

Maxwell's Equations and the Principles of Electromagnetism

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Abstract: Maxwell's equations are a set of four equations that theoretically predict the existence of "electromagnetic waves." Based on theoretical analysis and experimental verification, this study proves that Maxwell's "displacement current" hypothesis is untrue; that is, a changing electric field in a vacuum cannot induce a magnetic field. Similarly, a changing magnetic field in a vacuum cannot induce an electric field; that is, Faraday's law of electromagnetic induction is not true in a vacuum. **Therefore, there are no "electromagnetic waves" in a vacuum. It is the electric field waves, not the "electromagnetic waves" that achieve wireless communication.** As an approximate empirical formula, Faraday's law of electromagnetic induction is widely used in metal coils, but it is incomplete in theory. **In this study, a magnetic field current and an equipotential metal current ring with the "zero resistance" characteristic are proposed, which theoretically perfects Faraday's law of electromagnetic induction.** It is also revealed that a transformer, motor, and other components of electrical equipment have the highest electromagnetic energy conversion efficiency when the magnetic circuit and electric circuit are symmetrical. In Maxwell's equations, two out of the four equations are incomplete or untrue. Based on the physical origin relationship among the charge, current, electric field, and magnetic field, this study summarizes three fundamental laws of electromagnetics: Coulomb's law, Lorentz's law of magnetic field induction, and Lorentz's law of magnetic field force. An electric field, a magnetic field, and a light field are the position characteristic, velocity characteristic, and acceleration characteristic of a charge, respectively. Therefore, the propagation speeds of an electric field, a magnetic field, and a light field in a vacuum are equal, all of them are C , which is self-consistent and reasonable. An electric field, a magnetic field, and a light field cannot induce each other in a vacuum. However, an electric field, a magnetic field, and a light field can be indirectly induced each other with the participation of a charge. **This study denies the existence of "electromagnetic waves" and improves Faraday's law of electromagnetic induction, which will have a profound impact on scientific discovery and technological progress.**

Keywords: Maxwell's equations; displacement current; electromagnetic wave; equipotential metal current loop; magnetic field current; Faraday's law of electromagnetic induction; electric field wave; magnetic field wave; light field wave; three fundamental laws of electromagnetism; Coulomb's law; Lorentz's law of magnetic field induction; Lorentz law of magnetic field force.

1. Introduction

In 2004, the British scientific magazine *Physics World* organized an event in which the readers were invited to choose the greatest formula in the history of science. As a result, Maxwell's equations topped the list, overtaking Einstein's mass-energy equation, Newton's second law, the Schrödinger equation, and other equations. Maxwell's equations reveal the common relationship among charge, current, an electric field, and a magnetic field, unifying electricity and magnetism in a nearly perfect way, and predicting that light is an electromagnetic wave.

Maxwell's equations were the greatest scientific achievement of physics in the 19th century. Maxwell's equations were also the technological foundations of the second industrial revolution in the 19th century, which was characterized by electrification, and of modern society, which is characterized by microelectronics informatization. Maxwell unified light, electricity, and magnetism and completed the second great synthesis in the history of science. At the Centenary Celebration of Maxwell's birth, Einstein praised Maxwell for making the most profound and fruitful change in physics since Newton. The following are Maxwell's equations in integral form.

$$\oiint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0} \quad (1-1)$$

$$\oiint_S \mathbf{B} \cdot d\mathbf{s} = 0 \quad (1-2)$$

$$\oint_L \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\Phi_B}{dt} \quad (1-3)$$

$$\oint_L \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 \mathbf{I}_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad (1-4)$$

Eq. (1-1) is Gauss's law for an electric field. It states that the electric flux passing through a certain closed surface is proportional to the amount of charge Q enclosed by the closed surface, and the coefficient is $1/\epsilon_0$. The electric field is the active field.

Eq. (1-2) is Gauss's law for a magnetic field. It states that the magnetic flux passing through a certain closed surface must be equal to 0. Since there is no magnetic monopole in nature, the N pole and the S pole cannot be separated; that is, the magnetic field is a passive field.

Eq. (1-3) is Faraday's law of electromagnetic induction. The law states that a magnetic field induces an electric field; that is, the induced electromotive force in a closed coil is proportional to the rate of change of the magnetic flux passing through the cross-section of the coil, and the coefficient is -1 .

Eq. (1-4) is the Ampere-Maxwell law. According to Ampere's circuital law, the line integral of the magnetic induction intensity \mathbf{B} along a closed curve L is equal to μ_0 multiplied by the current passing through the closed curve L .

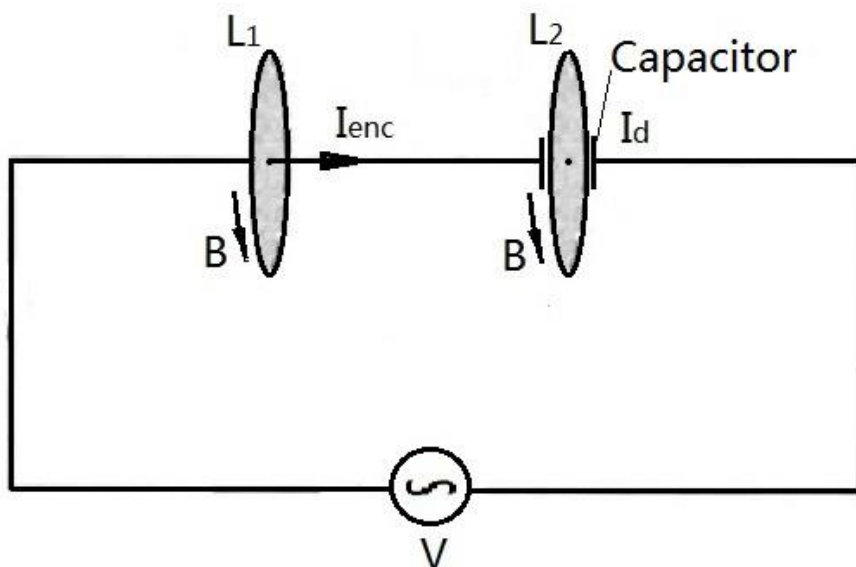


Figure 1.1 Schematic representation of the Ampere-Maxwell law

Figure 1.1 shows a simple circuit containing a capacitor. A conducting current I_{enc} flows through the cross-section of the closed curve L_1 ; hence,

$$\oint_{L_1} B \cdot d\ell = \mu_0 I_{\text{enc}}$$

The cross-section of the closed curve L_2 is between two plates of the capacitor, and there is no conducting current I_{enc} passing through, but there is an electric field and electric flux between the two plates. Therefore, Maxwell introduced the “displacement current” hypothesis in 1865 and defined the “displacement current” I_d as

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

There is a displacement current I_d passing through the cross-section of the closed curve L_2 . Then,

$$\oint_{L_2} B \cdot d\ell = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell introduced the “displacement current” hypothesis and extended Ampere’s circuital law to the full current law, that is, Ampere-Maxwell law.

$$\oint_L B \cdot d\ell = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad (1-4)$$

Ampere-Maxwell law reveals that a magnetic field can be induced by a “displacement current,” that is, by a changing electric field.

According to the Ampere-Maxwell law in Equation (1-4), a changing electric field induces a magnetic field. According to Faraday’s law of electromagnetic induction in Equation (1-3), a changing magnetic field induces an electric field. The electric field and the magnetic field are closely linked and induce each other to form unified electromagnetic waves, as shown in Figure 1.2.

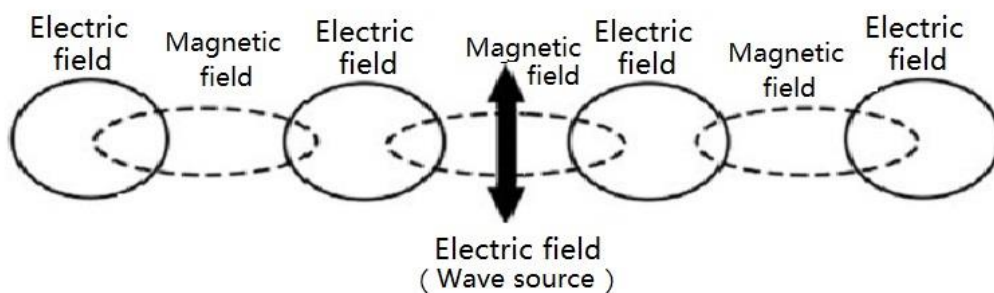


Figure 1.2 Schematic of electromagnetic waves

Maxwell introduced the “displacement current” hypothesis in 1865 that theoretically predicted the existence of electromagnetic waves and inferred from Maxwell’s equations that light was an electromagnetic wave. These ideas were astonishing to scientists throughout the world. The experimental conditions at that time could not prove that a changing electric field induced a magnetic field and a changing magnetic field induced an electric field. The electromagnetic wave theory was not accepted by most scientists. Only a few scientists in the world were willing to accept and support this theory, and the German physicist Hertz was one of them.

In 1887, eight years after Maxwell's death, Hertz experimentally proved the existence of electromagnetic waves. The experimental setup is shown in Figure 1.3.

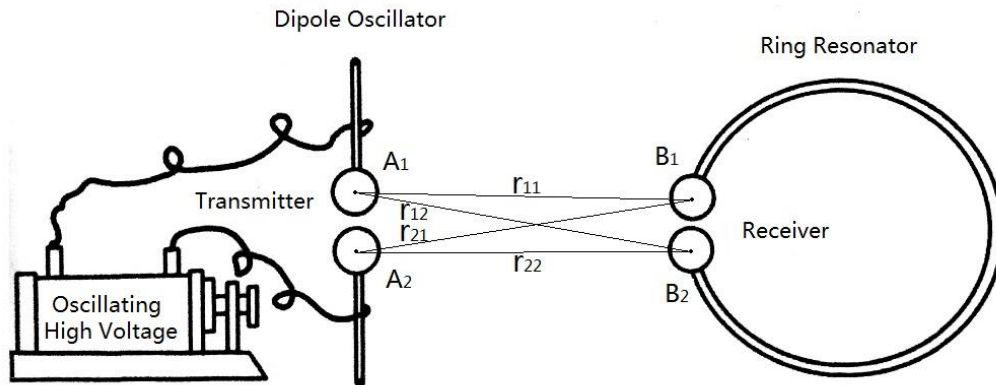


Figure 1.3 Diagram of the experimental device designed by Hertz for the proof of electromagnetic waves

The experimental device was very simple and was mainly composed of an electromagnetic wave transmitter and receiver. The electromagnetic wave transmitter used a dipole oscillator, as shown in Figure 1.3. The transmitter consisted of two metal rods that were equipped with copper balls A_1 and A_2 , and there was a gap between the two copper balls. An oscillating high-voltage electric field was generated between the copper balls A_1 and A_2 , and a dazzling spark was generated to excite an electromagnetic wave that propagated outward. The detector used a ring resonator, and the two ends of the ring were also two copper balls B_1 and B_2 with a gap. The Hertz experiment found that when there was a spark bounce in the gap between the two copper balls A_1 and A_2 of the transmitter resonator, there was also a spark bounce in the gap between the two copper balls B_1 and B_2 of the receiver resonator. In this way, Hertz achieved the transmission and reception of electromagnetic waves through experiments, which proved the existence of electromagnetic waves for the first time.

2. Theoretical analysis and experimental verification of the “displacement current” hypothesis

Could a changing electric field genuinely induce a magnetic field? Is the “displacement current” hypothesis true?

The experiment in which the German physicist Hertz proved the existence of electromagnetic waves in 1887 is reviewed below.

Referring to Figure 1.3, powered by the alternating high voltage of the oscillating dipole, the amount of charge $Q_{A_1}(t)$ and $Q_{A_2}(t)$ on the two brass balls A_1 and A_2 of the emitter oscillator varied periodically with time. Then $Q_{A_1}(t)$ and $Q_{A_2}(t)$ generated a periodic alternating high voltage between two brass balls A_1 and A_2 , producing a discharge spark. The distances from the brass ball A_1 to the two brass balls B_1 and B_2 of the receiver were r_{11} and r_{12} , respectively, corresponding to their unit vectors \mathbf{r}_{11} and \mathbf{r}_{12} . The distances from brass ball A_2 to brass balls B_1 and B_2 were r_{21} and r_{22} , respectively, and their unit vectors were \mathbf{r}_{21} and \mathbf{r}_{22} .

According to Coulomb's law, the electric field intensity on the brass ball B_1 was

$$E_{B1}(t) = k \left(\frac{Q_{A1}(t_1)}{r_{11}^2} \mathbf{r}_{11} + \frac{Q_{A2}(t_1)}{r_{21}^2} \mathbf{r}_{21} \right) \quad (2-1)$$

The electric field intensity on the brass ball B₂ was

$$E_{B2}(t) = k \left(\frac{Q_{A1}(t_1)}{r_{12}^2} \mathbf{r}_{12} + \frac{Q_{A2}(t_1)}{r_{22}^2} \mathbf{r}_{22} \right) \quad (2-2)$$

where $k = 1/4\pi\epsilon_0$; $t_1 = t - r_{11}/c$, and c is the velocity of the electric field, which is equal to the speed of light.

The electric field intensities $\mathbf{E}_{B1}(t)$ and $\mathbf{E}_{B2}(t)$ induced charges $Q_{B1}(t)$ and $Q_{B2}(t)$ on the brass balls B₁ and B₂ of the receiver. The charges $Q_{B1}(t)$ and $Q_{B2}(t)$ also varied periodically with time, and their frequencies and charge changes were synchronized with $Q_{A1}(t)$ and $Q_{A2}(t)$. By adjusting the structural parameters, the emitter oscillator and receiver could form resonance. Thus, a sufficient amount of charges $Q_{B1}(t)$ and $Q_{B2}(t)$ could accumulate on the two copper balls B₁ and B₂ of the receiver, and a periodic alternating high voltage could also form between the two copper balls B₁ and B₂ to generate discharge sparks.

In the Hertz experiment in 1887, general knowledge of wireless communication was almost zero. Today, one and a half centuries later, we re-examine the above experiment. In Equations (2-1) and (2-2), there is only a charge and an electric field, and there is no magnetic field. This illustrates the fact that the electric field waves can be generated by the change of the charge at the emitter side, the electric field waves can propagate independently in a vacuum (air), and the electric field waves can be received independently at the receiver side. The generation, propagation, and reception of electric field waves can be completed independently by the electric field itself without the participation of the magnetic field. **Therefore, the Hertz experiment did not prove the existence of electromagnetic waves; it only proved that wireless communication could be produced independently by electric field waves.**

In fact, if the magnetic field wave is involved in the experiment of Hertz, it is difficult to induce large enough varying electric fields $\mathbf{E}_{B1}(t)$ and $\mathbf{E}_{B2}(t)$ on the brass balls B₁ and B₂ of the receiver. Based on the electromagnetic wave theory, the changing electric field induces the vortex magnetic field, and the changing magnetic field induces the vortex electric field, as shown in Figure 1.2. These types of electromagnetic waves diverge and propagate in the air. The electric field energy received by the receiver copper balls B₁ and B₂ is very small, and it is difficult to generate a discharge spark between the two copper balls B₁ and B₂.

In the following, we theoretically calculate and analyze the “displacement current” hypothesis.

Assuming that the “displacement current” hypothesis is true, without losing generality, there is a point charge $q(t_1)$, and the amount of charge $q(t_1)$ changes as a sine function:

$$q(t_1) = q_0 \sin \omega t_1$$

There is a point A in space, and the distance between point A and the point charge is l . Then, the electric field intensity at point A is

$$\mathbf{E}_A(t) = \frac{q_0 \sin \omega t_1}{4 \pi \epsilon_0 l^2} \mathbf{n} \quad (2-3)$$

where $t_1 = t - l/c$, the speed of the electric field is equal to the speed of light C , \mathbf{n} is the unit vector, and its direction is from the charge $q(t_1)$ to the point A.

To calculate the magnetic induction intensity $\mathbf{B}_A(t)$ generated by the changing electric field $\mathbf{E}_A(t)$ at point A, a circular cross-section with a radius of Δr is selected at point A, and the circular cross-section is perpendicular to the electric field intensity $\mathbf{E}_A(t)$, as shown in Figure 2.1.

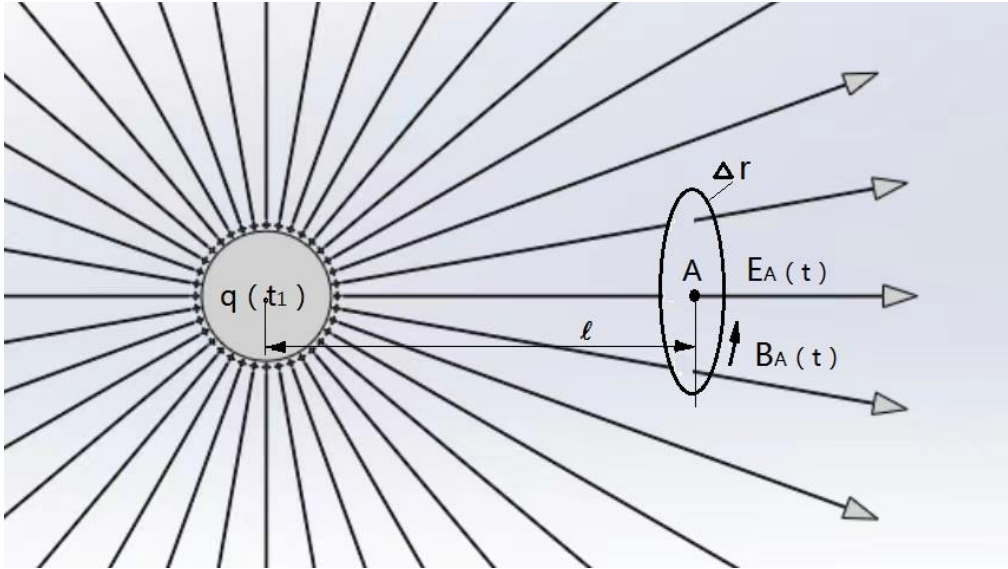


Figure 2.1 Electric and magnetic field waves induced by periodically varying charges

When Δr is small enough relative to ℓ , it can be approximately considered that the electric field and the magnetic field in the circular cross-section are uniform fields. Then the electric flux of the circular cross-section is

$$\Phi_{EA}(t) = E_A(t) \pi \Delta r^2$$

$$\Phi_{EA}(t) = \frac{\Delta r^2 q_0 \sin \omega t_1}{4 \epsilon_0 \ell^2} \mathbf{n}$$

Thus, its corresponding displacement current is

$$\begin{aligned} I_d(t) &= \epsilon_0 \frac{d\Phi_{EA}(t)}{dt} \\ &= \epsilon_0 \frac{\Delta r^2 q_0 \omega \cos \omega t_1}{4 \epsilon_0 \ell^2} \end{aligned}$$

$$I_d(t) = \frac{\Delta r^2 q_0 \omega \cos \omega t_1}{4 \ell^2}$$

Based on the “displacement current” assumption, the magnetic induction intensity $\mathbf{B}_A(t)$ is generated by the “displacement current,” and there is

$$\oint_{L_{\Delta r}} \mathbf{B}_A(t) \cdot d\boldsymbol{\ell} = \mu_0 I_d(t)$$

where $L_{\Delta r}$ is the closed curve of a circle with a radius Δr . According to the above equation, there is

$$2\pi \Delta r \mathbf{B}_A(t) = \mu_0 \frac{q_0 \Delta r^2 \omega}{4 \ell^2} \cos \omega t_1$$

$$\mathbf{B}_A(t) = \mu_0 \frac{q_0 \Delta r \omega}{8 \pi \ell^2} \cos \omega t_1$$

(2-4)

According to Equation (2-3), the maximum electric field intensity at point A is

$$\mathbf{E}_{A\text{Max}} = \frac{q_0}{4 \pi \epsilon_0 \ell^2} \quad (2-5)$$

The maximum energy density of the electric field at point A is

$$\begin{aligned} \mathbf{p}_{E\text{AMax}} &= \frac{1}{2} \epsilon_0 \mathbf{E}_{A\text{Max}}^2 \\ \mathbf{p}_{E\text{AMax}} &= \frac{q_0^2}{32 \pi^2 \epsilon_0 \ell^4} \end{aligned} \quad (2-6)$$

In accordance with Eq. (2-4), the maximum magnetic induction intensity at point A is

$$\mathbf{B}_{A\text{Max}} = \mu_0 \frac{q_0 \Delta r \omega}{8 \pi \ell^2} \quad (2-7)$$

The maximum energy density of the magnetic field at point A is

$$\begin{aligned} \mathbf{p}_{B\text{AMax}} &= \frac{1}{2} \mu_0 \mathbf{B}_{A\text{Max}}^2 \\ \mathbf{p}_{B\text{AMax}} &= \frac{\mu_0 q_0^2 \Delta r^2 \omega^2}{128 \pi^2 \ell^4} \end{aligned} \quad (2-8)$$

In order to intuitively understand the magnetic induction intensity generated by the “displacement current” at point A, and letting $q_0=10^{-6}$ C, $l=10$ m, $\Delta r=0.001$ m, $f=100$ MHz, $\omega=2\pi f = 200\pi \times 10^6$ Hz, due to $\Delta r \ll l$, according to Eq. (2-4), the maximum electric field intensity at point A is

$$\begin{aligned} \mathbf{E}_{A\text{max}} &= q_0 / (4 \pi \epsilon_0 \ell^2) \\ &= 10^{-6} / (4 \times 3.14 \times 8.854 \times 10^{-12} \times 10^2) \text{ v/m} \\ \mathbf{E}_{A\text{max}} &= 89.9 \text{ v/m} \end{aligned} \quad (2-9)$$

The maximum energy density of the electric field at point A is

$$\begin{aligned} \mathbf{P}_{E\text{AMax}} &= \epsilon_0 \mathbf{E}_{A\text{max}}^2 / 2 \\ &= 8.854 \times 10^{-12} \times 89.9^2 / 2 \quad \text{J/m}^3 \\ \mathbf{P}_{E\text{AMax}} &= 3.578 \times 10^{-8} \text{ J/m}^3 \end{aligned} \quad (2-10)$$

In accordance with Eq. (2-7), the maximum magnetic induction intensity at point A is

$$\begin{aligned} \mathbf{B}_{A\text{Max}} &= \mu_0 \frac{q_0 \Delta r \omega}{8 \pi \ell^2} \\ &= \frac{4 \times 3.14 \times 10^{-7} \times 10^{-6} \times 0.001 \times 200 \times 10^6 \times 3.14}{8 \times 3.14 \times 10^2} \\ \mathbf{B}_{A\text{Max}} &= 3.14 \times 10^{-10} \text{ T} \end{aligned}$$

(2-11)

The maximum energy density of the magnetic field at point A is

$$\begin{aligned} P_{BAMax} &= B_{Amax}^2 / 2\mu_0 \\ &= (3.14 \times 10^{-10})^2 / (2 \times 4\pi \times 3.14 \times 10^{-7}) \\ P_{BAMax} &= 3.92 \times 10^{-12} \text{ J/m}^3 \end{aligned} \quad (2-12)$$

Comparing Equations (2-10) and (2-12), the magnetic field generated by the excitation of the changing electric field at point A has very little energy density relative to the electric field energy density. **Therefore, even if the “displacement current” hypothesis were true, based on the energy conversion, the energy of the changing electric field couldn’t be effectively converted into the energy of a magnetic field in a vacuum.**

In fact, based on the energy conversion and spatial divergence propagation of electromagnetic waves, the electric field energy density and the magnetic field energy density of the alternating part at any point in space must be equal. In the field of modern application technology, it is impossible to achieve high-power (hundreds of watts) long-distance (several meters) wireless charging to fulfill microwave directional communication.

Furthermore, taking the radius of the circular cross-section as $\Delta r = 0.002$ m, the maximum magnetic induction intensity at point A is obtained from Equation (2-7):

$$\begin{aligned} B_{AMax} &= \mu_0 \frac{q_0 \Delta r \omega}{8 \pi \ell^2} \\ &= \frac{4 \times 3.14 \times 10^{-7} \times 10^{-6} \times 0.002 \times 200 \times 10^6 \times 3.14}{8 \times 3.14 \times 10^2} \\ B_{AMax} &= 6.28 \times 10^{-10} \text{ T} \end{aligned} \quad (2-13)$$

Comparing the results of Equations (2-11) and (2-13), the maximum magnetic induction intensity at point A is related to the size of the radius Δr of the selected circular cross-section. When the radius of the selected circular cross-section Δr is different, the maximum magnetic induction intensity B_{Amax} that is obtained is also different. Moreover, it can be directly determined from Equation (2-7) that when the radius of the selected circular section Δr approaches zero, the maximum magnetic induction intensity B_{Amax} at point A is zero. **Therefore, Maxwell's “displacement current” hypothesis is not self-consistent in theory.**

In fact, according to Equation (2-7), For the condition that the maximum electric field intensity remains unchanged, the maximum magnetic induction intensity B_{Amax} at point A is related not only to the radius of the selected circular cross-section Δr but also to the angular frequency ω .

Maxwell introduced the “displacement current” hypothesis in 1861. After a century and a half, no experiment has genuinely proved its correctness so far.

Figure 2-2a shows a photograph of the experimental setup used by the author to verify the “displacement current” hypothesis, and Figure 2-2b shows a schematic diagram of the experimental setup. In the figures, the capacitor C is composed of two round aluminum plates. The radius of the plate is $r_c = 100$ mm, and

the distance between the two plates is $d_c=60$ mm. A magnetic field induction coil L_{BE} is set on the side of the gap between the two plates. The radius is $r_{be}=25$ mm, the height is $height=10$ mm, and the number of turns is $n=1000$. An induction resistor $R_{BE}=50 \Omega$ is connected between the two lead wires of the coils L_{BE} . The distance from the center of the induction coils to the center of two plates of capacitor C is $l=130$ mm. A closed curve of the induction magnetic field circle L_c is selected with the radius l centered around the capacitor C .

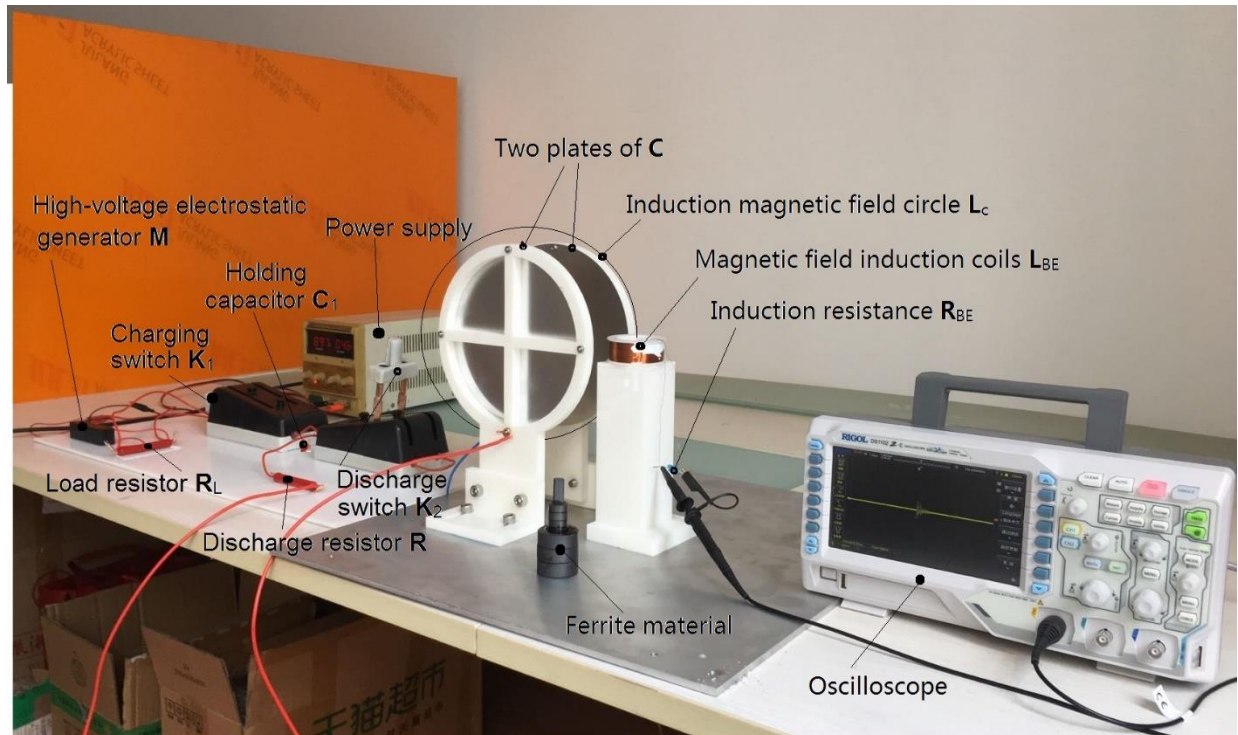


Figure 2-2a Photograph of the experimental device proving the “displacement current” hypothesis

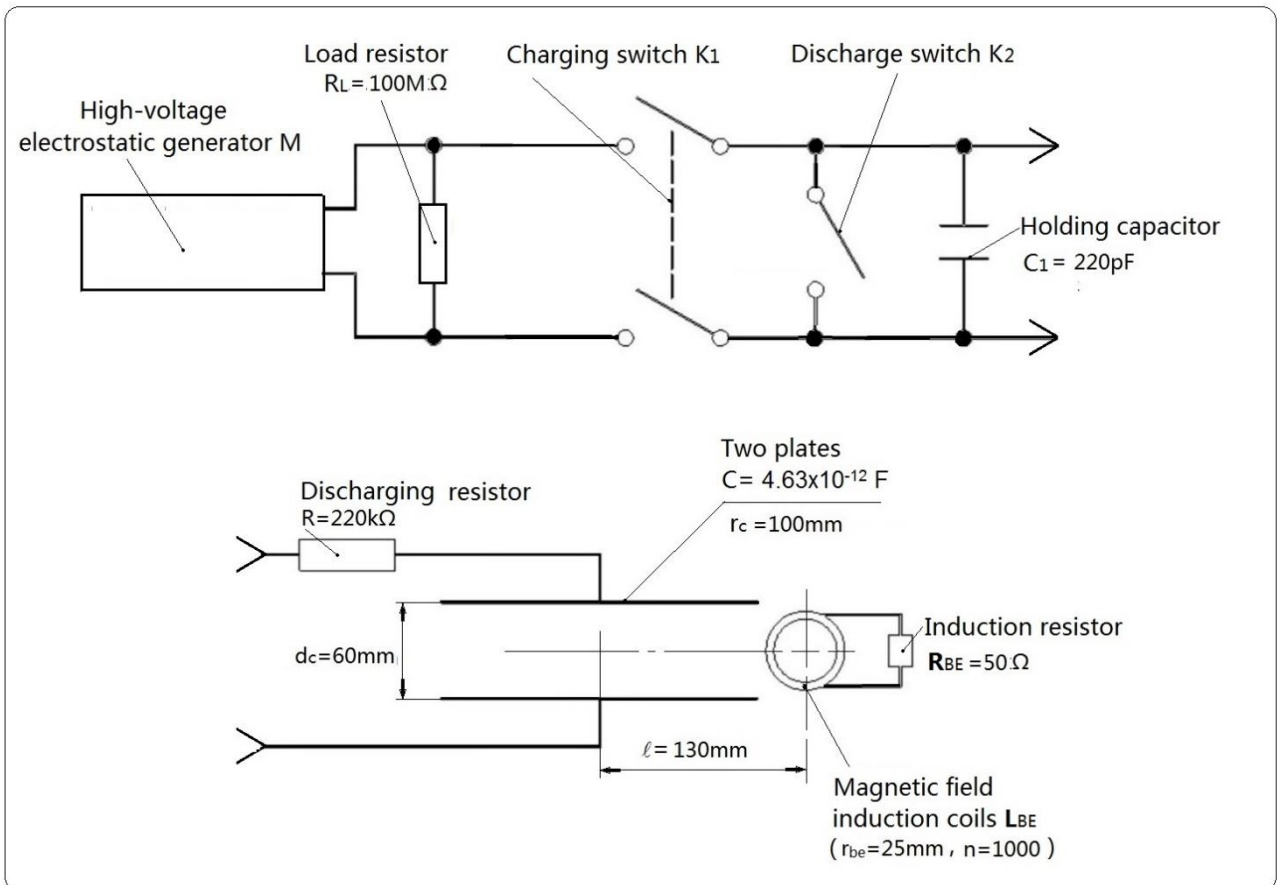


Figure 2-2b Schematic diagram of the experimental device proving the “displacement current” hypothesis

The voltage between the two plates of capacitor C is $V(t)$, the electric field intensity is $E(t)$, and the electric flux is $\Phi_E(t)$. The high-voltage electrostatic generator M generates the high static voltage $V_0=20000$ V. The charging switch K_1 is turned on to charge the capacitor C, and the discharge switch K_2 is turned on to discharge the capacitor C through a resistor R ($R=220$ k Ω). When the capacitor C is discharged, a varying electric field is generated between the two plates of the capacitor. According to Maxwell’s “displacement current” hypothesis, a varying magnetic field $B(t)$ is generated along the closed curve L_C . This varying magnetic field induces an electromotive force $V_{BE}(t)$ on the induction coils L_{BE} , which can be detected in the resistor R_{BE} .

The capacitance of the capacitor C is

$$\begin{aligned}
 C &= \epsilon_0 \pi r_c^2 / d_c \\
 &= 8.85 \times 10^{-12} \times 3.14 \times 0.1^2 / 0.06 \\
 &= 4.63 \times 10^{-12} \text{ F}
 \end{aligned}$$

The time constant is

$$RC = 220 \times 1000 \times 4.63 \times 10^{-12} = 1.0 \times 10^{-6} \text{ s} = 1.0 \mu\text{s}$$

The electric flux between the two plates of the capacitor C is

$$\begin{aligned}\Phi_E(t) &= E(t) \pi r_c^2 \\ &= \pi r_c^2 V(t) / d_c \\ &= 0.523 V(t)\end{aligned}$$

$$\text{Then } d\Phi_E(t) / dt = 0.523 dV(t) / dt \quad (2-14)$$

When the switch K_1 is turned off and the switch K_2 is turned on, the capacitor C is discharged via the resistor R , the voltage $V(t)$ on the two plates of the capacitor decreases exponentially, and there is

$$V(t) = V_0 e^{-t/RC} \quad (2-15)$$

According to Eqs. (2-14) and (2-15), the change rate of the electric flux between the two plates of the capacitor C is obtained:

$$\begin{aligned}d\Phi_E(t) / dt &= - (0.523 V_0 / RC) e^{-t/RC} \\ d\Phi_E(t) / dt &= - 1.046 \times 10^{10} e^{-t/RC}\end{aligned} \quad (2-16)$$

If the "displacement current" hypothesis is true, according to Maxwell's equation (1-4), the following expression is obtained:

$$\oint_{L_C} \mathbf{B}(t) \cdot d\ell = \mu_0 \epsilon_0 \frac{d\Phi_E(t)}{dt} \quad (2-17)$$

Considering the magnetic induction intensity $\mathbf{B}(t)$ passing through induction coils L_{BE} as a uniform magnetic field, the approximate calculation is obtained from Eq. (2-17).

$$\begin{aligned}\mathbf{B}(t) (2 \pi \ell) &= \mu_0 \epsilon_0 \frac{d\Phi_E(t)}{dt} \\ \mathbf{B}(t) &= \frac{\mu_0 \epsilon_0}{2 \times 3.14 \times 0.130} \times (- 1.046 \times 10^{10}) e^{-t/RC} \\ \mathbf{B}(t) &= -1.28 \times 10^{10} \mu_0 \epsilon_0 e^{-t/RC}\end{aligned}$$

The magnetic flux of the induction coils L_{BE} and the change rate of the magnetic flux are

$$\begin{aligned}\Phi_B(t) &= \mathbf{B}(t) (\pi r_{be}^2) \\ &= -1.28 \times 10^{10} \times 3.14 \times 0.025^2 \mu_0 \epsilon_0 e^{-t/RC} \\ \Phi_B(t) &= -2.51 \times 10^7 \mu_0 \epsilon_0 e^{-t/RC} \\ \frac{d\Phi_B(t)}{dt} &= 2.51 \times 10^{13} \mu_0 \epsilon_0 e^{-t/RC}\end{aligned} \quad (2-18)$$

The alternating magnetic flux in Eq. (2-18) generates an induced voltage in the induction coils L_{BE} , and according to Maxwell's equation, Eq. (1-3),

$$\oint_{L_{BE}} \mathbf{E}_{BE}(t) \cdot d\ell = - \frac{d\Phi_B(t)}{dt}$$

The left side of the above equation is the induced voltage of a single turn of the induction coils L_{BE} . Then:

$$\begin{aligned}
 V_{BE1}(t) &= - \frac{d\Phi_B(t)}{dt} \\
 &= - 2.51 \times 10^{13} \mu_0 \epsilon_0 e^{-t/RC} \\
 &= - 2.51 \times 10^{13} \times 8.854 \times 4 \times 3.14 \times 10^{-19} e^{-t/RC} \\
 V_{BE1}(t) &= - 0.000279 e^{-t/RC} \text{ V}
 \end{aligned}$$

The positive and negative poles of the magnetic field induction coils L_{BE} are switched, with the number of turns being $n=1000$, so the total induced voltage of L_{BE} is

$$\begin{aligned}
 V_{BE}(t) &= 1000 \times 0.000279 e^{-t/RC} \text{ V} \\
 V_{BE}(t) &= 0.279 e^{-t/RC} \text{ V} \qquad (2-19)
 \end{aligned}$$

According to Equation (2-19), the voltage on the induction resistance R_{BE} decreases exponentially. The voltage waveform is shown in Figure 2.3.

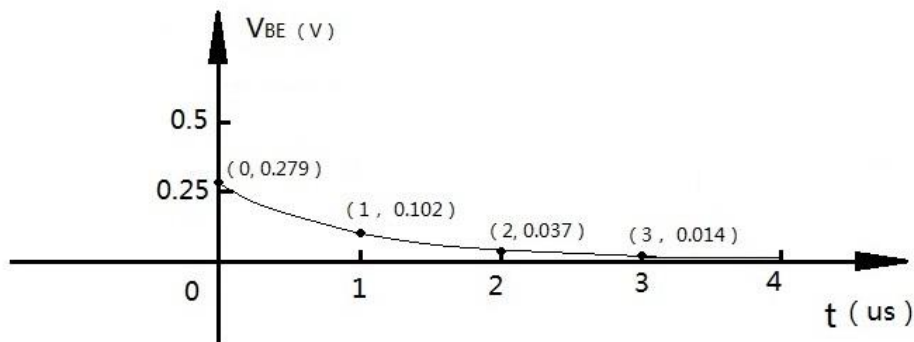


Figure 2.3 Induced voltage waveform calculated based on the “displacement current” hypothesis

As shown in Figure 2.3, $V_{BE} = 0.279 \text{ V}$ at $t=0$; $V_{BE} = 0.102 \text{ V}$ at $t = 1 \mu\text{s}$; $V_{BE} = 0.037 \text{ V}$ at $t=2 \mu\text{s}$; and $V_{BE} = 0.014 \text{ V}$ at $t = 3 \mu\text{s}$.

Figure 2-4 shows the actual voltage waveform obtained with the oscilloscope for the induction resistor R_{BE} . There is no induced voltage waveform calculated according to the “displacement current” hypothesis, as shown in Figure 2-3. **This indicates that when the capacitor C is discharged, the change of the electric flux between the two plates of the capacitor C does not induce a magnetic field as predicted under the “displacement current” hypothesis. Therefore, the above experiment directly proves that the “displacement current” hypothesis is not true.**

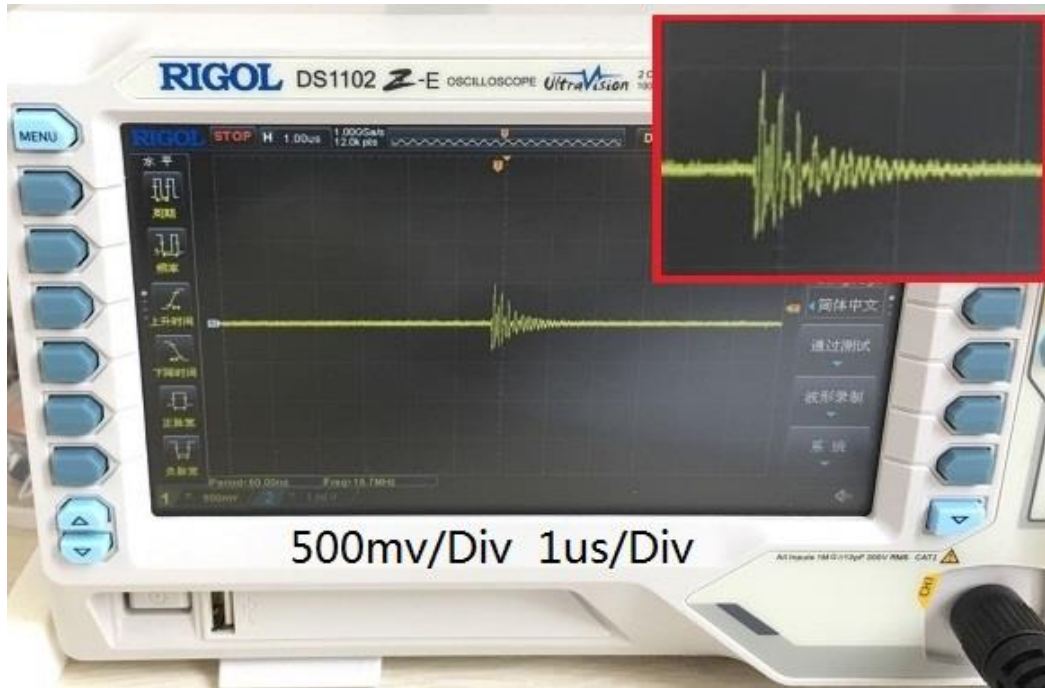


Figure 2.4 Voltage waveform detected by oscilloscope (hollow induction coils)

As shown in Fig. 2-4, the voltage waveform on R_{BE} shows a symmetrical oscillation waveform with a maximum peak value of 0.4 V, and a $1 \mu s$ later, the oscillation waveform has almost decayed to zero. When we insert a ferrite material into the hollow of the induction coils L_{BE} , the symmetrical oscillation waveform still exists, and the waveform has only a slight change, as shown in Fig. 2-5. Therefore, we can determine that the oscillation waveform is not caused by the induction of the alternating magnetic field generated by the “displacement current” hypothesis, and the symmetrical oscillation waveforms should be the interference signal generated by the electric field when the capacitor C is discharged rapidly.

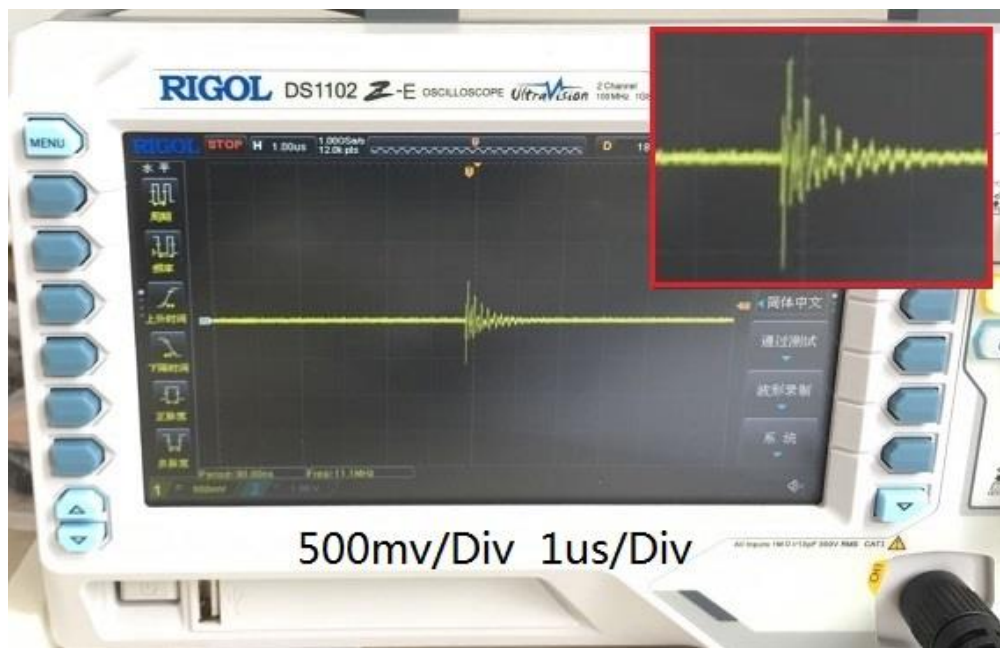


Figure 2.5 Voltage waveform detected by oscilloscope (ferrite induction coils)

In summary, we obtain the following conclusions:

- (1) The generation, propagation, and reception of the electric field waves in the vacuum can be completed independently without the participation of the magnetic field waves.
- (2) If the “displacement current” hypothesis were true in the vacuum, based on the energy conversion, the energy of the changing electric field could not be effectively converted into the energy of the magnetic field.
- (3) If the “displacement current” hypothesis were true in the vacuum, based on the propagation mode, the “electromagnetic waves” would propagate by swirling outward, and it would be impossible to achieve directional communication.
- (4) Maxwell's “displacement current” hypothesis is not self-consistent in theory. For the same point in space, when the radius of the selected circular cross-section or the angular frequency is different, the calculated magnetic induction intensity is also different.
- (5) The experiment directly proves that Maxwell’s “displacement current” hypothesis is not true, that is, a changing electric field cannot induce a magnetic field in a vacuum.
- (6) There are no “electromagnetic waves” in a vacuum. The light must not be an “electromagnetic wave,” and the light is a light field wave itself. It is the electric field wave, not the “electromagnetic wave,” that achieves wireless communication.

3. Theoretical analysis and experimental verification of magneto-electric induction

In a vacuum, a changing electric field cannot induce a magnetic field. The question is whether a changing magnetic field can induce an electric field. This is the third equation, Equation (1-3), of Maxwell’s equations, and it is also Faraday’s law of electromagnetic induction. Its integral form is

$$\oint_L E \cdot d\ell = -\frac{d\Phi_B}{dt} \quad (3-1)$$

Without loss of generality, we use symmetrical cylindrical electromagnetic coils L_B to generate an alternating magnetic field with an alternating current. Then we demonstrate the electric field generated by the alternating magnetic field induced at point A in a vacuum (air), as shown in Figure 3.1.

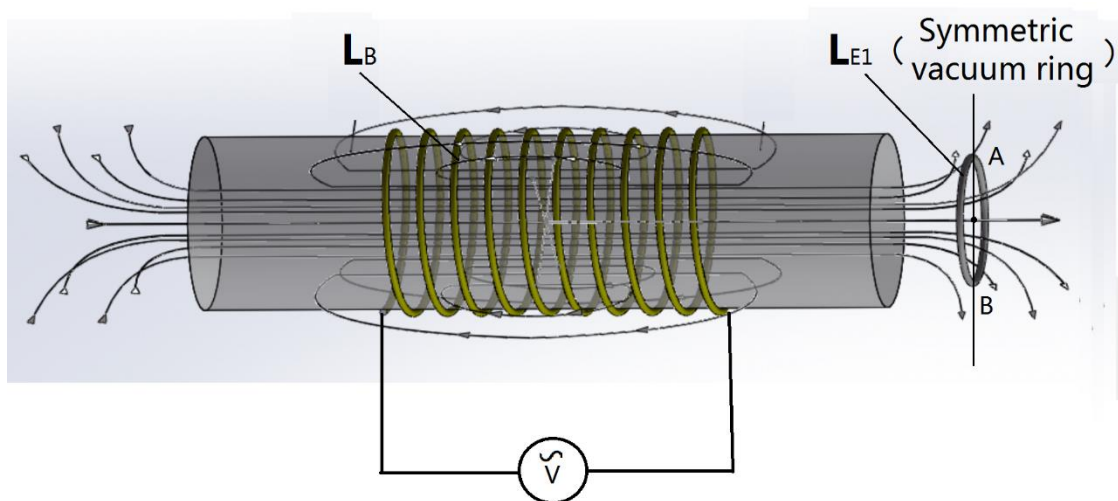


Figure 3.1 Induced electric field of symmetric vacuum ring

Assuming that equation (3-1) is true, a closed circular ring L_{E1} with radius r_1 is selected in the vacuum passing through point A. The center of the circular ring is located on the central axis of the electromagnetic coils L_B , and the closed circular ring L_{E1} is perpendicular to the central axis of the electromagnetic coils L_B . Since the alternating magnetic field generated by the cylindrical electromagnetic coils L_B is completely symmetric with respect to the closed circular ring L_{E1} , the electric field intensity E_0 at any point on the closed circular ring L_{E1} must be equal in a vacuum.

On the closed circular ring L_{E1} , taking another point B relative to point A, AB is the diameter of the closed circular ring L_{E1} . Then the voltage at point B compared to point A is

$$U_{BA} = \pi r_1 E_0 \quad (3-2)$$

Similarly, according to the principle of symmetry, the potentials at point A and point B must be equal. Then

$$U_{BA} = 0 \quad (3-3)$$

According to Equations (3-2) and (3-3), the following expression is obtained:

$$E_0 = 0$$

That is, on the vacuum closed circular ring L_{E1} , the electric field intensity generated by the cylindrical electromagnetic coils L_B is identical and equal to zero at any point, and the closed circular ring L_{E1} is **an equipotential circular ring. Therefore, an alternating magnetic field in a vacuum cannot induce an electric field.**

It is assumed that the closed circular ring L_{E1} in the vacuum is replaced by a closed metal circular ring L_{E2} with a radius of r_1 , as shown in Figure 3.2.

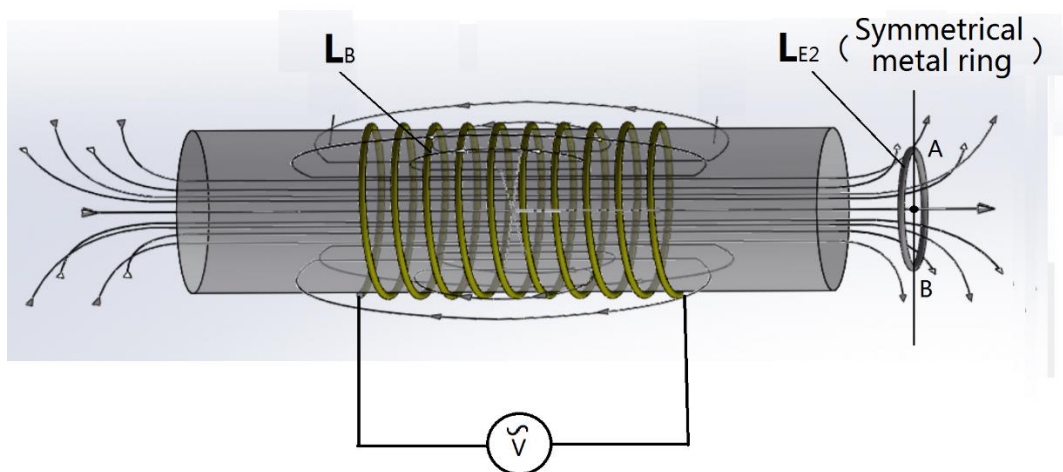


Figure 3.2 Induced electric field of symmetric metal ring

Similarly, in the closed metal ring L_{E2} , the electric field intensity is identical and equal to zero everywhere, so the closed metal ring L_{E2} is also an equipotential metal ring. However, according to the Lenz metal ring experiment, it is confirmed that there is an alternating current in the closed metal ring L_{E2} , and the magnetic field generated by the alternating current is repelled by the alternating magnetic field of L_B . In

the closed metal ring L_{E2} , the electric field intensity is zero everywhere and there is current in the ring. This metal ring L_{E2} shows the characteristic of “zero resistance.” We define this type of closed metal current ring as **an equipotential metal current ring**.

According to an analysis of the above phenomenon, the alternating magnetic field of the L_B coils causes the magnetic flux in the metal ring L_{E2} to change. The physical essence is that the alternating magnetic field lines generated by the L_B coils cut the metal ring L_{E2} , which is equivalent to the motion of the charge in the metal ring L_{E2} relative to the magnetic field.

According to Lorentz’s law of magnetic field force, the magnetic field force drives the charge in the L_{E2} metal ring to make the directional movement form a current. We define the current generated by the magnetic field force as the **magnetic field current I_B** . To distinguish the currents, the current generated by the electric field force is designated as the electric field current I_E . In Figure 3-2, due to the symmetry, the magnetic field current I_B in the metal ring L_{E2} flows uniformly, and the charge is also uniformly distributed in the metal ring L_{E2} , so the electric field intensity in the metal ring L_{E2} is zero everywhere

The magnetic field current I_B in the closed metal ring L_{E2} described above is determined using the Lorentz law of magnetic field force, the vector expression of which is

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{3-4}$$

The directions of the magnetic field force \mathbf{F} , the charge velocity \mathbf{v} , and the magnetic induction intensity \mathbf{B} follow the right-handed spiral rule.

For the above equipotential metal current ring L_{E2} , it is assumed that the resistance of the metal ring is R . The question then becomes whether the thermal power loss of the magnetic field current I_B satisfies Ohm's law, that is, whether $W = I_B^2 R$ is true. The characteristics of the equipotential metal current ring need further study.

For the experiment shown in Figure 3.2, the closed metal ring L_{E2} is moved and the center of the metal ring L_{E2} is allowed to deviate from the central axis of L_B , as shown in Figure 3.3.

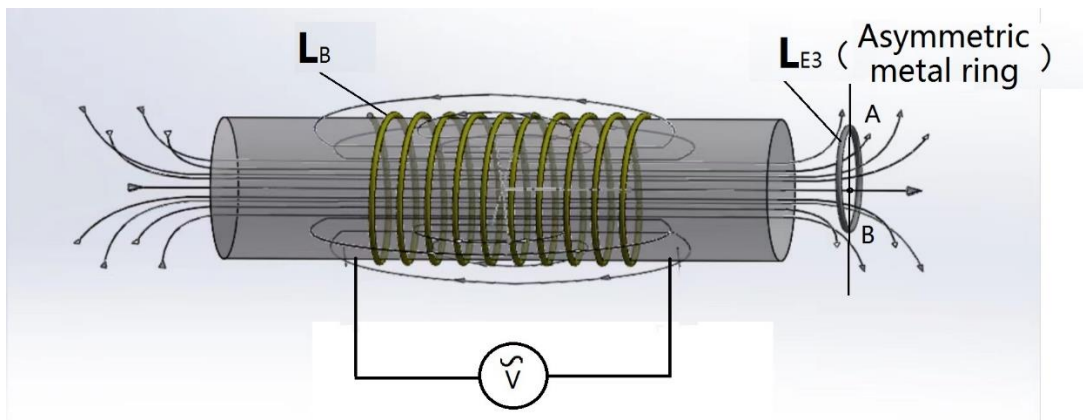


Figure 3.3 Induced electric field of asymmetric metal ring

The closed metal ring after translation is L_{E3} . The magnetic induction intensity B_A at point A is less than the magnetic induction intensity B_B at point B, so the Lorentz magnetic field forces on the charge at points A and B are different. Because the Lorentz magnetic field force on the charge in the closed metal ring L_{E3} is different at different positions and the charge distribution in the metal ring L_{E3} is uneven, there is an

electric field and a potential difference among the points of the metal ring L_{E3} . Therefore, at this time, the charge in the metal ring L_{E3} is subject to both the magnetic field force and the electric field force. The metal ring L_{E3} has both the magnetic field current I_B and the electric field current I_E , and the magnetic field current I_B and the electric field current I_E oppose each other.

The equipotential metal current ring has only the magnetic field current I_B , and its conversion efficiency between the magnetic field energy and the electric field energy is the highest. Therefore, under the conditions that the technology permits, for electrical equipment, such as transformers and motors, the magnetic circuit and the electric circuit should be symmetrical as much as possible.

The magnetic induction intensity through the closed metal rings L_{E2} and L_{E3} is adjusted and the change rates of the magnetic fluxes of the two metal rings are equalized; that is, the right side of Equation (3-1) is equal. However, the closed metal ring L_{E2} only has the magnetic field current I_B , while the closed metal ring L_{E3} has both the magnetic field current I_B and the electric field current I_E , that is, Equation (3-1) **Faraday's law of electromagnetic induction is not only related to the change rate of the magnetic flux of a closed metal ring but also related to a structure of the closed metal ring. Therefore, Faraday's law of electromagnetic induction is only an approximate empirical formula for metal rings and it is incomplete in theory.**

For the experiment of a symmetrical metal ring in Figure 3.2, it can be imagined that there is an elastic metal ring L_{E4} in a uniform magnetic field and L_{E4} uniformly shrinks to the center, as shown in Figure 3.4. It can be intuitively concluded that the magnetic flux in the elastic metal ring L_{E4} changes, and there is current in the L_{E4} ring. However, due to the symmetry of the elastic metal ring L_{E4} , the electric field intensity in the L_{E4} ring is zero everywhere, and the potential between any two points must also be zero. Therefore, the elastic metal ring L_{E4} is **an equipotential metal current ring.**

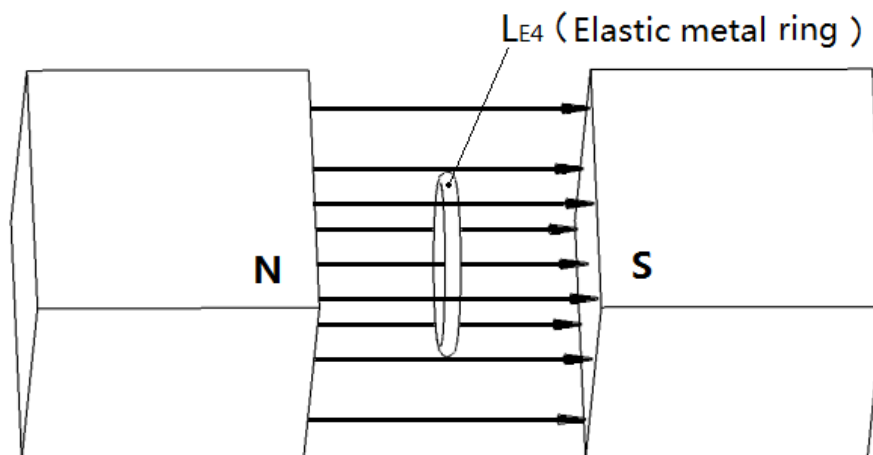


Figure 3.4 Induced electric field of symmetric elastic metal ring

In the 200 years of electromagnetism study since Faraday, either the induced electromotive force generated when the magnetic field lines cut the metal wires, or the motional electromotive force generated when the metal wires cut the magnetic field lines, researchers have never considered this special symmetrical structure of an equipotential metal current ring. This incompleteness has led to the incompleteness of Faraday's law of electromagnetic induction in theory.

For Faraday's law of electromagnetic induction in Equation (3-1), the line integral of the electric field in the closed loop is the electric potential difference at the same point, which must be zero according to the definition of electric field and the potential. It can be assumed that there is a small gap in the metal rings L_{E2} and L_{E3} and the charge is driven by the magnetic field force to accumulate at the two ends of the gap, thereby generating an electric field and an electric potential in the metal ring. However, according to the above analysis, this potential is related not only to the rate of change in the magnetic flux of the metal ring but also to the structure of the metal ring. Therefore, Faraday's law of electromagnetic induction is only an approximate empirical formula for the metal ring that is not complete in theory.

In summary, we obtain the following conclusions:

(1) In a vacuum, a changing magnetic field cannot induce an electric field; that is, in a vacuum, Faraday's law of electromagnetic induction is not true.

(2) The equipotential metal current ring only has the magnetic field current I_B , which has the characteristic of "zero resistance," and the conversion efficiency between the magnetic field energy and the electric field energy is the highest. Under the conditions that the technology permits, for electrical equipment, such as transformers and motors, the magnetic circuit and the electric circuit should be symmetrical as much as possible

(3) As an approximate empirical formula, Faraday's law of electromagnetic induction is applicable to metal coils, but it is incomplete in theory.

(4) The physical essence of Faraday's law of electromagnetic induction is the Lorentz law of magnetic field force; that is, the electric charge moving in the magnetic field is affected by the Lorentz magnetic field force.

4. Three fundamental laws of electromagnetism

Through the above theoretical calculation and experimental verification, it is concluded that in Maxwell's equations, two out of the four equations are incomplete or untrue. In this paper, based on the physical origin relationship among the charge, current, electric field, and magnetic field, electromagnetism is summarized into three fundamental laws.

4.1 Coulomb's law

Coulomb's law is one of the fundamental laws of electromagnetism. We consider a single point charge $q(t_1)$ in a boundless free space, and we take a point A at a distance of $r(t_1)$ from the charge $q(t_1)$. At point A, the electric field intensity $\mathbf{E}_A(t)$ is

$$\mathbf{E}_A(t) = k \frac{q(t_1)}{r(t_1)^2} \mathbf{r} \quad (4-1)$$

Where k is called the electrostatic force constant, with $k=9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$ in a vacuum, and \mathbf{r} is the unit vector of $r(t_1)$. The speed of the electric field is equal to the light velocity c , so $t_1 = t - r(t_1) / c$.

The charge $q(t_1)$ and the distance $r(t_1)$ can change with time, so the electric field intensity $\mathbf{E}_A(t)$ also changes with time.

The electric field, electric field intensity, electric flux, electric potential, electric potential energy, and Gauss's law of electric field are the mathematical expansion and application promotion of Coulomb's law.

4.2 Lorentz's law of magnetic field induction: The moving charges induce magnetic fields

Moving charges induce magnetic fields and the moving charges are the only direct cause for the generation of magnetic fields.

In a vacuum, it is assumed that the velocity of a moving charge $q(t_1)$ is $v(t_1)$, and the distance between the point A and the charge $q(t_1)$ is $r(t_1)$. The moving charge $q(t_1)$ induces a magnetic induction intensity $\mathbf{B}(t)$ at point A:

$$\mathbf{B}(t) = \frac{\mu_0}{4\pi} \frac{q(t_1) \mathbf{v}(t_1) \times \mathbf{r}}{r(t_1)^3} \quad (4-2)$$

Where \mathbf{r} is the unit vector of $r(t_1)$, and the direction of $\mathbf{B}(t)$ is perpendicular to the plane determined by $\mathbf{v}(t_1)$ and \mathbf{r} .

The Biot-Savart law and Ampere's circuital law are mathematical extensions and general applications of Lorentz's law for a magnetic field.

4.3 Lorentz law of magnetic field force: The moving charge in the magnetic field is affected by the magnetic field force

In a vacuum, a charge $q(t)$ with velocity $\mathbf{v}(t)$ is moving in a magnetic field $\mathbf{B}(t)$, the charge $q(t)$ is affected by a Lorentz magnetic field force $\mathbf{F}(t)$, and the magnetic field force is

$$\mathbf{F}(t) = q(t) \mathbf{v}(t) \times \mathbf{B}(t) \quad (4-3)$$

Ohm's law, Faraday's law of electromagnetic induction, mass spectrometers, and cyclotrons are the mathematical expansion and technology application of Lorentz's law of magnetic field force.

5. Electric fields, magnetic fields, light fields, and charges

In a vacuum, a changing electric field cannot generate a magnetic field and a changing magnetic field cannot generate an electric field. Therefore, there are no "electromagnetic waves" in a vacuum, and light must not be an "electromagnetic wave"; rather, light is the light field itself.

5.1 Charge and electric field

For a charge q , the electric field intensity at any point A in space:

$$\mathbf{E}_A = k \frac{q}{r^2} \mathbf{r} \quad (5-1)$$

where k is called the electrostatic force constant, r is the distance between the charge q and the point A, and \mathbf{r} is the unit vector of r .

For a specific charge q , the electric field intensity E_A at point A is only related to r , the position of the charge. Therefore, the electric field is a characteristic of the position of the charge.

The charge is the only direct factor that generates an electric field. An electric field and an electric field wave propagate independently in a vacuum, and their propagation speed is C , which is equal to the speed of light.

5.2 Charge and magnetic field

A moving charge generates a magnetic field, and the magnetic field intensity B_A at any point A in space is as follows.

$$\mathbf{B}_A = \frac{\mu_0}{4\pi} \frac{q \mathbf{v} \times \mathbf{r}}{r^3} \quad (5-2)$$

\mathbf{v} is the velocity of charge q , r is the distance from point A to charge q , and \mathbf{r} is the unit vector of r .

The motionless charge q does not generate a magnetic field, and only the moving charge q can generate a magnetic field. The magnetic induction intensity is proportional to the moving speed of the charge q . Therefore, the magnetic field is a characteristic of the velocity of the charge.

The moving charge is the only direct factor that generates a magnetic field. The magnetic field and the magnetic field wave propagate independently in a vacuum, and the propagation speed is C , which is equal to the speed of light.

5.3 Charge and light field

When an electron jumps from a high energy level to a low energy level, the electron emits a photon. When an electron absorbs a photon, the electron jumps from a low energy level to a high energy level. Therefore, the light field is a characteristic of the acceleration/deceleration of the charge.

The generation mechanism of X-rays and γ -rays is also due to the transition or acceleration/deceleration of a charge, so X-rays and γ -rays are the light fields.

A light field and a light wave propagate independently in a vacuum, and the propagation speed is C .

5.4 Mutual transformation of an electric field, a magnetic field, and a light field through an electric charge

An electric field, a magnetic field, and a light field are the position characteristic, velocity characteristic, and acceleration characteristic of a charge, respectively. Therefore, the propagation speeds of an electric field, a magnetic field, and a light field in a vacuum are equal, all of them are c , which is self-consistent and reasonable. An electric field, a magnetic field, and a light field cannot induce each other in a vacuum.

An electric field cannot directly induce a magnetic field. An electric field can generate a magnetic field indirectly with the participation of a charge. The electric field drives the charge to move, and the charge movement induces the magnetic field.

An electric field cannot directly induce a light field. An electric field can generate a light field indirectly with the participation of charge. The electric field drives the charge transition and acceleration/deceleration and then emits or absorbs the light field.

A magnetic field cannot directly induce an electric field. A magnetic field can generate an electric field indirectly with the participation of charge. The magnetic field drives the charge to move, and the change of the charge position induces the electric field.

A magnetic field cannot directly induce a light field. A magnetic field can generate a light field indirectly with the participation of a charge. The magnetic field drives the charge transition and acceleration/deceleration and then emits or absorbs the light field.

A light field cannot directly induce an electric field. A light field can generate an electric field indirectly with

the participation of charge. The light field drives the charge to move, and the change of the charge position induces the electric field.

A light field cannot directly induce a magnetic field. A light field can generate a magnetic field indirectly with the participation of a charge. The light field drives the charge to move, and the charge movement induces the magnetic field.

There are no “electromagnetic waves” in the real physical world. Electric field waves, magnetic field waves, and light field waves propagate independently. Electric field waves are suitable for long-distance wireless communication, and magnetic field waves are suitable for short-distance power conversion.

6. Conclusion

Maxwell's equations are the greatest scientific accomplishment of physics in the 19th century. They contain a set of four equations that theoretically predict the existence of “electromagnetic waves.” Based on theoretical analysis and experimental verification, this paper proves that Maxwell's “displacement current” hypothesis is not true, that is, a changing electric field in a vacuum cannot induce a magnetic field. The fourth equation, Equation (1-4), of Maxwell's equations is not true.

Similarly, a changing magnetic field in a vacuum cannot induce an electric field; that is, Faraday's law of electromagnetic induction is not true in a vacuum. As an approximate empirical formula, Faraday's law of electromagnetic induction is widely used for metal coils, but it is incomplete in theory. In this paper, a magnetic field current and an equipotential metal current ring with a “zero resistance” characteristic are proposed, which theoretically perfects Faraday's law of electromagnetic induction. It is also revealed that transformers, motors, and other types of electrical equipment have the highest electromagnetic energy conversion efficiency when the magnetic circuit and the electric circuit are symmetrical. The third equation, Equation (1-3), of Maxwell's equations is not true in a vacuum and is incomplete in metal coils.

In Maxwell's equations, two out of the four equations are incomplete or untrue. Based on the physical origin relationship among the charge, current, electric field, and magnetic field, this paper summarizes three fundamental laws of electromagnetics: Coulomb's law, Lorentz's law of magnetic field induction, and Lorentz's law of magnetic field force.

An electric field, a magnetic field, and a light field are the characteristics of the position, velocity, and acceleration of the charge, respectively. The velocity of an electric field, a magnetic field, and a light field in the vacuum is c . An electric field, a magnetic field, and a light field cannot be directly induced and transformed. However, an electric field, a magnetic field, and a light field can be indirectly induced and transformed with the participation of charge. There are no “electromagnetic waves” in a vacuum. Theoretically, wireless communication can be independently achieved with electric field waves, magnetic field waves, and light field waves. In practical application, electric field waves are suitable for long-distance wireless communication, and magnetic field waves are suitable for short-range power conversion.

This study denies the existence of “electromagnetic waves” and improves Faraday's law of electromagnetic induction, which will have a profound impact on scientific discovery and technological progress.

7. Further steps and prospects

7.1 Scientific prospects

Electromagnetism is the most profound and effective scientific achievement in physics since Newton. It

has laid a theoretical foundation for the Special Theory of Relativity and quantum mechanics, and it is the precursor of modern physics.

Based on theory and experiments, this study proves there are no “electromagnetic waves” in a vacuum. which will have a profound impact on modern microscopic quantum science and macroscopic universe exploration. The subdivision of “electromagnetic waves” into independent electric, magnetic, and light fields waves would expand and deepen the understanding of fundamental areas of science such as microscopic particles, lasers, black-body radiation, cosmic background radiation, cosmic galaxies, black holes, neutron stars, and pulsars.

7.2 Technology and application prospects

Electromagnetism is the technological foundation of the second industrial revolution in the 19th century, which was characterized by electrification.

In this paper, a magnetic field current and an equipotential metal current ring with “zero resistance” characteristics are proposed, which theoretically improves Faraday's law of electromagnetic induction, which will greatly improve the performance and energy consumption of key electrical components such as transformers and motors.

Electromagnetism also is the technological foundation of modern society, which is characterized by microelectronics information technologies.

Based on theory and experiment, this study proves that there are no “electromagnetic waves” in the world. It is the electric field waves, not “electromagnetic waves” that produce wireless communication. This will have a great impact on modern wireless microelectronics technology and improve the design theory of large scientific devices such as radio telescopes. This work provides a new theoretical basis for wireless communication technology, radar technology, stealth technology, and microelectronic digital technology, including the 4G/5G mobile phones people use every day.

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