Knot in geometrical optics

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We propose there exists a knot in geometrical optics as a solution of eikonal equation. We study the relations of a knot-refractive index and topology-quantized electric charge.

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A knot is by definition a smooth-embedding of a circle in $\mathbb{R}^3$[1]. Two knots are equivalent if one knot can be deformed continuously into the other without crossing itself[1]. Here, $\mathbb{R}^3$ is a 3-dimensional Euclidean (flat) space. The knot appears everywhere in physics e.g. in Maxwell's electrodynamics, Skyrme theory, quantum chromodynamics (QCD), fluid dynamics, atomic physics, plasma physics, polymer physics, condensed matter physics, even in Einstein's theory[2].

The existence of knot in the Maxwell's theory so far has not been well known, because people believed that the theory, being *linear*, has no structure for topological object[2]. It is because of a topological theory must be *non-linear*[4]. In electrodynamics, a knot could be formed by bending the electric and magnetic field lines so that they could form *closed loops* and thus be *linked*[2], and this linking could provide the topological structure[2]. The electric and magnetic potentials are related to the electic and magnetic helicities respectively and in turn these (integer) helicities are related to a knot.

How to obtain topological object such as a knot from the Maxwell's theory, a linear theory? The standard electromagnetic equations (the Maxwell's theory) can be derived from *an underlying structure which is both nonlinear and topological*[4]. There is thus *a hidden nonlinearity*, shown by the fact that although the electric and magnetic fields obey the linear Maxwell equations, *not all its solutions are admissible*, since they must verify the condition that *the magnetic helicity be an integer number*[4] as written below

$$h_m = \int_{\mathbb{R}^3} \vec{A} \cdot \vec{B} \; d^3 r = n \qquad (1)$$

where h_m is the magnetic helicity, $\vec{A}$ is the vector potential, $\vec{B}$ is the magnetic vector field and n is integer number. The conditions (1) is called *the topological quantum conditions*[4].

In order to construct an underlying structure which is both non-linear and topological, suppose that we have *a subset of the electric and magnetic field* which is *locally equivalent* to the fields of Maxwell standard theory but *globally different* since this a subset of fields obey the topological quantum conditions (1) whereas the fields of Maxwell theory are not at all admissible as the solutions[5].

Because there exists a knot in the Maxwell's theory[2,4] *and in fact the geometrical optics (the eikonal equation) can be derived from the Maxwell's theory (Maxwell equations)*[3]*, we propose that there exists a knot in the geometrical optics as a solution of the eikonal equation. What is the form of a knot-refractive index relation?*

How is the quantization of electric charge viewed from topology?

The work is still in progress.

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