

# Ultimate Acceleration Causes Quark to Obey SU(n) Symmetry

Huaiyang Cui

Department of Physics, Beihang University, Beijing, 102206, China

Email: hycui@buaa.edu.cn, hycui.blogspot.com

(July 27, 2022, submitted to viXra)

**Abstract:** In analogy with the ultimate speed  $c$ , there is an ultimate acceleration  $\beta$ , nobody's acceleration can exceed this limit  $\beta$ , for electrons and quarks,  $\beta=2.327421e+29(m/s^2)$ . Because this ultimate acceleration is a large number, any effect connecting to  $\beta$  will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, it is found that quark's matter wave must obey the SU(n) symmetry in a strong interaction process. It is completely a new aspect to quark model for its relativistic matter wave to contain SU(n) symmetry.

## 1. Introduction

This year is 99th anniversary of the initiative of de Broglie's matter wave [1-3], it is a good time for rediscovering the matter wave.

In analogy with the ultimate speed  $c$ , there is an ultimate acceleration  $\beta$ , nobody's acceleration can exceed this limit  $\beta$ , for electrons and quarks,  $\beta=2.327421e+29(m/s^2)$ . Because this ultimate acceleration is a large number, any effect connecting to  $\beta$  will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, it is found that quark's matter wave must obey the SU(n) symmetry in a strong interaction process. It is completely a new aspect to quark model for its relativistic matter wave to contain SU(n) symmetry.

## 2. How to connect the ultimate acceleration with quantum theory

In the relativity, the speed of light  $c$  is an ultimate speed, nobody's speed can exceed this limit  $c$ . The relativistic velocity  $u$  of a particle in the coordinate system  $(x_1, x_2, x_3, x_4=ict)$  satisfies

$$u_1^2 + u_2^2 + u_3^2 + u_4^2 = -c^2 \quad (1)$$

No matter what particles (electrons, molecules, neutrons, quarks), their 4-vector velocities all have the same magnitude:  $|u|=ic$ . All particles gain **equality** because of the same magnitude of the 4-velocity  $u$ . The acceleration  $a$  of a particle is given by

$$a_1^2 + a_2^2 + a_3^2 = a^2; \quad (a_4 = 0; \quad \because x_4 = ict) \quad (2)$$

Assume that particles have an ultimate acceleration  $\beta$  as limit, no particle can exceed this acceleration limit  $\beta$ . Subtracting the both sides of the above equation by  $\beta^2$ , we have

$$a_1^2 + a_2^2 + a_3^2 - \beta^2 = a^2 - \beta^2; \quad a_4 = 0 \quad (3)$$

It can be rewritten as

$$[a_1^2 + a_2^2 + a_3^2 + 0 + (i\beta)^2] \frac{1}{1 - a^2 / \beta^2} = -\beta^2 \quad (4)$$

Now, the particle subjects to an acceleration whose five components are specified by

$$\begin{aligned} \alpha_1 &= \frac{a_1}{\sqrt{1 - a^2 / \beta^2}}; & \alpha_2 &= \frac{a_2}{\sqrt{1 - a^2 / \beta^2}}; \\ \alpha_3 &= \frac{a_3}{\sqrt{1 - a^2 / \beta^2}}; & \alpha_4 &= 0; & \alpha_5 &= \frac{i\beta}{\sqrt{1 - a^2 / \beta^2}}; \end{aligned} \quad (5)$$

where  $\alpha_5$  is the newly defined acceleration in five dimensional space-time  $(x_1, x_2, x_3, x_4 = ict, x_5)$ . Thus, we have

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 + \alpha_5^2 = -\beta^2; \quad \alpha_4 = 0 \quad (6)$$

It means that the magnitude of the newly defined acceleration  $\alpha$  for every particle takes the same value:  $|\alpha| = i\beta$  (constant imaginary number), all particle accelerations gain **equality** for the sake of the same magnitude.

How to resolve the velocity  $u$  and acceleration  $\alpha$  into  $x, y$ , and  $z$  components? In realistic world, a hand can rotate a ball moving around a circular path at constant speed  $v$  with constant centripetal acceleration  $a$ , as shown in Fig.1(a).

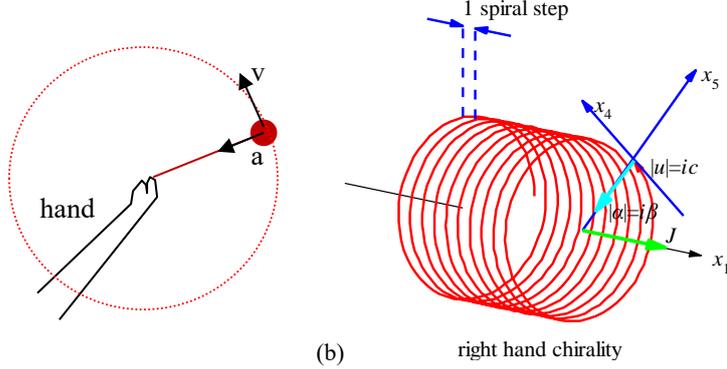


Fig.1 (a) A hand rotates a ball moving around a circular path at constant speed  $v$  with constant centripetal acceleration  $a$ . (b) The particle moves along the  $x_1$  axis with the constant speed  $|u|=ic$  in the  $u$  direction and constant centripetal force in the  $x_5$  axis at the radius  $iR$  (imaginary number).

```
<Clet2020 Script>// Clet is a C compiler [17]
double D[100],S[2000];int i,j,R,X,N;
int main(){R=50;X=50;N=600;D[0]=-50;D[1]=0;D[2]=0;D[3]=X;D[4]=0;D[5]=0;D[6]=-50;D[7]=R;D[8]=0;
D[9]=600;D[10]=10;D[11]=R;D[12]=0;D[13]=3645;
Lattice(SPIRAL,D,S);SetViewAngle(0,80,-50);DrawFrame(FRAME_NULL,1,0xfffff);
Draw("LINE,0,2,XYZ,0","-150,0,0,-50,0,0");Draw("ARROW,0,2,XYZ,10","50,0,0,150,0,0");
SetPen(2,0xff0000);Plot("POLYLINE,0,600,XYZ",S[9]);i=9+3*N-6;Draw("ARROW,0,2,XYZ,10",S[i]);
TextHang(S[i],S[i+1],S[i+2]," #if|u|=ic#t");TextHang(150,0,0," #ifx#sd1#t");SetPen(2,0x005fff);
Draw("LINE,1,2,XYZ,8","-50,0,50,-50,0,100");Draw("LINE,1,2,XYZ,8","-40,0,50,-40,0,100");
Draw("ARROW,0,2,XYZ,10","-80,0,100,-50,0,100");Draw("ARROW,0,2,XYZ,10","-10,0,100,-40,0,100");
TextHang(-50,0,110,"1 spiral step");i=9+3*N;S[i]=50;S[i+1]=10;S[i+2]=10;
Draw("ARROW,0,2,XYZ,10","50,0,0,50,80,80");TextHang(50,80,80," #ifx#sd5#t");
Draw("ARROW,0,2,XYZ,10","50,72,0,50,0,72");TextHang(50,0,72," #ifx#sd4#t");
SetPen(3,0x00ffff);Draw("ARROW,0,2,XYZ,15",S[i-3]);TextHang(S[i],S[i+1],S[i+2]," #if|alpha|=i*beta#t");
SetPen(3,0x00ff00);Draw("ARROW,0,2,XYZ,15","50,0,0,120,0,0");TextHang(110,5,5," #ifj#t");
TextHang(-60,0,-80," right hand chirality");#v07=?>A
```

In analogy with the ball in a circular path, consider a particle in one dimensional motion along the  $x_1$  axis at the speed  $v$ , in the Fig.1(b) it moves with the constant speed  $|u|=ic$  almost along the  $x_4$  axis and slightly along the  $x_1$  axis, and the constant centripetal acceleration  $|\alpha|=i\beta$  in the  $x_5$  axis at

the constant radius  $iR$  (imaginary number); the coordinate system  $(x_1, x_4=ict, x_5=iR)$  establishes a cylinder coordinate system in which this particle moves spirally at the speed  $v$  along the  $x_1$  axis. According to usual centripetal acceleration formula  $a=v^2/r$ , the acceleration in the  $x_4$ - $x_5$  plane is given by

$$a = \frac{v^2}{r} \Rightarrow i\beta = \frac{|u|^2}{iR} = -\frac{c^2}{iR} = i\frac{c^2}{R} . \quad (7)$$

Therefore, the track of the particle in the cylinder coordinate system  $(x_1, x_4=ict, x_5=iR)$  forms a shape, called as **acceleration-roll**. The faster the particle moves along the  $x_1$  axis, the longer the spiral step is.

As like a steel spring with elastic wave, the track in the acceleration-roll in Fig.1(b) can be described by a wave function whose phase changes  $2\pi$  for one spiral step. **Apparently, this wave is the de Broglie's matter wave for electrons, protons and quarks, etc.**

**Proof:** The wave function phase changes  $2\pi$  for one spiral circumference  $2\pi(iR)$ , then a small displacement of the particle on the spiral track is  $|u|d\tau=icd\tau$  in the 4-vector  $u$  direction, thus this wave phase along the spiral track is evaluated by

$$phase = \int_0^\tau \frac{2\pi}{2\pi(iR)} icd\tau = \int_0^\tau \frac{c}{R} d\tau . \quad (8)$$

Substituting the radius  $R$  into it, the wave function  $\psi$  is given by

$$\psi = \exp(-i \cdot phase) = \exp(-i \int_0^\tau \frac{c}{R} d\tau) = \exp(-i \frac{\beta}{c} \int_0^\tau d\tau) . \quad (9)$$

In the theory of relativity, we known that the integral along  $d\tau$  needs to transform into realistic line integral, that is

$$\begin{aligned} d\tau &= -c^2 \frac{d\tau}{-c^2} = (u_1^2 + u_2^2 + u_3^2 + u_4^2) \frac{d\tau}{-c^2} \\ &= (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4) \frac{1}{-c^2} \end{aligned} \quad (10)$$

Therefore, the wave function  $\psi$  is evaluated by

$$\begin{aligned} \psi &= \exp(-i \frac{\beta}{c} \int_0^\tau d\tau) \\ &= \exp(i \frac{\beta}{c^3} \int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)) \end{aligned} \quad (11)$$

This wave function may have different explanations, depending on the particle under investigation. If the  $\beta$  is replaced by the Planck constant for electrons, the wave function is given by

$$\begin{aligned} \text{assume: } \beta &= \frac{mc^3}{\hbar} \\ \psi &= \exp(\frac{i}{\hbar} \int_0^x (mu_1 dx_1 + mu_2 dx_2 + mu_3 dx_3 + mu_4 dx_4)) \end{aligned} \quad (12)$$

where  $mu_4 dx_4 = -Edt$ , it strongly suggests that the wave function is just the **de Broglie's matter wave**

[1,2,3].

In Fig.1(b), the acceleration-roll of particle moves with two distinctions: right-hand chirality and left-hand chirality. The direction of the angular momentum  $J$  would be slightly different from the  $x_1$  due to spiral precession. It is easy to calculate the ultimate acceleration  $\beta$ , the radius  $R$  and the angular momentum  $J$  in the plane  $x_4$ - $x_5$  for a spiraling electron as

$$\begin{aligned}\beta &= \frac{c^3 m}{\hbar} = 2.327421e+29 \text{ (M/s}^2\text{)} \\ R &= \frac{c^2}{\beta} = 3.861593e-13 \text{ (M)} \\ J &= \pm m |u| iR = \mp \hbar\end{aligned}\tag{13}$$

```
<Clet2020 Script>// Clet is a C compiler [17]
double beta,R,J,m,D[10];char str[200];
int main(){m=ME;beta=SPEEDC*SPEEDC*SPEEDC*m/PLANCKBAR;
R=SPEEDC*SPEEDC/beta;J=PLANCKBAR;Format(str,"beta=%e, #nR=%e, #nJ=%e",beta,R,J);
TextAt(50,50,str);ClipJob(APPEND,str);}#v07=#
```

Considering another explanation to  $\psi$  for planets in the solar system, no Planck constant can be involved. But, in a many-body system with the total mass  $M$ , the data-analysis [14] tells us that the ultimate acceleration can be rewritten in terms of **Planck-constant-like constant**  $h$  as

$$\begin{aligned}\text{assume: } \beta &= \frac{c^3}{hM} \\ \psi &= \exp\left(\frac{i}{hM} \int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)\right)\end{aligned}\tag{14}$$

The constant  $h$  will be determined by experimental observations. The papers [15,16] showed that this wave function is applicable to several many-body systems in the solar system, the wave function is called as the **acceleration-roll wave**.

Consider third explanation to  $\psi$  for atoms in cells and viruses. Typically, sound speed in water is  $v=1450\text{m/s}$ , according to  $v=f\lambda$ , the sound wavelength  $\lambda$  for frequency  $1\text{kHz}--1\text{Mhz}$  is  $1.5\text{m}--1.45\text{mm}$ . In general, the sound wavelength is larger than cell size, because cell size is about 1 micro. Thus, almost all cells and viruses live in a smaller space which is not sensitive to the sound. The acceleration-roll can provide a kind of wave with a shorter wavelength and lower frequency for various cells and viruses beyond human-sensitive sound wave [14].

Of cause, there is "ultimate distance" [14] from which the Hubble law can be derived out.

Tip: actually, ones cannot get to see the acceleration-roll of a particle in the relativistic space-time  $(x_1, x_2, x_3, x_4=ict)$ ; only get to see it in the cylinder coordinate system  $(x_1, x_4=ict, x_5=iR)$ .

### 3. Matter wave with SU(n) symmetry

In an electromagnetic field, single electron's matter wave is modified as follows

$$\phi = \exp\left(\frac{i}{\hbar} \int_{x_0}^x (mu_\mu + qA_\mu) dx_\mu\right)\tag{15}$$

where  $A$  represents the electromagnetic 4-vector potential, the integral takes from the initial point  $x_0$  to the final point  $x$  by an arbitrary mathematical path in the coherent zone. Similarly, in a closed  $N$  particle system, the  $j$ -th particle moves in an interacting field  $A$  which is the sum of the contributions from other  $N-1$  particles.

$$A_\mu(x^{(j)}) = \sum_{k=1, k \neq j}^N A_\mu^{(k)} = \sum_{k=1}^N S^{(jk)} p_\mu^{(k)} . \quad (16)$$

where superscript denotes the particle index,  $S^{(jk)}$  is the couple coefficient of momenta between the  $j$ -th particle and the  $k$ -th particle. Then  $j$ -th quark's matter wave is given by

$$\phi^{(j)} = \exp\left(\frac{i}{\hbar} \int_{x_0}^x (p_\mu^{(j)} + \sum_{k=1}^N S^{(jk)} p_\mu^{(k)}) dx_\mu^{(j)}\right) \quad (17)$$

In other words, quark's acceleration-roll only allows the matter wave to change its phase in an interaction process. For convenience, we define the **riding-wave momentum**  $R_\mu$  by

$$R_\mu = m u_\mu + q A_\mu . \quad (18)$$

then we have

$$R_\mu^{(j)} = p_\mu^{(j)} + \sum_{k=1}^N S^{(jk)} p_\mu^{(k)} , \quad (19)$$

In matrix form, that is

$$\begin{aligned} R &= (1 + S) p \\ \text{or} \\ \begin{bmatrix} R^{(1)} \\ R^{(2)} \\ \dots \\ R^{(N)} \end{bmatrix} &= (1 + \begin{bmatrix} S^{(11)} & S^{(12)} & \dots & S^{(1N)} \\ S^{(21)} & S^{(22)} & \dots & S^{(2N)} \\ \dots & \dots & \dots & \dots \\ S^{(N1)} & S^{(N2)} & \dots & S^{(NN)} \end{bmatrix}) \begin{bmatrix} p^{(1)} \\ p^{(2)} \\ \dots \\ p^{(N)} \end{bmatrix} . \end{aligned} \quad (20)$$

where 1 denotes unitary matrix, now, the matrix  $S$  represents the interactions between each other, actually its entries are regarded as smaller quantities for most particle interactions. In most cases for larger  $N$  system, the coefficient matrix  $S$  is an array hard to calculate like the 4-vector potential, so in practice  $S$  is regarded as unknown quantity determined by experiments.

**Theorem 1:** The interaction matrix  $S$  in the riding-wave momentum formula is a Hermitian matrix:  $S^+ = S$ .

**Proof:** The wave function of the  $j$ -th particle is given by

$$\begin{aligned} \phi^{(j)} &= \exp\left(\frac{i}{\hbar} \int_{0(L)}^x R_\mu^{(j)} dx_\mu\right) \\ &= \exp\left(\frac{i}{\hbar} \int_{0(L)}^x [p_\mu^{(j)}(\mathbf{x}, t) + S^{(jk)} p_\mu^{(k)}] dx_\mu\right) , \end{aligned} \quad (21)$$

where, the duplicated indices imply summation over (Einstein summation convention). We define

$$\begin{aligned} \psi^{(j)} &= \exp\left(\frac{i}{\hbar} \int_{0(L)}^x p_\mu^{(j)} dx_\mu\right) , \\ |\psi^{(j)}|^2 &= \psi^{(j)} \psi^{(j)*} = 1 \quad (\text{no sum over } j) \end{aligned} \quad (22)$$

regarding  $S$  as smaller quantities for the interactions, then we have

$$\begin{aligned}
\phi^{(j)} &= \exp\left(\frac{i}{\hbar} \int_{0(L)}^x [p_\mu^{(j)}(\mathbf{x}, t) + S^{(jk)} p_\mu^{(k)}] dx_\mu\right) \\
&= \psi^{(j)} \exp\left(\frac{i}{\hbar} \int_{0(L)}^x S^{(jk)} p_\mu^{(k)} dx_\mu\right) \\
&= \psi^{(j)} \left(1 + \frac{i}{\hbar} \int_{0(L)}^x S^{(jk)} p_\mu^{(k)} dx_\mu\right) + O(S^2)
\end{aligned} \tag{23}$$

Typically, the acceleration-roll demands the matter wave function to meet the unity requirement:

$$|\phi^{(j)}|^2 = \phi^{(j)} \phi^{(j)*} = 1 \quad (\text{no sum over } j) \tag{24}$$

Using

$$\begin{aligned}
\phi^{(j)} &= \psi^{(j)} \left(1 + \frac{i}{\hbar} \int_{0(L)}^x S^{(jk)} p_\mu^{(k)} dx_\mu\right) \\
\phi^{(j)*} &= \phi^{(j)+} = \left(1 - \frac{i}{\hbar} \int_{0(L)}^x [p_\mu^{(k)*} [S^{(kj)*} d[x_\mu]^*]] \psi^{(j)*}\right)
\end{aligned} \tag{25}$$

The transpose operation used in the above expression is a preparation for the consistency of matrix calculation, we have

$$\begin{aligned}
|\phi^{(j)}|^2 &= \phi^{(j)*} \phi^{(j)} \quad (\text{no sum over } j) \\
&= \left(1 - \frac{i}{\hbar} \int_{0(L)}^x [p_\mu^{(k)*} [S^{(kj)*} d[x_\mu]^*]] \psi^{(j)*} \psi^{(j)} \left(1 + \frac{i}{\hbar} \int_{0(L)}^x S^{(jk)} p_\mu^{(k)} dx_\mu\right)\right) \\
&= \left(1 - \frac{i}{\hbar} \int_{0(L)}^x [p_\mu^{(k)*} [S^{(kj)*} d[x_\mu]^*]] \right) \left(1 + \frac{i}{\hbar} \int_{0(L)}^x S^{(jk)} p_\mu^{(k)} dx_\mu\right) \\
&= 1 - \frac{i}{\hbar} \int_{0(L)}^x [p_\mu^{(k)*} [S^{(kj)*} d[x_\mu]^*]] + \frac{i}{\hbar} \int_{0(L)}^x S^{(jk)} p_\mu^{(k)} dx_\mu + O(S^2)
\end{aligned} \tag{26}$$

We know

$$[p_\mu^{(k)*} d[x_\mu]^*] = p_\mu^{(k)} dx_\mu \tag{27}$$

thus

$$\phi^{(j)+} \phi^{(j)} = 1 + \frac{i}{\hbar} \int_{0(L)}^x [-S^{(kj)*} + S^{(jk)}] p_\mu^{(k)} dx_\mu \tag{28}$$

The integral path is an arbitrary mathematical path (not particle tracks); therefore, the unity requirement leads to the conclusion

$$\begin{aligned}
\phi^{(j)+} \phi^{(j)} = 1 &\rightarrow -S^{(kj)*} + S^{(jk)} = 0 \\
S^+ &= S
\end{aligned} \tag{29}$$

Proof done.

**Theorem 2:** Since  $S$  is a Hermitian matrix, according to the theory of group, for  $N=3$ , the Hermitian  $S$  is a linear combination of the Gell-mann matrix set in terms of  $SU(3)$  symmetry:

$$S_{3 \times 3} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \sum_{a=1}^8 \frac{1}{2} c_a \lambda_a \quad (30)$$

where  $c_a$  are eight real small coefficients of the expansion,  $\lambda_a$  are the eight independent Gell-Mann matrices called as the eight generators of SU(3) group, the Gell-mann matrices are

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (31)$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \quad \lambda_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad (32)$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \quad (33)$$

**Proof:** This is a mathematical theorem, was proved in the theory of group. Proof omitted here.

**Theorem 3:** Since  $S$  is a Hermitian matrix, according to the theory of group, for  $N=2$ , the Hermitian  $S$  is a linear combination of the Pauli matrix set in terms of SU(2) symmetry:

$$\begin{bmatrix} R^{(1)} \\ R^{(2)} \end{bmatrix} = (1 + S) \begin{bmatrix} P^{(1)} \\ P^{(2)} \end{bmatrix} = (1 + c_1 \sigma_1 + c_2 \sigma_2 + c_3 \sigma_3) \begin{bmatrix} P^{(1)} \\ P^{(2)} \end{bmatrix}. \quad (34)$$

Where the Pauli matrices (SU(2) group) are given by (corresponding to  $\lambda_1, \lambda_2, \lambda_3$  in SU(3) group)

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (35)$$

and  $c_1, c_2, c_3$  are three independent real first order small parameters.

**Proof:** This is a mathematical theorem, was proved in the theory of group. Proof omitted here.

**Theorem 4:** The riding-wave momentum  $\mathbf{R}$  is recognized as the canonical momentum in the analytical mechanics.

**Proof:** by definition, we know

$$\begin{aligned} \mathbf{R}^{(j)} &= \mathbf{p}^{(j)} + q^{(j)} \mathbf{A}^{(j)} \\ iH^{(j)} / c &= R_4^{(j)} = p_4^{(j)} + q^{(j)} A_4^{(j)} \end{aligned} \quad (36)$$

If the  $j$ -th particle in the  $N$  particle system is in its stationary state, then the fourth component of the riding-wave momentum of the  $j$ -th particle is a constant. It is called as the Hamiltonian of the  $j$ -th particle  $H^{(j)}$ , or called as the energy of the  $j$ -th particle  $E^{(j)}$ , given by

$$R_4^{(j)} = (1 + \sum_k S^{(jk)}) p_4^{(k)}$$

$$R_4^{(j)} = p_4^{(j)} + q^{(j)} A_4^{(j)} \equiv \frac{iH^{(j)}}{c} \equiv \frac{iE^{(j)}}{c} , \quad (37)$$

where  $A_4^{(j)}$  is the electric potential at where the  $j$ -th particle experiences. In the followings, we drop the superscript (j), substituting them into the total momentum formula

$$p^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = -m^2 c^2 , \quad (38)$$

we get

$$(\mathbf{R} - q\mathbf{A})^2 - (H / c + iqA_4)^2 = -m^2 c^2 . \quad (39)$$

That is

$$H = c\sqrt{|\mathbf{R} - q\mathbf{A}|^2 + m^2 c^2} - icqA_4 \quad (40)$$

This Hamiltonian is completely the same as that in usual textbooks of electrodynamics, under this square root,  $\mathbf{R}$  is recognizable as the canonical momentum of analytical mechanics, rather than  $\mathbf{p}$ . In quantum mechanics, a stationary state definitely means the energy (or the Hamiltonian) to be a constant, so  $R_4 = \text{const}$ . Proof done.

## 4. Conclusions

This year is 99th anniversary of the initiative of de Broglie's matter wave [1-3], it is a good time for rediscovering the matter wave. In analogy with the ultimate speed  $c$ , there is an ultimate acceleration  $\beta$ , nobody's acceleration can exceed this limit  $\beta$ , for electrons and quarks,  $\beta = 2.327421e+29(\text{m/s}^2)$ . Because this ultimate acceleration is a large number, any effect connecting to  $\beta$  will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, it is found that quark's matter wave must obey the SU(n) symmetry in a strong interaction process. It is completely a new aspect to quark model for its relativistic matter wave to contain SU(n) symmetry.

## References

- [1] de Broglie, L., CRAS,175(1922):811-813, translated in 2012 by H. C. Shen in Selected works of de Broglie.
- [2] de Broglie, Waves and quanta, Nature, 112, 2815(1923): 540.
- [3] de Broglie, Recherches sur la théorie des Quanta, translated in 2004 by A. F. Kracklauer as De Broglie, Louis, On the Theory of Quanta. 1925.
- [4] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete, Phys. Rev., 1935, 47, 777-780,
- [5] D.Bohm, A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables. I, Phys. Rev., 1952, 85, 166-179,
- [6] D.Bohm, A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables. II, Phys. Rev., 1952, 85, 180-193,
- [7] R. G. Chambers, Shift of an electron interference pattern by enclosed magnetic flux[J], Phys. Rev. Lett., 1960, 5:3-5.,
- [8] A.Tonomura , N.Osakabe , T.Matsuda , T.Kawasaki , J.Endo , S.Yano , H.Yamada , Evidence for Aharonov-Bohm Effect with

- Magnetic Field Completely Shielded from Electron wave[J], Phys. Rev. Lett., 1986, 56, 8:792-795.,
- [8] N.Osakabe , T. Matsuda, T.Kawasaki , J.Endo , A.Tonomura , Experimental confirmation of Aharonov-Bohm effect using a toroidal magnetic field confined by a superconductor[J], Phys. Rev. , 1986, A 34, 2:815-819.,
- [9] L.I. Schiff, quantum mechanics, McGraw-Hill Book Company, 1968, third ed.,
- [10] E.G. Harris , Introduction to Modern Theoretical Physics, John Wiley & Sons, 1975, Vol.1 and 2,  
<https://doi.org/10.1007/BF00207145>
- [11] D.J.Griffiths, Introduction to quantum mechanics, Pearson Education,Inc., 2005,
- [12] F. Halzen, A.D.Martin, Quarks and Lepton: An Introductory Course in Modern Particle Physics, John Wiley & Sons,Inc., 1984,
- [13] Huaiyang Cui, Relativistic Matter Wave and Its Explanation to Superconductivity: Based on the Equality Principle, Modern Physics, 10,3(2020)35-52. <https://doi.org/10.12677/MP.2020.103005>
- [14] Huaiyang Cui, Relativistic Matter Wave and Quantum Computer, Amazon Kindle ebook, 2021.
- [15] Huaiyang Cui, Evidence of Planck-Constant-Like Constant in Five Planetary Systems and Its Significances, viXra:2204.0133, 2022.
- [16] Huaiyang Cui, Approach to enhance quantum gravity effects by ultimate acceleration, viXra:2205.0053, 2022.
- [17] Clet Lab, Clet: a free C compiler, download at <https://drive.google.com/file/d/1OjKqANcgZ-9V56rgcoMtOu9w4rP49sgN/view?usp=sharing>