A Proof of the Erdös-Straus Conjecture

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Abstract

In this article, we classify gradually positive integers≥2, and express every class of positive integers into a sum of 3 unit fractions.

First, divide positive integers ≥2 into 8 kinds, and then formulate each of 7 kinds within them into a sum of 3 unit fractions.

For the unsolved kind, divide it into 3 genera, and then formulate each of 2 genera within them into a sum of 3 unit fractions.

For the unsolved genus, further divide it into 5 sorts, and formulate each of 3 sorts within them into a sum of 3 unit fractions.

For two unsolved sorts $\frac{4}{49+120c}$ and $\frac{4}{121+120c}$ where c≥0, let each of them be expressed as a sum of an unit fraction plus a true fraction, and that take out the unit fraction and call it $\frac{1}{Y}$. After that, if the true fraction can equal an unit fraction, then we follow the formula that Ernst G. Straus made to transform this unit fraction or $\frac{1}{Y}$ into the sum of two each other's- distinct unit fractions, such that each unsolved sort becomes a sum of 3 unit fractions; if the true fraction can not equal an unit fraction,

then we let it to equal the sum of an unit fraction plus another true fraction, and that take out the unit fraction and call it $\frac{1}{Z}$, next, prove that another proper fraction can be identically converted to $\frac{1}{X}$.

Due to c≥0, above two cases exist surely when c is taken different values.

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1. Introduction

The Erdös-Straus conjecture relates to Egyptian fractions. In 1948, Paul Erdös conjectured that for any integer n≥2, there are $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ invariably, where X, Y and Z are positive integers; [1].

Later, Ernst G. Straus conjectured that X, Y and Z satisfy X≠Y, Y≠Z and Z≠X, because there are the convertible formulas $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ and $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ where r≥1; [2].

Thus, the Erdös conjecture and the Straus conjecture are equivalent from each other, and they are called the Erdös-Straus conjecture collectively.

As a general rule, the Erdös-Straus conjecture states that for every integer n≥2, there are positive integers X, Y and Z, such that $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

Yet, it remains a conjecture that has neither is proved nor disproved; [3].

2. Divide integers≥2 into 8 kinds and formulate 7 kinds of them

First, divide integers≥2 into 8 kinds, i.e. 8k+1with k≥1, and 8k+2, 8k+3, 8k+4, 8k+5, 8k+6, 8k+7, 8k+8, where k≥0, and arrange them as follows:

K\n:	8k+1,	8k+2,	8k+3,	8k+4,	8k+5,	8k+6,	8k+7,	8k+8
0,	①,	2,	3,	4,	5,	6,	7,	8,
1,	9,	10,	11,	12,	13,	14,	15,	16,
2,	17,	18,	19,	20,	21,	22,	23,	24,
…,	…,	…,	…,	…,	…,	…,	…,	…,

Excepting n=8k+1, formulate each of other 7 kinds into $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$:

(1) For n=8k+2, there are $\frac{4}{8k+2}=\frac{1}{4k+1}+\frac{1}{4k+2}+\frac{1}{(4k+1)(4k+2)}$;

(2) For n=8k+3, there are $\frac{4}{8k+3}=\frac{1}{2k+2}+\frac{1}{(2k+1)(2k+2)}+\frac{1}{(2k+1)(8k+3)}$;

(3) For n=8k+4, there are $\frac{4}{8k+4}=\frac{1}{2k+3}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+1)(2k+2)}$;

(4) For n=8k+5, there are $\frac{4}{8k+5}=\frac{1}{2k+2}+\frac{1}{(8k+5)(2k+2)}+\frac{1}{(8k+5)(k+1)}$;

(5) For n=8k+6, there are $\frac{4}{8k+6}=\frac{1}{4k+3}+\frac{1}{4k+4}+\frac{1}{(4k+3)(4k+4)}$;

(6) For n=8k+7, there are $\frac{4}{8k+7}=\frac{1}{2k+3}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+2)(8k+7)}$;

(7) For n=8k+8, there are $\frac{4}{8k+8}=\frac{1}{2k+4}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+3)(2k+4)}$.

By this token, above 7 kinds of integers are suitable to the conjecture.

3. Divide the unsolved kind into 3 genera and formulate 2 genera of them

For the unsolved kind when n=8k+1 with k≥1, let us divide it by 3 and get 3 genera: 1. the remainder is 0, when k=1+3t, where t≥0;

2. the remainder is 2, when k=2+3t, where t≥0;

3. the remainder is 1, when k=3+3t, where t≥0.

These 3 genera of odd numbers are shown below:

k:	1, 2, 3,	4, 5, 6,	7, 8, 9,	10, 11, 12,	13, 14, 15, ...
8k+1:	9, 17, 25,	33, 41, 49,	57, 65, 73,	81, 89, 97,	105, 113, 121, ...
the remainder:	0, 2, 1,	0, 2, 1,	0, 2, 1,	0, 2, 1,	0, 2, 1, ...

Excepting the genus (3), we formulate other 2 genera as follows:

(8) For $\frac{8k+1}{3}$ where the remainder is equal to 0, there are

$$\frac{4}{8k+1}=\frac{1}{\frac{8k+1}{3}}+\frac{1}{8k+2}+\frac{1}{(8k+1)(8k+2)}$$

Due to k=1+3t and t≥0, there are $\frac{8k+1}{3}=8t+3$, so we confirm that $\frac{8k+1}{3}$ in the above equation is an integer.

(9) For $\frac{8k+1}{3}$ where the remainder is equal to 2, there are

$$\frac{4}{8k+1}=\frac{1}{\frac{8k+2}{3}}+\frac{1}{8k+1}+\frac{1}{\frac{(8k+1)(8k+2)}{3}}$$

Due to k=2+3t and t≥0, there are $\frac{8k+2}{3}=8t+6$, so we confirm that $\frac{8k+2}{3}$ and $\frac{(8k+1)(8k+2)}{3}$ in the above equation are two integers.

4. Divide the unsolved genus into 5 sorts and formulate 3 sorts of them

For the unsolved genus $\frac{8k+1}{3}$, its remainder equals 1 where k=3+3t and t≥0, as listed above 8k+1=25, 49, 73, 97, 121 etc. So divide them into 5 sorts: 25+120c, 49+120c, 73+120c, 97+120c and 121+120c where c≥0, *ut infra*.

C\n:	25+120c,	49+120c,	73+120c,	97+120c,	121+120c,
0,	25,	49,	73,	97,	121,
1,	145,	169,	193,	217,	241,
2,	265,	289,	313,	337,	361,
…,	…,	…,	…,	…,	…,

Excepting n=49+120c and n=121+120c, formulate other 3 sorts, they are:

(10) For n=25+120c, there are $\frac{4}{25+120c}=\frac{1}{25+120c}+\frac{1}{50+240c}+\frac{1}{10+48c}$;

(11) For n=73+120c, there are

$$\frac{4}{73+120c}=\frac{1}{(73+120c)(10+15c)}+\frac{1}{20+30c}+\frac{1}{(73+120c)(4+6c)};$$

(12) For n=97+120c, there are

$$\frac{4}{97+120c}=\frac{1}{25+30c}+\frac{1}{(97+120c)(50+60c)}+\frac{1}{(97+120c)(10+12c)}.$$

For each of preceding 12 equations which express $\frac{4}{n}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$, please each reader self to make a check respectively.

5. Prove the sort $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

For a proof of the sort $\frac{4}{49+120c}$, it means that when c is equal to each of positive integers plus 0, there are always $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

After c is given each of values from smallest to largest, the fraction $\frac{4}{49+120c}$ can be substituted by the sum of an unit fraction plus a proper fraction, and that there are infinite more identical sums like this, yet there is no a repetition in all such fractions, as listed below:

$$\frac{4}{49+120c}$$

$$=\frac{1}{13+30c}+\frac{3}{(13+30c)(49+120c)}$$

$$=\frac{1}{14+30c}+\frac{7}{(14+30c)(49+120c)}$$

$$=\frac{1}{15+30c}+\frac{11}{(15+30c)(49+120c)}$$

…

$$=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$$, where α and c ≥ 0.

…

As listed above, undoubtedly, we can first let $\frac{1}{13+\alpha+30c}=\frac{1}{Y}$.

If $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ can be expressed as an unit fraction $\frac{1}{W}$ where W is a positive integer, and α and c ∈positive integers plus 0, then there are $\frac{4}{49+120c}=\frac{1}{Y}+\frac{1}{W}$. Next, we follow the formula $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ or $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ to transform $\frac{1}{W}$ or $\frac{1}{Y}$ as two each other's-distinct unit fractions, such that $\frac{4}{49+120\mathrm{c}}$ is equal to $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ in form.

For example, when α=1 and c=0, $\frac{4}{49+120c}=\frac{4}{49}$, $\frac{1}{13+\alpha+30\mathrm{c}}=\frac{1}{14}$, and

$$\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}=\frac{1}{2\times49}=\frac{1}{2(2\times49)}+\frac{1}{2(2\times49)}=\frac{1}{2\times49+1}+\frac{1}{2\times49(2\times49+1)},$$

then we get $\frac{4}{49}=\frac{1}{14}+\frac{1}{2\times49+1}+\frac{1}{2\times49(2\times49+1)}$.

For another example, when α=1 and c=7, $\frac{4}{49+120c}=\frac{4}{889}$, $\frac{1}{13+\alpha+30c}=\frac{1}{224}$ and

$$\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}=\frac{1}{224\times127}=\frac{1}{2\times28448}+\frac{1}{2\times28448}=\frac{1}{28448+1}+\frac{1}{28448(28448+1)},$$

then we get $\frac{4}{889}=\frac{1}{224}+\frac{1}{28448+1}+\frac{1}{28448(28448+1)}$.

It is necessary to note that $\frac{4}{49+120c}$ cannot be expressed as the sum of two identical unit fractions, because either of two identical fractions which

$\frac{4}{49+120c}$ is divided into, its numerator is an even number 2, yet its denominator is an odd number.

If $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ cannot be expressed as an unit fraction $\frac{1}{W}$,then from

$\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}=\frac{1}{(13+\alpha+30c)(49+120c)}+\frac{4\alpha+2}{(13+\alpha+30c)(49+120c)}$ to let

$\frac{1}{(13+\alpha+30c)(49+120c)}=\frac{1}{Z}$, next, let us prove $\frac{4\alpha+2}{(13+\alpha+30c)(49+120c)}=\frac{1}{X}$

where α and c ∈positive integers plus 0, *ut infra*.

***Proof*·** First, let us compare the values of the numerator 4α+2 and the denominator (13+α+30c)(49+120c) .

Since there is c ∈positive integers plus 0, then 13+α+30c can always be greater than 4α+2. And then, we just take 13+α+30c as the denominator temporarily, while reserve 49+120c for later.

In the fraction $\frac{4\alpha+2}{13+\alpha+30c}$, since the numerator 4α+2 is an even number, in addition, the reserved 49+120c is an odd number, so the denominator 13+α+30c must be an even numbers. Only in this case, it can reduce the fraction to become possibly an unit fraction, so α in the denominator 13+α+30c can only be each of positive odd numbers.

After α is assigned to odd numbers 1, 3, 5 and otherwise, the numerator and the denominator of the fraction $\frac{4\alpha+2}{13+\alpha+30c}$ divided by 2, then the fraction $\frac{4\alpha+2}{13+\alpha+30c}$ is turned into the fraction $\frac{3+4k}{7+k+15c}$ identically, where c∈positive integers plus 0, $\alpha=2k+1$ and $k\geq1$. If let k=0, then the numerator is 3 and the denominator is 7+15c, since 15c are integral multiples of 3 and 7 is not, then $\frac{3+4k}{7+k+15c}$ cannot become an unit fraction, so we abandon $\alpha=1$, i.e. k=0.

After assigning values of k from small to large to the fraction $\frac{3+4k}{7+k+15c}$, there are $\frac{3+4k}{7+k+15c}=\frac{7}{8+15c}$, $\frac{11}{9+15c}$, $\frac{15}{10+15c}$ ...

Such being the case, letting the numerator and the denominator of the fraction $\frac{3+4k}{7+k+15c}$ divided by 3+4k, then we get an indeterminate unit fraction, and its denominator is $\frac{7+k+15c}{3+4k}$, and its numerator is 1.

Thus, we are necessary to prove that the denominator $\frac{7+k+15c}{3+4k}$ be able to become a positive integer in the case where $k\geq1$ and c ∈positive integers plus 0.

For the fraction $\frac{7+k+15c}{3+4k}$, its numerator 7+k+15c is able to form infinite more consecutive integers after k and c are assigned values, while the denominator is only equal to each individual odd number of 3+4k.

After k is assigned to values from small to large, $\frac{7+k+15c}{3+4k}$ is equal to $\frac{8+15c}{7}$, $\frac{9+15c}{11}$, $\frac{10+15c}{15}$, ...

For each of 3+4k after k is assigned values, it can always seek own integral multiples within infinite more consecutive positive integers of 7+k+15c after k and c are given their respective values, such that $\frac{7+k+15c}{3+4k}$ becomes at least one batch of positive integers, no matter what positive odd number that 3+4k is equal to, where k≥1.

For example, when k=1, there are $\frac{7+k+15c}{3+4k}=\frac{8+15c}{7}=14+15s$ where c=6+7s, and s is equal to each of positive integers plus 0.

For another example, when k=8, there are $\frac{7+k+15c}{3+4k}=\frac{15+15c}{35}=3+3s$ where c=6+7s, and s is equal to each of positive integers plus 0.

It is obvious that when c is equal to a kind of positive integers, corresponding $\frac{7+k+15c}{3+4k}$ is equal to at least one kind of positive integers.

If c is equal to every kind of positive integers, then corresponding $\frac{7+k+15c}{3+4k}$ is equal to its all kinds of positive integers, and vice versa.

That is to say, for $\frac{7+k+15c}{3+4k}$ as the denominator of the aforementioned indeterminate unit fraction, when k is assigned any integer≥1, it can become a kind of positive integers on the premise that c takes its due values, and each such positive integer is represented by μ.

So, the fraction $\frac{3+4k}{7+k+15c}$ becomes infinite more unit fractions of $\frac{1}{\mu}$.

For each unit fraction $\frac{1}{\mu}$, multiply its denominator by 49+120c reserved, we get the unit fraction $\frac{1}{\mu(49+120c)}$, and let $\frac{1}{\mu(49+120c)}=\frac{1}{X}$.

To sum up, in the case where $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ cannot be expressed as an unit fraction $\frac{1}{W}$, we have proved that there are affirmatively $\frac{4}{49+120c}=\frac{1}{\mu(49+120c)}+\frac{1}{13+\alpha+30c}+\frac{1}{(13+\alpha+30c)(49+120c)}$ where μ is a positive integer and $\mu=\frac{7+k+15c}{3+4k}$, k≥1, α=2k+1and c∈positive integers plus 0.

Because enable $\frac{4}{49+120c}$ to become the sum of two terms, there are only two cases where these two terms are unit fractions and only one within these two terms is an unit fraction, and that we have proved that these two cases can be identically transformed to $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ where c≥0.

6. Prove the sort $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

The proof in this section is exactly similar to that in the section 5.

Namely, for a proof of the sort $\frac{4}{121+120c}$, it means that when c is equal to each of positive integers plus 0, there always is $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

After c is given each of values from smallest to largest, the fraction $\frac{4}{121+120c}$ can be substituted by the sum of an unit fraction plus a proper fraction, and that there are infinite more identical sums like this, yet there is no a repetition in all such fractions, as listed below:

$$\frac{4}{121+120c}$$

$$=\frac{1}{31+30\mathrm{c}}+\frac{3}{(31+30c)(121+120c)}$$

$$=\frac{1}{32+30\mathrm{c}}+\frac{7}{(32+30c)(121+120c)}$$

$$=\frac{1}{33+30\mathrm{c}}+\frac{11}{(33+30c)(121+120c)}$$

…

$$=\frac{1}{31+\alpha+30\mathrm{c}}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$$, where α and c ≥ 0.

…

As listed above, undoubtedly, we can first let $\frac{1}{31+\alpha+30c}=\frac{1}{Y}$.

If $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ can be expressed as an unit fraction $\frac{1}{V}$ where V is a positive integers, and α and c ∈positive integers plus 0, then there are $\frac{4}{121+120\mathrm{c}}=\frac{1}{Y}+\frac{1}{V}$. Next, we follow the formula $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ or $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ to transform $\frac{1}{V}$ or $\frac{1}{Y}$ as two each other's-distinct unit fractions, such that $\frac{4}{121+120c}$ is equal to $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ in form.

For example, when α=2 and c=0, $\frac{4}{121+120c}=\frac{4}{121}$, $\frac{1}{31+\alpha+30c}=\frac{1}{33}$, and

$$\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}=\frac{1}{3\times 121}=\frac{1}{2(3\times 121)}+\frac{1}{2(3\times 121)}=\frac{1}{3\times 121+1}+\frac{1}{3\times 121(3\times 121+1)},$$

then we get $\frac{4}{121}=\frac{1}{33}+\frac{1}{3\times 121+1}+\frac{1}{3\times 121(3\times 121+1)}$.

For another example, when α=1 and c=5, $\frac{4}{121+120c}=\frac{4}{721}$, $\frac{1}{31+\alpha+30c}=\frac{1}{182}$ and

$$\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}=\frac{1}{182\times 103}=\frac{1}{2\times 18746}+\frac{1}{2\times 18746}=\frac{1}{18746+1}+\frac{1}{18746(18746+1)},$$

then we get $\frac{4}{721}=\frac{1}{182}+\frac{1}{18746+1}+\frac{1}{18746(18746+1)}$.

It is necessary to note that $\frac{4}{121+120c}$ cannot be expressed as the sum of two identical unit fractions, because either of two identical fractions which $\frac{4}{121+120c}$ is divided into, its numerator is an even number 2, yet its denominator is an odd number.

If $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ cannot be expressed as an unit fraction $\frac{1}{V}$, then from

$$\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}=\frac{1}{(31+\alpha+30c)(121+120c)}+\frac{4\alpha+2}{(31+\alpha+30c)(121+120c)}$$ to

let $\frac{1}{(31+\alpha+30c)(121+120c)}=\frac{1}{Z}$, next prove $\frac{4\alpha+2}{(31+\alpha+30c)(121+120c)}=\frac{1}{X}$

where α and c ∈positive integers plus 0, *ut infra*.

Proof**·** First, let us compare the values of the numerator 4α+2 and the denominator (31+α+30c)(121+120c) .

Since there is c $\in$positive integers plus 0, then, 31+α+30c can always be greater than 4α+2. And then, we just take 31+α+30c as the denominator temporarily, while reserve 121+120c for later.

In the fraction $\frac{4\alpha+2}{31+\alpha+30c}$, since the numerator 4α+2 is an even number, in addition, the reserved 121+120c is an odd number, so the denominator 31+α+30c must be an even numbers. Only in this case, it can reduce the fraction to become possibly an unit fraction, so α in the denominator 31+α+30c can only be each of positive odd numbers.

After α is assigned to odd numbers 1, 3, 5 and otherwise, the numerator and the denominator of the fraction $\frac{4\alpha+2}{31+\alpha+30c}$ divided by 2, then the fraction $\frac{4\alpha+2}{31+\alpha+30c}$ is turned to the fraction $\frac{3+4k}{16+k+15c}$ identically, where c $\in$positive integers plus 0, k≥1, and α=2k+1. If let k=0, then the denominator is 16+15c and the numerator is 3, since 15c are integral multiples of 3 and 16 is not, then $\frac{3+4k}{16+k+15c}$ cannot become an unit fraction, so we abandon α=1, i.e. k=0.

After assigning values of k to from small to large to the fraction $\frac{3+4k}{16+k+15c}$, there are $\frac{3+4k}{16+k+15c}=\frac{7}{17+15c}$, $\frac{11}{18+15c}$, $\frac{15}{19+15c}$...

Such being the case, letting the numerator and the denominator of the fraction $\frac{3+4k}{16+k+15c}$ divided by 3+4k, then we get an indeterminate unit fraction, and its denominator is $\frac{16+k+15c}{3+4k}$, and its numerator is 1.

Thus, we are necessary to prove that the denominator $\frac{16+k+15c}{3+4k}$ be able to become a positive integer in the case where k≥1 and c ∈positive integers plus 0.

For the fraction $\frac{16+k+15c}{3+4k}$, its numerator 16+k+15c is able to form infinite more consecutive positive integers after k and c are assigned values, while the denominator is only equal to each individual odd number of 3+4k.

After k is assigned values to from small to large, $\frac{16+k+15c}{3+4k}$ is equal to

$\frac{17+15c}{7}$, $\frac{18+15c}{11}$, $\frac{19+15c}{15}$, ...

For each of 3+4k after k is assigned values, it can always seek own integral multiples within infinite more consecutive positive integers of 16+k+15c after k and c are given their respective values, such that $\frac{16+k+15c}{3+4k}$ becomes at least one batch of positive integers, no matter what positive odd number that 3+4k is equal to, where k≥1.

For example, when k=2, there are $\frac{16+k+15c}{3+4k}=\frac{18+15c}{11}=3+15s$ where c=1+11s, and s is equal to each of positive integers plus 0.

Also, when k=4, there are $\frac{16+k+15c}{3+4k}=\frac{20+15c}{19}=5+30s$ where c=5+38s, and s is equal to each of positive integers plus 0.

It is obvious that when c is equal to a kind of positive integers, corresponding $\frac{16+k+15c}{3+4k}$ is equal to at least one kind of positive integers. If c is equal to every kind of positive integers, then corresponding $\frac{16+k+15c}{3+4k}$ is equal to its all kinds of positive integers, and vice versa.

That is to say, for $\frac{16+k+15c}{3+4k}$ as the denominator of the aforementioned indeterminate unit fraction, when k is assigned any integer≥1, it can become a kind of positive integers on the premise that c takes its due values, and each such positive integer is represented by λ.

So, the fraction $\frac{3+4k}{16+k+15c}$ becomes infinite more unit fractions of $\frac{1}{\lambda}$.

For each unit fraction $\frac{1}{\lambda}$, multiply its denominator by 121+120c reserved, we get the unit fraction, $\frac{1}{\lambda(121+120c)}$ and let $\frac{1}{\lambda(121+120c)}=\frac{1}{X}$.

To sum up, in the case where $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ cannot be expressed as an unit fraction $\frac{1}{V}$, we have proved that there are affirmatively $\frac{4}{121+120c}=\frac{1}{\lambda\,(121+120c)}+\frac{1}{31+\alpha+30c}+\frac{1}{(31+\alpha+30c)(121+120c)}$ where λ is a positive integer and $\lambda=\frac{16+k+15c}{3+4k}$, k≥1, α=2k+1 and c∈positive integers plus 0.

Because enable $\frac{4}{121+120c}$ to become the sum of two terms, there are only two cases where these two terms are unit fractions and only one within these two terms is an unit fraction, and that we have proved that these two cases can be identically transformed to $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ where c≥0.

The proof was thus brought to a close. As a consequence, the Erdös-Straus conjecture is tenable.

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References

[1] J. W. Sander, *On* $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ *and Iwaniec' Half Dimensional Sieve*; Journal of Number Theory, Volume 46, Issue 2, February 1994, Pages 123-136;

https://www.sciencedirect.com/science/article/pii/S0022314X84710080?via%3Dihub

[2] Wolfram Math world, the web's most extensive mathematics resource;

https://mathworld.wolfram.com/Erdos-StrausConjecture.html

[3] Konstantine Zelator, *An ancient Egyptian problem: the diophantine equation* $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$, *n>or=2*; arXiv: 0912.2458.