MINKOWSKI COMPLETE SPACE-TIME ROTATION HYPOTHESIS

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Abstract. This model uses rotation of Minkowski space-time, but assumption is that there can be time flowing in any direction not just in normal time direction, from it comes S tensor field that represents possible states of light cones.

Contents

1. Minkowski rotated complete space-time

1.1. **S field.** For given n dimensional Minkowski space-time I can create a mixed tensor field that splits light cones [1] into two parts, past light cone and future light cone and so does split all vectors into past part and future part. Upper indexes represent just normal space-time coordinate, down indexes represent past or future part of light cone- so of space-time itself. This field can be written in a matrix form:

$$S^{\alpha}_{\beta}\left(\mathbf{x}\right) = \begin{bmatrix} S^{1}_{-}\left(\mathbf{x}\right) & S^{1}_{+}\left(\mathbf{x}\right) \\ ... & ... \\ S^{\alpha}_{-}\left(\mathbf{x}\right) & S^{\alpha}_{+}\left(\mathbf{x}\right) \end{bmatrix} \tag{1.1}$$

Where coordinate $\mathbf{x}$ that whole field depends on is split into $2n$ dimensional coordinate like field itself it has future and past part:

$$\left(\mathbf{x}\right) = \left(\mathbf{x}^{1}_{-}, \mathbf{x}^{1}_{+}, ..., \mathbf{x}^{\alpha}_{-}, \mathbf{x}^{\alpha}_{+}\right) \tag{1.2}$$

Base vectors in this space-time can be any base vectors that are n orthogonal coordinates but they are split into two parts, base vectors can be expressed as:

$$\hat{\mathbf{x}}^{1}_{-} = \int_{-\frac{1}{2}}^{0} dx^{1}\hat{\mathbf{e}}_{\mathbf{1}} \quad \hat{\overline{\mathbf{x}}}^{1}_{-} = \int_{0}^{-\frac{1}{2}} dx^{1}\hat{\mathbf{e}}_{\mathbf{1}} \tag{1.3}$$

$$\hat{\mathbf{x}}^{1}_{+} = \int_{0}^{\frac{1}{2}} dx^{1}\hat{\mathbf{e}}_{1} \quad \hat{\overline{\mathbf{x}}}^{1}_{+} = \int_{\frac{1}{2}}^{0} dx^{1}\hat{\mathbf{e}}_{\mathbf{1}} \tag{1.4}$$

$$... \tag{1.5}$$

$$\hat{\mathbf{x}}^{\alpha}_{-} = \int_{-\frac{1}{2}}^{0} dx^{\alpha}\hat{\mathbf{e}}_{\alpha} \quad \hat{\overline{\mathbf{x}}}^{\alpha}_{-} = \int_{0}^{-\frac{1}{2}} dx^{\alpha}\hat{\mathbf{e}}_{\alpha} \tag{1.6}$$

$$\hat{\mathbf{x}}^{\alpha}_{+} = \int_{0}^{\frac{1}{2}} dx^{\alpha}\hat{\mathbf{e}}_{\alpha} \quad \hat{\overline{\mathbf{x}}}^{\alpha}_{+} = \int_{\frac{1}{2}}^{0} dx^{\alpha}\hat{\mathbf{e}}_{\alpha} \tag{1.7}$$

I used overline notation for anti-coordinates that represent the opposite of normal coordinates. So S mixed tensor field is a tensor field that have coordinates it moves split into present and past.

1.2. **Light cones.** Light cones direction does not have to be in one direction, more precise time direction is not fixed in this hypothesis. I can create for n dimensional space-time an n orthogonal time directions so there are n light cones, so i can re-write matrix S field as:

$$S^{\alpha_k}_{\beta_k}(\mathbf{x}) = \left[S^{\alpha_1}_{\beta_1}(\mathbf{x}) \quad ... \quad S^{\alpha_k}_{\beta_k}(\mathbf{x}) \right] \tag{1.8}$$

Where each of those matrix inside final matrix represents one possible orthogonal direction of time axis so of all light cones and all other axis. It means that now this matrix has $2n^2$ components. So does coordinates now change from being $2n$ to being $2n^2$:

$$(\mathbf{x}) = \left(\mathbf{x}^{1_1}_{-_1}, \mathbf{x}^{1_1}_{+_1}, ..., \mathbf{x}^{\alpha_1}_{-_1}, \mathbf{x}^{\alpha_1}_{+_1}, ..., \mathbf{x}^{1_k}_{-_k}, \mathbf{x}^{1_k}_{+_k}, ..., \mathbf{x}^{\alpha_k}_{-_k}, \mathbf{x}^{\alpha_k}_{+_k} \right) \tag{1.9}$$

Those light cones are connect to spin, each part of light cone contributes to spin by one half or saying it another full past and future part gives spin one for each part of that field. So spin is just sum of two numbers N_- that represents all past cones and N_+ that represents all future cones:

$$\sigma = \frac{1}{2}(N_- + N_+) \tag{1.10}$$

From S field I will create a more complex tensor field, that is equal to, where I used sum with tilde symbol $\tilde{+}$, I will explain it's meaning in next subsection:

$$T^{\alpha_k \gamma_k}_{\mu_k \nu_k} = \partial_{\mu_k} \left(S^{\alpha_k}_{-_k}(\mathbf{x}) \tilde{+} S^{\alpha_k}_{+_k}(\mathbf{x}) \right) \otimes \partial_{\nu_k} \left(S^{\gamma_k}_{-_k}(\mathbf{x}) \tilde{+} S^{\gamma_k}_{+_k}(\mathbf{x}) \right) \tag{1.11}$$

$$\overline{T}^{\alpha_k \gamma_k}_{\mu_k \nu_k} = \partial_{\mu_k} \left(S^{\alpha_k}_{+_k}(\mathbf{x}) \tilde{+} S^{\alpha_k}_{-_k}(\mathbf{x}) \right) \otimes \partial_{\nu_k} \left(S^{\gamma_k}_{+_k}(\mathbf{x}) \tilde{+} S^{\gamma_k}_{-_k}(\mathbf{x}) \right) \tag{1.12}$$

This field is how S field changes in any direction and will be useful later when I create field equation. For now I can write distance in space-time is equal to:

$$ds^2 = \eta_{\alpha_k \gamma_k} T^{\alpha_k \gamma_k}_{\mu_k \nu_k} dx^{\mu_k} dx^{\nu_k} \tag{1.13}$$

There is need for space-time interval to be true in all frame of reference[2][3] so I can write it as:

$$ds'^2 = ds^2 = \eta_{\alpha_k \gamma_k} T^{\alpha_k \gamma_k}_{\mu_k \nu_k} dx^{\mu_k} dx^{\nu_k} = \eta_{\alpha_k \gamma_k} dx^{\mu_k} dx^{\nu_k} \tag{1.14}$$

That means if field does not change anything it has to give same space-time interval as for changed one.

1.3. **Spin.** In previous subsection I used notation $\tilde{+}$ that means for two base vectors going from one vector to another:

$$S_{-_k}^{\alpha_k}(\mathbf{x})\tilde{+}S_{+_k}^{\alpha_k}(\mathbf{x})=S^{\alpha_k}(\mathbf{x}) \tag{1.15}$$

$$S_{+_k}^{\alpha_k}(\mathbf{x})\tilde{+}S_{-_k}^{\alpha_k}(\mathbf{x})=\overline{S}^{\alpha_k}(\mathbf{x}) \tag{1.16}$$

So i go from past to future or from future to past, that represents field and ant-field. In terms of basis vectors this operation is equal to:

$$\hat{\mathbf{x}}_{-_k}^{\alpha_k}\tilde{+}\hat{\mathbf{x}}_{+_k}^{\alpha_k}=\int_{-\frac{1}{2}}^{\frac{1}{2}}dx^{\alpha_k}\hat{\mathbf{e}}_{\alpha_\mathbf{k}} \tag{1.17}$$

$$\hat{\mathbf{x}}_{+_k}^{\alpha_k}\tilde{+}\hat{\mathbf{x}}_{-_k}^{\alpha_k}=\int_{\frac{1}{2}}^{-\frac{1}{2}}dx^{\alpha_k}\hat{\mathbf{e}}_{\alpha_\mathbf{k}} \tag{1.18}$$

Now I can move to rotation and spin, rotation operator [4] that acts on parts of those vectors can be split into that sum operation:

$$R_{\alpha_k}^{\mu_k}(\phi)=R_{\alpha_k}^{\mu_k}\left(\phi_{-_k}\right)\tilde{+}R_{\alpha_k}^{\mu_k}\left(\phi_{+_k}\right) \tag{1.19}$$

$$R_{\alpha_k}^{\mu_k}(-\phi)=R_{\alpha_k}^{\mu_k}\left(\phi_{+_k}\right)\tilde{+}R_{\alpha_k}^{\mu_k}\left(\phi_{-_k}\right) \tag{1.20}$$

So rotation operator acting on vector that has only one part gives half of rotation, First i write whole equation for acting of full vector then on only one part, where rotation angle sign can't be define for half rotation:

$$R_{\alpha_k}^{\mu_k}\left(\phi_{-_k}\right)\tilde{+}R_{\alpha_k}^{\mu_k}\left(\phi_{+_k}\right)S_{-_k}^{\alpha_k}(\mathbf{x})\tilde{+}S_{+_k}^{\alpha_k}(\mathbf{x})=R_{\alpha_k}^{\mu_k}(\phi)\,S^{\alpha_k}(\mathbf{x}) \tag{1.21}$$

$$R_{\alpha_k}^{\mu_k}\left(\phi_{+_k}\right)\tilde{+}R_{\alpha_k}^{\mu_k}\left(\phi_{-_k}\right)S_{-_k}^{\alpha_k}(\mathbf{x})\tilde{+}S_{+_k}^{\alpha_k}(\mathbf{x})=R_{\alpha_k}^{\mu_k}(-\phi)\,\overline{S}^{\alpha_k}(\mathbf{x}) \tag{1.22}$$

$$R_{\alpha_k}^{\mu_k}\left(\phi_{-_k}\right)S_{-_k}^{\alpha_k}(\mathbf{x})=R_{\alpha_k}^{\mu_k}\left(\pm\frac{\phi}{2}\right)S_{-_k}^{\alpha_k}(\mathbf{x}) \tag{1.23}$$

$$R_{\alpha_k}^{\mu_k}\left(\phi_{+_k}\right)S_{+_k}^{\alpha_k}(\mathbf{x})=R_{\alpha_k}^{\mu_k}\left(\pm\frac{\phi}{2}\right)S_{\beta_{+_k}}^{\alpha_k}(\mathbf{x}) \tag{1.24}$$

$$\tag{1.25}$$

For anti-field i can write same relations :

$$\hat{\overline{\mathbf{x}}}_{+_k}^{\alpha_k}\tilde{+}\hat{\overline{\mathbf{x}}}_{-_k}^{\alpha_k}=\int_{-\frac{1}{2}}^{\frac{1}{2}}dx^{\alpha_k}\hat{\mathbf{e}}_{\alpha_\mathbf{k}} \tag{1.26}$$

$$\hat{\overline{\mathbf{x}}}_{-_k}^{\alpha_k}\tilde{+}\hat{\overline{\mathbf{x}}}_{+_k}^{\alpha_k}=\int_{\frac{1}{2}}^{-\frac{1}{2}}dx^{\alpha_k}\hat{\mathbf{e}}_{\alpha_\mathbf{k}} \tag{1.27}$$

$$R_{\alpha_k}^{\mu_k}(-\phi)=R_{\alpha_k}^{\mu_k}\left(\overline{\phi}_{-_k}\right)\tilde{+}R_{\alpha_k}^{\mu_k}\left(\overline{\phi}_{+_k}\right) \tag{1.28}$$

$$R_{\alpha_k}^{\mu_k}(\phi)=R_{\alpha_k}^{\mu_k}\left(\overline{\phi}_{+_k}\right)\tilde{+}R_{\alpha_k}^{\mu_k}\left(\overline{\phi}_{-_k}\right) \tag{1.29}$$

1.4. **Field equation.** Field equation will state equality between rotation and T tensor field. It can be written:

$$ds^2 = \eta_{\alpha_k\gamma_k} T^{\alpha_k\gamma_k}_{\mu_k\nu_k} dx^{\mu_k} dx^{\nu_k} = \eta_{\alpha_k\gamma_k} R^{\alpha_k}_{\mu_k} \otimes R^{\gamma_k}_{\nu_k} dx^{\mu_k} dx^{\nu_k} \tag{1.30}$$

Now rotation operator needs to preserve space-time distance [5]:

$$ds'^2 = ds^2 = \eta_{\alpha_k\gamma_k} R^{\alpha_k}_{\mu_k} \otimes R^{\gamma_k}_{\nu_k} dx^{\mu_k} dx^{\nu_k} = \eta_{\mu_k\nu_k} dx^{\mu_k} dx^{\nu_k} \tag{1.31}$$

So all operation that are acting on field need to be rotation in $2n^2$ dimensional space-time that comes from splitting future and past of n dimensional Minkowski space-time. There are two rules that field has to obey, first one is that filed is written in Planck units, second that change in field is never more than one or less than minus one. Second rule can be written as:

$$-u^{\alpha_k}_{\mu_k}(\mathbf{x}) \leq \partial_{\mu_k}\left(S^{\alpha_k}_{-k}(\mathbf{x}) \tilde{+} S^{\alpha_k}_{+k}(\mathbf{x})\right) \leq u^{\alpha_k}_{\mu_k}(\mathbf{x}) \tag{1.32}$$

$$-u^{\gamma_k}_{\nu_k}(\mathbf{x}) \leq \partial_{\nu_k}\left(S^{\gamma_k}_{-k}(\mathbf{x}) \tilde{+} S^{\gamma_k}_{+k}(\mathbf{x})\right) \leq u^{\gamma_k}_{\nu_k}(\mathbf{x}) \tag{1.33}$$

Where $u^{\gamma_k}_{\delta_k}(\mathbf{x})$ is just a field that has everywhere value one. It can be expressed in matrix form:

$$u^{\alpha_k}_{\mu_k}(\mathbf{x}) = u^{\gamma_k}_{\nu_k}(\mathbf{x}) = \begin{bmatrix} 1 & ... & 1 \\ \vdots & \ddots & \vdots \\ 1 & ... & 1 \end{bmatrix} \tag{1.34}$$

From it follows that for whole T tensor field same rule apply. That can be expressed:

$$-u^{\alpha_k}_{\mu_k}(\mathbf{x}) \otimes u^{\gamma_k}_{\nu_k}(\mathbf{x}) \leq T^{\alpha_k\gamma_k}_{\mu_k\nu_k} \leq u^{\alpha_k}_{\mu_k}(\mathbf{x}) \otimes u^{\gamma_k}_{\nu_k}(\mathbf{x}) \tag{1.35}$$

Field and anti-field space-time interval has to be equal that can be written as:

$$ds^2 = \eta_{\alpha_k\gamma_k} T^{\alpha_k\gamma_k}_{\mu_k\nu_k} dx^{\mu_k} dx^{\nu_k} = \eta_{\alpha_k\gamma_k} \overline{T}^{\alpha_k\gamma_k}_{\mu_k\nu_k} d\overline{x}^{\mu_k} d\overline{x}^{\nu_k} \tag{1.36}$$

Field and anti-field in reality differ only by that coordinates for anti-field are coordinate for field with minus sign. Field can be real or imaginary but can't be complex. For imaginary field I will get opposite sign of space-time :

$$ds^2_{\mathrm{Re}} = -ds^2_{\mathrm{Im}} \tag{1.37}$$

1.5. **Probability.** Last part of this hypothesis is probability that is crucial in any quantum theory. I can re-write field equation for space-time interval by adding new term, first i start with T tensor:

$$\sum_{p}\psi_{p}\left(\mathbf{x}\right)\psi_{p}^{*}\left(\mathbf{x}\right)T_{\mu_{k}\nu_{k}p}^{\alpha_{k}\gamma_{k}}=\sum_{p}\psi_{p}\left(\mathbf{x}\right)\partial_{\mu_{k}}\left(S_{-_{k}p}^{\alpha_{k}}\left(\mathbf{x}\right)\tilde{+}S_{+_{k}p}^{\alpha_{k}}\left(\mathbf{x}\right)\right)$$
$$\otimes\psi_{p}^{*}\left(\mathbf{x}\right)\partial_{\nu_{k}}\left(S_{-_{k}p}^{\gamma_{k}}\left(\mathbf{x}\right)\tilde{+}S_{+_{k}p}^{\gamma_{k}}\left(\mathbf{x}\right)\right) \tag{1.38}$$

$$\sum_{p}\overline{\psi}_{p}\left(\mathbf{x}\right)\overline{\psi}_{p}^{*}\left(\mathbf{x}\right)\overline{T}_{\mu_{k}\nu_{k}p}^{\alpha_{k}\gamma_{k}}=\sum_{p}\overline{\psi}_{p}\left(\mathbf{x}\right)\partial_{\mu_{k}}\left(S_{+_{k}p}^{\alpha_{k}}\left(\mathbf{x}\right)\tilde{+}S_{-_{k}p}^{\alpha_{k}}\left(\mathbf{x}\right)\right)$$
$$\otimes\overline{\psi}_{p}^{*}\left(\mathbf{x}\right)\partial_{\nu_{k}}\left(S_{+_{k}p}^{\gamma_{k}}\left(\mathbf{x}\right)\tilde{+}S_{-_{k}p}^{\gamma_{k}}\left(\mathbf{x}\right)\right) \tag{1.39}$$

Now I can move to field equation:

$$\sum_{p}\psi_{p}\left(\mathbf{x}\right)\psi_{p}^{*}\left(\mathbf{x}\right)ds_{p}^{2}=\sum_{p}\psi_{p}\left(\mathbf{x}\right)\psi_{p}^{*}\left(\mathbf{x}\right)\eta_{\alpha_{k}\gamma_{k}}T_{\mu_{k}\nu_{k}p}^{\alpha_{k}\gamma_{k}}dx_{p}^{\mu_{k}}dx_{p}^{\nu_{k}}$$
$$=\sum_{p}\psi_{p}\left(\mathbf{x}\right)\psi_{p}^{*}\left(\mathbf{x}\right)\eta_{\alpha_{k}\gamma_{k}}R_{\mu_{k}p}^{\alpha_{k}}\otimes R_{\nu_{k}p}^{\gamma_{k}}dx_{p}^{\mu_{k}}dx_{p}^{\nu_{k}} \tag{1.40}$$

Probability sum of all wave functions multiply with its complex conjugate over whole space has to be equal to one:

$$\sum_{p}\int\psi_{p}\left(\mathbf{x}\right)\psi_{p}^{*}\left(\mathbf{x}\right)d\mathbf{r}=1 \tag{1.41}$$

When there is measurement done all values of wave function go to zero expect that one that was measured, this can be written using knocker delta, where measured outcome is q index, that knocker delta can vary across time that means that q value can be different for given point of time or more precise for point of time this function can have different q value so q is a function of time $q(\mathbf{t})$:

$$\sum_{p}\psi_{p}\left(\mathbf{x}\right)\psi_{p}^{*}\left(\mathbf{x}\right)\mapsto\sum_{p}\delta_{pq(\mathbf{t})} \tag{1.42}$$

References

[1] https://mathworld.wolfram.com/LightCone.html

[2] https://mathworld.wolfram.com/LorentzTransformation.html

[3] https://mathworld.wolfram.com/MinkowskiMetric.html

[4] https://mathworld.wolfram.com/RotationMatrix.html

[5] https://www.sciencedirect.com/science/article/pii/S221137971832076X