Extension of an imaginary triangle through Complex Variables

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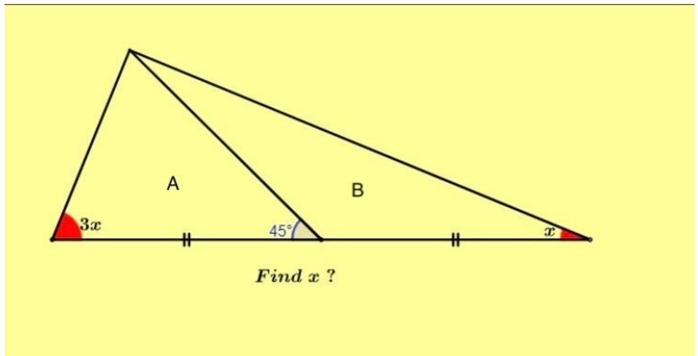
Abstract: Complex Variables has a link to general geometry in placing the geometry of squares. Given is a problem in geometry where a short-cut is taken to solve what the angle is in the given situation.



Question: If the angle opposite 45° and the angle opposite x° when added is equal to $x^4 + y^4 + z^4 = (a + b)$ given that $x^2 + y^2 + z^2 = \sqrt{\pi}$ and $x + y + z = 0$, find x° and organize the triangles to form a proof of the Pythagorean Theorem.

Answer: Given a sphere $x^2 + y^2 + z^2 = r^2$, where r=$\pi^{1/4}$ we have $x^2 + y^2 + z^2 = \sqrt{\pi}$

If $x + y + z = 0$, what is $(a + b)\ from\ x^4 + y^4 + z^4 = (a + b)$, if we use understanding from complex variables we know that π exists in a square root. By correlating the imaginary or complex component iv, we can take $z = w = 0$, so $w = u + iv \rightarrow$ $\sqrt{\sqrt{\pi}}/\sqrt{2} = x,\ -\sqrt{\sqrt{\pi}}/\sqrt{2} = y,\ 0 = z,\ so\ x + y + z = 0$, then $x^2 + y^2 + z^2 = \frac{\sqrt{\pi}}{2} + \frac{\sqrt{\pi}}{2} + 0 = \sqrt{\pi}$, this satisfies the condition so $x^4 + y^4 + z^4 = \frac{\pi}{4} + \frac{\pi}{4} + 0 = \pi/2 = a + b$. Complex Variables is used since $u\ and\ v$ cancel, the i part implies negation.

Triangle A: $45^{\circ} + 3x^{\circ} + a = 180^{\circ}$, Triangle B: $135^{\circ} + x^{\circ} + b = 180^{\circ},\ a + b = \pi/2 = 90^{\circ}$

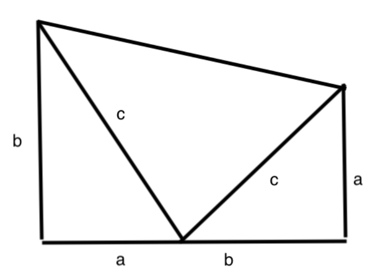
Triangle B+Triangle A= $270^{\circ} + 4x^{\circ} = 360^{\circ},\ so\ x^{\circ} = \frac{90}{4}\ or\ \frac{45}{2}$, then $x^{\circ} = 22.5^{\circ}$

Final Part 1 Answer: Angle $a + b = \frac{\pi}{2},\ so\ x^{\circ} = 22.5^{\circ} or\ \frac{\pi}{8}$

How would we check this?

Check: $Triangle\ A = 180^{\circ} = 45^{\circ} + 3 \cdot 22.5^{\circ} + a,\ a = 67.5^{\circ},\ b = 22.5^{\circ},\ Triangle\ B = 135^{\circ} + 22.5^{\circ} + b = 180^{\circ}$ ⋄

Part 2 Proof: Rearranging the structure of triangles (A and B), we create this diagram. An imaginary triangle (C) is inverted.



Using the law of similar trapezoid areas

$$\frac{1}{2}(a + b)(a + b) = ab + \frac{1}{2}c^2$$

$$\frac{1}{2}(a^2 + 2ab + b^2) = ab + \frac{1}{2}c^2$$

$$(a^2 + 2ab + b^2) = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

Thus the Pythagorean Theorem is proved by law of similar Trapezoids