already argued in section 3.1. As already hinted in [6,7], matter and cosmological constant have opposite effects. So adding matter, at least fermions and scalars, directly interacting with gravity, results into moving from a UV fixed point, that a priori would be positive without fields per [7], to a lower λ, and then to a negative values one. With this, we recover the results of [8,11,12]).

If too much matter is added, we know that fermions fight off gravity (i.e., think about the Pauli exclusion principle), which amounts to a collapse of g_N and/or an explosion of λ, at very small scales: no UV fixed point will be able to exist. This reasoning explains the observation of [11,12] of incompatibilities of asymptotically safe gravity with too many field beyond the SM.

This analysis is can be extended to situations where more dimensions are involved. In such cases, we must consider the following:

- The arguments presented in section 3.1.
- Compact or non-compact dimensions have little impact on any analysis as we are working at, or near, a UV fixed point, where even compact dimensions should therefore ultimately appear large.
- The Schwarzschild radius in different dimensions is provided by equation (25) in [44]. Between 4D and 12D, it decreases with the dimensions, which amounts to a weakened initial (at the UV fixed point) g_N and stronger λ impact. When matter is present its accelerate the effect that would have to start from a more problematic point that ultimately cannot exist.
- Considering [45], we see that increasing dimension beyond 5 or 6 has already a significant effects on the radius changes.

We again recover the results of [11,12].

Therefore we claim that such results are quite well explainable, independently of the detailed numerical analysis whose methodology is disputed by some.

At this stage, we believe that it is also worth further pointing out that, beyond the work done with the asymptotically safe gravity problem, and its perturbative or non-perturbative models, other analyses have shown the possibility of well-behaved gravity. For example, [49] provides another approach, arguably non-perturbative, that shows well behaved gravity, again scale free at trans-Planckian scales, and as stated by the authors, a proof of the principle that the quantum effective action comes with sufficient freedom to accommodate the physics requirements of asymptotic safety, and have actual well-behaved truncations due to compensations across the

[10] This type of diagram may also explain the possible confusion between screening and anti-screening effects discussed in [33]. Our analysis handles it without encountering the issues.

Feynman diagrams associated to the perturbative models. In other words, the concerns reviewed and discussed in section 5, that essentially relied on arguing that the mathematics wouldn't compute, or that the effects wouldn't actually cancel, probably do compute, and cancel, when all effects are considered. While in the next section, we will explain why, in multi-fold universe, asymptotic safety is all but certain; these considerations add to our bewildering in the abilities of mathematics to characterize nature: approximate QFT theories of quantum gravity and macroscopic Hilbert Einstein Action seem to correctly model 1) asymptotic safety 2) microscopic mechanisms like 2D random walk and multi-fold mechanisms a priory not at all considered by the theories. Wigner was right to marvel [50]. *Note added 8/22/22: [54] should bewilder the reader even more: GR seems to encounter multi-folds at Planck scales. Later work, besides [6], also show how even Yang Mills and the Standard Model seem to also be contained (and contain, it's a duality) in GR / the Hilbert Einstein action [5,55,59,60].*

In any case, and without dependency on multi-fold, [53] provides a different non-perturbative proof of the asymptotic safety of GR-based gravity. It should settle this discussion in favor of asymptotic safety of GR-based gravity.

7. Conclusions for conventional Physics and superstrings

At this stage of the paper, we have shown that:

- Quantum gravity asymptotic safety is quite likely for QFT models, where we encounter dimensional collapse to 2D, something encountered in most quantum gravity approaches.
- The asymptotic safe gravity fixed point corresponds roughly to the phase transition to 2D processes where quantum gravity is renormalizable.
- Around, and at, 2D, multiple models seem to converge to model gravity with 2D gravity, non-local actions, Polyakov-like actions, multi-fold random walk etc. It hints that indeed superstrings are such an equivalent model around the fixed point. Also, they predict asymptotic safety of gravity, contrary to the views of the many fans who believe that such prediction is not supported by superstrings.
- Coexistence of superstrings can be understood as a good approximation, or as a results of well-behaved (continuous) transition, when evolving from spacetime to the dual tangent AdS.
- This way, it is possible to reconcile de Sitter being in string Swampland and superstrings, if equations characterizing the trajectories of the cosmological coupling are correct in estimating the UV fixed point as being in an AdS spacetime that later evolve to dS as scales increase, and if the UV fixed point corresponds to 2D gravity.
- However suitability or relevance of this result is still open ended as incompatibilities of superstring and supersymmetry with SM, due to asymptotic safety [52], reappear when moving away from the UV fixed point (to larger scales). And in any case supersymmetry is then also forbidden in an asymptotic de Sitter spacetime [7]. Arguments of supersymmetry breaking resolve noting as the theory then still keep super partners, even if at high energy, and those render everything incompatible with the would be established asymptotic safety of gravity [52].
- While estimating the evolution from the fixed point is tricky, models mimicking (2+ε)D to 2D gravity are implying suitability of the approach despite the criticism, raised against the asymptotic safety of gravity program; noting that such criticisms typically do not consider the dimensionality reduction to 2D.
- Our non-perturbative reasonings also predict a UV fixed point, and trajectories of gravity couplings that match actual estimation of the trajectory equations. They also predict the behavior in the presence of additional fields and additional dimensions. This continues to be a problem for superstrings and supersymmetry theories at larger scales; something that 2D convergence across the theories does not

resolve, as already mentioned these are incompatible with the SM, if asymptotic safety of gravity is in place.

8. Multi-fold Universe Considerations

Let us now make sure that all these arguments and results obtained so far continue to fit in multi-fold theories.

8.1 Original Arguments in Favor of Asymptotic Safe Gravity in Multi-fold Universes

In a multi-fold universe, we already argued asymptotic safety of gravity [52], with different sets of reasonings (from [9]):

- A) The spacetime is discrete, fractal and random (built by random walks) [1]. [35] proposes an approach to renormalize the spacetime geometry modeled by a random fractal graph (which is how a multi-fold spacetime can be modeled), which is another way to see how a reconstructed multi-fold universe behaves as a macroscopic spacetime that follows GR ([1] instead simply achieved that same analysis with the microscopic black holes surrounding every concretized spacetime location and every particles.). That approach leads to UV fixed points.
- B) The multi-fold spacetime is non-commutative and (multi)fractal ,which also leads to fixed points [36].
- C) Starting from the Ultimate Unification reasoning around falsifying the strict Weak Gravity Conjecture presented in [1,29]:
    - at very small scales all entangled virtual pairs are massless and with same ranges of interaction. At smaller scale, it becomes apparent that they propagate via random walk leading to a fractal (and Lorentz invariant and non-commutative) spacetime. When reaching the scale where the random walks are visible, the process becomes scale independent and essentially 2-D (which is asymptotically safe / renormalizable [9, 30, 33, 34,37]).
    - Unoccupied (but concretized – see [1]) spacetime points are simply following a scale independent random fractal structure, which has therefore a UV fixed point.
    - Spacetime points occupied by particles are within a microscopic blackhole where no singularity is involved because of discreteness, torsion and dark energy effects, and have their effects visible only externally, beyond the horizon(s), at a fixed scale defined by the nature of the particle. Therefore, effects become also scale independent. Somehow, this, and UU, match the arguments made in [25] about simplification of interactions at very high energies.
- D) Spacetime is fractal at the discrete scale, and so pure gravity corresponds to a critical fixed point (RG) [9], where universality takes over for the effective theories at larger scales. There formulations above the scale of the critical point are asymptotically safe and these scales are the scales that matter for SM.
- E) [1] showed that spacetime and gravity in a multi-fold universe becomes a 2-D process at small scales (where random fractal walks dominate). 2-D gravity is renormalizable [9, 30, 33,34,37]
- F) Combining all, or some, of the arguments A)- through E)

These different reasonings demonstrate a UV fixed point and asymptotic safe in a multi-fold universe. C) is in our view the most rigorous and complete argument but each are probably sufficient and A) is the most direct.

8.2 Explaining the Running Couplings from the Literature in a Multi-fold Universe

Section 8.1 explained why gravity is expected to be asymptotically safe in multi-fold universes. Now, let us try to repeat the arguments of section 6, but for multi-fold universes.

Dark energy effects in a multi-fold universe [1,7,23] correspond to a positive cosmological constant. Note the mentions in [1,46] of the entanglement of (some) concretized spacetime points (those concretized by random walk of a same particle or from a path from another point). These are assumed to go away rapidly (e.g. because of the quantum fluctuations taking place at the location once concretized). However they also imply a very short scale spacetime "self-attraction", or gravity field felt by anything passing in between[11], that would correspond to a negative cosmological constant(coupling) (i.e. a negative dark energy effect), and possibly dominate the multi-fold dark energy effects of [1,7,23] at very small scales (where entanglement exists between concretized neighboring spacetime points)[12].

The 2D dimension reduction is due to a random walk process, as described in [1]. Furthermore, if the conditions were such that the {dominant} particle(s) at very small scale is(are) massless, or all particles are massless and appear with similar effects as predicted by UU, and the Higgs field proposal [1,29,46], then it is interesting to know that we recover another property of [22], where we find that equation 4 in [22], implies a 2D conformant field associated to a free massless boson path integral in 2D [48], fully aligned with UU or Higgs random walks, as a 2D processes; a quite interesting corroborating result. *Note added on 8/22/22: Per [57,61], we see that at such scales, fermions are probably just implemented by combinations of massless Higgs bosons. As such the boson comment made above is sufficient.*

For the rest:

- When emerging from 2D processes, we expect that, as in [1,23], the multi-fold dark energy effect dominates. Therefore, we expect a UV fixed point with a positive λ (remember this is in the absence of any other field). Without fields, a tough concept for a random walk model, only fluctuations take place and local spacetime attraction has no meaning as nothing is really being attracted by anything. Therefore, the outcome is expected to give a positive λ at the UV fixed point.
- Increases of positive λ amount to allowing virtual particles to move further away, therefore associated to a reduction of g_N. However it may first come with a localized increase as doing so first enables more

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[11] [1], and some following papers [5,59,60], mentioned such attraction/entanglement of neighboring spacetime locations visited, or concretized by the same particle's random walk, but we never actually explored if attraction existed, or what it could imply. It is to be kept in mind when reading earlier multi-fold papers as tracked in [5]. We should have mentioned this earlier… It took us re-reading [8] to realize the importance of such a discussion.

[12] As a result, we should be clear that when we discuss the ability o the multi-fold theory to support big bounces, big crunches, or to prevent any gravity, or cosmological, singularity, we so far listed the multi-fold dark energy effects as contributing to it (although not essential as torsion and discreteness of spacetime as well as uncertainty fluctuations are the main effects). We should also have listed this effect, as countering these other effects. However, it does not change our analysis that no singularity, gravitational or cosmological, should exists in multi-fold universes; except maybe at the big bang, if localized instead of being spread across a wide region, depending on its origin as, for example, discussed in [47]). With respect to [47], this effect may contribute, and help "close the deal" at very small scales, to ensure the total collision (within an uncertainty radius and minimum length) needed to achieve an N-body gravity driven explosive total collision.

entangled virtual particles to reach further (hence stronger g_N) before weakening the effect (r is increased for a given point in the mechanism and attractive effective potential from [1]).

- Decreases of positive λ correspond to increase of g_N for the same reasons.
- Decreases of negative λ amount to increase of g_N, now dominantly due to the extra spacetime attraction.
- UV fixed point always exists, because all of these models are approximation of a discrete fractal spacetime.

With this, we recover all the behaviors of section 6. In our view, it is remarkable that, in a multifold universe, we can assign physical multi-fold explanations to the trajectories of the running couplings/constants, and the UV fixed point, based on the 2D random walks, UU, multi-fold mechanisms, and entanglement of neighboring concretized spacetime locations.

Finally, in terms of superstrings, the equivalence of superstrings at the 2D UV fixed point comes from the fact that at those scales, the Hilbert Einstein action matches the string actions (See [1,6,7]), and that in 2D this amounts to the propagation of strings through random walk along a 1D axis: the effect appears at larger scales as equivalent to the actual random jumps of particles in a discrete fractal spacetime. So yes the mathematics can coincide at such scales just like we explained in [1] that the fluctuations of particles at the horizon of a blackhole may appear also as strings. This is again the interpretation found in [1] for alleged superstrings appearing on the horizon of black holes [43] (See section 5).

The arguments that AdS(5) leaks into the multi-fold spacetime so that the 2D approximation by superstrings match random walks spacetime reconstruction to provide a smooth transition between the dual tangent spaces applies best to multi-fold universes, where the AdS/CFT correspondence is factual, and AdS(5) is physically the dual tangent space to the multi-fold spacetime [1,6,56,62,63].

9. Conclusions

We have provided an analysis that indicates that gravity is asymptotically safe, in conventional QFT approaches, superstrings and multi-fold universes. Our approach includes non-perturbative reasonings, that we argue, avoids the typical criticism of, and concerns with, the asymptotic safety program for gravity. These reasoning are non-perturbative, and relevant to many other situations.

On one end, the paper re-affirms the challenges that superstrings, supersymmetry, and most related GUTs and TOEs, are incompatible with SM (or SM_G). On the other hand, it surprisingly reintroduced superstrings as a possible model for gravity in its 2D dominant phase, where one could see a smooth (continuous) transition between the multi-fold spacetime and the dual AdS(5). This transition disappears at larger scales.

As part of our analysis, we called out for the first time an additional event at very smalls scales (in the 2D phase) of multi-fold universes: entanglements between concretized spacetime locations that can result into a negative cosmological constant/coupling or rather a negative total dark energy effect at these very small scales. At larger scale, the quantum fluctuations are expected to eliminate this attractive contribution. These effects should be tracked throughout our analyses of non-singularities in multi-fold universes, where they remain avoided.

We assert asymptotic safety of gravity for GR-based gravity in conventional universe, i.e. without any multi-fold assumptions and in multi-fold universes.

Note added on August 22, 2022: The reader will find a differently derived confirmation of these results in a subsequent papers [52].

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