A new sky for the Collatz conjecture

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This small excerpt from our paper on the Collatz conjecture is intended to give new directions to those who are working on the understanding of this problem, but, above all, for those who have invested in the solution of this problem. We have limited ourselves in this document to raising awareness of the approach we have adopted, which was certainly not the right one. This is what makes such a simple problem seem so difficult to solve. As this man said: it is difficult to paint the real landscape when you accidentally end up on another landscape.

In this paper, we make a contribution to the understanding that the trivial cycle of basis 1 2 4 is only one case among a whole set of trivial cycles. And that in this generality, the trivial cycle of Collatz has the same properties as the other cycles and that it should not always be taken as a basic element. In this sense, trying to understand it does not bring us back to determining the behavior of this cycle because, other notions must be understood beforehand. Hence; in our opinion, all the difficulties that mathematicians and others encounter today and since.

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I- What is the sequence of Collatz (Let's find it in Wikipedia)

In mathematics, we call a sequence of natural integers defined in the following way : we start from a strictly positive integer ; if it is even, we divide it by 2 ; if it is odd, we multiply it by 3 and we add 1. By repeating the operation, we obtain a sequence of strictly positive integers, each of which depends only on its predecessor.

For example, from 14, we build the following sequence of numbers : 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2... This is called the Syracuse sequence of the number 14.

After the number 1 has been reached, the sequence of values (1,4,2,1,4,2...) repeats itself indefinitely in a cycle of length 3, called trivial cycle.

If we had started from another integer, by applying the same rules, we would have obtained a different sequence of numbers. A priori, it would be possible that the Syracuse sequence of certain starting values never reaches the value 1, either that it leads to a cycle different from the trivial cycle, or that it diverges towards infinity. However, we have never found an example of a sequence obtained according to the given rules which does not end up at 1, and then at the trivial cycle.

The Syracuse conjecture, also called the Collatz conjecture, the Ulam conjecture, the Czech conjecture or the 3x + 1 problem, is the mathematical hypothesis that the Syracuse sequence of any strictly positive integer reaches 1.

The problem to be solved until now known for this sequence is indeed the demonstration which would make it possible to show that all the Syracuse sequences always reach 1. Until today, nobody has dared to propose a satisfactory answer. Some have finally tried to undertake a generalization of this problem by proposing other trivial cycles different from 1 2 4. At this level again, the efforts did not lead to concrete results. For some, it was necessary to look for another trivial cycle on which a cycle of a sequence of an integer would lead. The latter in our opinion is possible but not as we envisage it.

Obviously, when we approached this problem, we started out with the same intuition as that of understanding each element or each whole that acts in this problem. Because, as time went by, we realized that nothing in mathematics seems to prevent chance. Even when this has been done without any prior foundation. As if things were constituted before we were there, we must therefore try to understand them at all costs.

Thus, our final question before addressing this conjecture and finding the reasons for its existence was this: Why does the trivial cycle impose on us the numbers 1 2 4 and not other known integers? Even far away, we have by simple joke asked: Why this cycle is not of the form 1 4 2 or 4 1 2 ? Because in the end, all integers can be represented here as well. In the end, we stayed on the cycle 1 2 4 and we undertook to study it.

II- Mill of Surielle

When we observe the trivial cycle that here 1 2 4, we will notice that these three integers are linked by a strong link. That it was not by chance that we had this trivial cycle. Maybe Collatz didn't realize it because sometimes we do something that makes sense, but we don't know the basis for this logic. I myself made a lot of discoveries about this by doing something that was logical, and then, afterwards, I was trying to find out how it was possible.

In order to try to understand this conjecture and solve it as easily as possible, we needed to understand each element involved in the trivial cycle. As I was saying, when we master all the ingredients, we can make multiple dishes. If, on the other hand, we know or even taste the dish, it is difficult to make another one or even to smell all the ingredients. Therefore, we must look for the ingredients at the base and understand if the dish cannot be prepared differently. You know the famous question of content and container. Between a one Euro coin and a loaf of bread; what contains the other? It is therefore what contains our trivial cycle and is capable of producing it that will be our objective here. How can we do this?

Looking at the trivial cycle 1 2 4, we came to this conclusion:

- The number 4 is the double if not the square of 2
- The number 2 is therefore half if not the square root of 4
- The number 2 is the double of 1 (only, at this level, we just have to notice that it is not valid for any integer)
- For the number 1, we did not say anything.

How can we then deduce another cycle from the data we have? To do so, we must rely on a single element capable of giving the other two elements. Here, it is the number 2 that we call **the basic integer or arithmetic variable**. Thus, if we take 2 in our trivial cycle 1 2 4, we will say that its square gives us 4. Similarly, we go even further and say that dividing 2 over itself gives us 1. So, we have from 2 our trivial cycle which is e 1 2 4. We can then from this rather simple approach generalize all this through a tool that we have just stated and which is the **Surielle mill**.

A- Understanding the Moulin de Surielle

In an even simpler way, we say that Surielle's mill is a tool that allows us to make a basic trivial cycle out of any "strictly positive" natural number because the inverse is true. So, if we consider x as any basic integer, we just have to determine its square which will give us x^2 and do the division of x to get 1. Note that the division of x means that the integer x is divided by itself. So in the end we get a basic model for any trivial cycle which is :

$$1 \ x \ x^2$$

To be able to check all this, we will take an example from a basic integer. If we take a natural integer like 3, we can determine for this integer the trivial cycle that goes with it. We just have to determine the square of 3 which gives us 9 and then divide 3 which gives us 1. Thus, we end up with a trivial cycle of the integer 3 of the form **1 3 9.**

For the case of the cycle that we all know and that is for the integer of base 2, we can proceed in the same way. If we consider that 2 is our base integer, we will determine its square which is 4. Dividing 2 gives us 1 and this brings us to a trivial cycle of this kind 1 2 4. So now we understand how the trivial cycle 1 2 4 was formed. Not obtained but we suppose that it already existed in this universe of numbers.

Now, everything is done to obtain the trivial cycles of any natural number. So we can say that there is an infinity of trivial cycles. But it doesn't stop there. Because in reality, each trivial cycle is accompanied by two major rules that we call **rules of ascendance and rules of descent**, we will not go into too much detail because this is not the place for all this. Our goal is just to show that the 1 2 4 trivial cycle is only a tiny part of an infinite number of trivial cycles that exist and that this exclusivity that we have granted it does not make it the major cycle. By the way, this rule is valid even for integers with commas. In fact for any integer of any known set. Except that with decimal numbers, you will have to taste a bit of complexity as we did.

So, for our descent rule, if we rely on what we already knew with the trivial cycle 1 2 4, we will say that it is of the form $\frac{n}{2}$. And for the descent rule, we will say that it is of the form Bn + 1

1- **Let's understand and determine the rule of descent**

The latter is rather easy, in the ancestry rule we have seen that it is the base integer that imposes it. Because, to get 1, you saw that we divided the base integer by itself. Thus, implicitly, this operation allows any integer to be verified if it is of the same nature as the basic integer. Hence, the form of the rule in the trivial cycle 1 2 4 which gives us $\boldsymbol{\frac{n}{2}}$**.**

So, if we consider our base integer to be **x**, for any **n** that we consider to be any integer, the ancestry rule for that integer will give us this:

$$\boldsymbol{\frac{n}{x}}$$

With **n** any integer and **x** our base integer.

Hence, in the Collatz cycle, as we have said, we have the form of the rule of ascendancy in this way:

$$\boldsymbol{\frac{n}{2}}$$

If we now return to the trivial cycle where the base integer is 3 and the cycle in this form 1 3 9, our ancestry rule will be easily applicable here:

$$\boldsymbol{\frac{n}{3}}$$

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So, we will check each time a number **n** if it is divisible by 3, we divide it by 3, if it is the opposite, then we have to determine the descent rule for this case.

2- **Let's understand and determine the rule of descent**

Still based on the Collatz trivial cycle model, we already have the form of the descent rule which is 3n + 1. But, how is this rule possible? Or how did the coefficient 3 end up in this place. Because, the grandfather Collatz or no one else could tell us why the coefficient 3 is at this level. Finally, it is true that we did not go through all the literature but we did not see this proposal. So we will proceed to another understanding of the coefficient B and then make the calculations for all B.

In the trivial Collatz cycle which is 1 2 4, we have seen that the coefficient 3 can be easily obtained by posing this difference: 4 - 1, which gives us 3. So the coefficient of B here is 3 and that gave us 3n + 1. This approach in its generality will give us this:

If we consider that the absolute top (this is how we call these values and we will explain one day why) 4 is the square of the base integer, then, we say that the coefficient B is obtained by the following difference:

$$B = x^2 - 1$$

Thus, we can write the descent rule in this way for all B :

$$(x^2 - 1)n + 1$$

Thus, each time we have the square of a base integer, we can determine the coefficient B that leads us to obtain the descent rule in a trivial cycle.

Coming back with the trivial cycle of the basic integer 3 which is of the form 1 3 9, we can calculate the coefficient B of this cycle.

Coefficient B of the basic integer 3 = 9 - 1

Coefficient B of the base integer 3 = 8

So the descent rule of this trivial cycle 1 3 9 is of this form :

8n + 1

So, we have just gathered here all the necessary elements to establish what we qualify by a "Surielle Universe". A Surielle universe is a universe composed of a trivial cycle and the set of two rules that allow us to verify that any integer in a so-called Syracuse sequence can always reach 1. Staying on these notions of cycles, we can therefore state the following:Prenez un nombre entier positif, et appliquez-lui le traitement suivant :

- If it is odd and divisible by 3, you divide it by 3, that is n / 3 ;
- If it is odd and not divisible by 3, you multiply it by 8 and add 1, so 8n + 1.
- If it is even and not divisible by 3, you multiply it by 8 and add 1, that is 8n + 1.

- If it is even and divisible by 3, you divide it by 3, that is n / 3 ;

Thus, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 3 9.

You will notice that we have just added another rule because among the natural integers we have integers that are at the same time divisible by 3 and divisible by 2, that is to say even integers. In the same way that among the odd integers you have integers divisible by 3 and those that are not. This is not the case with the sequence of Collatz which has for base integer 2 and which is the smallest divisor of all even integers. In this case, the n/2 rule remains absolute for any even integer. If you go a little further and consider that your base integer is 4, you will upset this whole rule. We have just given you the means, test it and you will see.

Now we have a trivial cycle which will bring all the sequences to 1, in the same way as the trivial cycle we know which is 1 2 4.

With all this, we can then give ourselves any trivial cycle and define the two rules that go with it using the properties given by the Surielle Mill. Let's start at the base.

Cycle One

Take a positive integer, and apply the following treatment to it:

- If it is even, you divide it by 2, that is n / 2 ;
- If it is odd, you multiply it by 3 and add 1, that is 3n + 1.

Second cycle

Take a positive integer, and apply the following treatment to it:

- If it is odd and divisible by 13, you divide it by 13, that is n / 13 ;
- If it is odd and not divisible by 13, you multiply it by 168 and add 1, so 168n + 1.
- If it is even, you multiply it by 168 and add 1, that is 168n + 1.

Thus, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 13 169.

Third cycle

Take a positive integer, and apply the following treatment to it:

- If it is odd and divisible by 7, you divide it by 7, that is n / 7 ;
- If it is odd and not divisible by 7, you multiply it by 48 and add 1, so 48n + 1.
- If it is even, you multiply it by 48 and add 1, that is 48n + 1.

Thus, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 7 49.

Fourth cycle

Take a positive integer, and apply the following treatment to it:

- If it is odd and divisible by 131, you divide it by 131, i.e. n / 131 ;
- If it is odd and not divisible by 131, you multiply it by 17160 and add 1, so 17160n + 1.
- If it is even, you multiply it by 131 and add 1, that is 17160n + 1.

Thus, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 131 17161.

Fifth cycle

Take a positive integer, and apply the following treatment to it:

- If it is odd and divisible by 11, you divide it by 11, i.e. n / 11 ;
- If it is odd and not divisible by 11, you multiply it by 120 and add 1, so 120n + 1.
- If it is even, you multiply it by 120 and add 1, that is 120n + 1.

Thus, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 11 121.

Sixth cycle

Take a positive integer, and apply the following treatment to it:

- If it is odd and divisible by 3, you divide it by 3, i.e. n / 3 ;
- If it is odd and not divisible by 3, multiply it by 8 and add 1, i.e. 8n + 1.
- If it is even and not divisible by 3, you multiply it by 8 and add 1, that is 8n + 1.
- If it is even and divisible by 3, you divide it by 3, that is n / 3 ;

Thus, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 3 9.

Seventh cycle

Take a positive integer, and apply the following treatment to it:

- If it is even and divisible by 16, you divide it by 16, i.e. n / 16 ;
- If it is even and not divisible by 16, you multiply it by 255 and add 1, so 255n + 1.
- If it is odd, you multiply it by 255 and add 1, that is 255n + 1.

So, whatever number you choose, you will always end up with 1 and in a trivial cycle of 1 16 256.

B- Function of the generalization

All of this has finally led us to what? In fact, solving the Collatz sequence should not go through everything we have seen so far. In my humble opinion, given that we have a tool that allows us to obtain trivial cycles and that the trivial cycle 1 2 4 is nothing more than a case among an infinity of cases. So the most essential question for us would be this:

Why and how does each cycle always return to 1?

It is on the latter that we have proposed a resolution.

This is not to trample on everything that has been done, because I have respect especially for all good will. But, it is not in the understanding of the flight time, the landing time or other notion in this kind that we will manage to solve this problem. If it is really in this case, we will certainly be tempted to solve this problem for all cycles. You will test these cycles and we will also give you some durations, you will see that the flight durations for an integer vary from one cycle to another. So we realized that the problem was elsewhere. So I looked elsewhere.

I already said it in some forums where I just wanted to show that there was much more than the trivial cycle of Collatz and that many people took a long way to solve this conjecture. I'm not belittling anyone or minimizing anyone, I'm just saying what I think to try to make my contribution.

For years, I have been browsing through documents, curious to see for example if there were other trivial cycles to see how the resolution will be approached, I have not had any. We talk about the 5n + 1 or 5x + 1 rule. As I said earlier, this rule is far from being a descent rule in the strict sense. It can be important but not in the sense of understanding the resolution of Collatz.

I had said that to check that a descent rule is true, we just have to add to the coefficient B the value 1 and check that the result gives us a perfect square. If in our case of 5x + 1, we add 1 to 5 which is the coefficient B, we get 6, while 6 is not a perfect square, we simply say that this rule can only be limited and you know it. You have others to test, do that and you will see. We are only talking about the 5x + 1 rule. Others have also tried other cycles and unfortunately it doesn't work. In the end, all the cycles I have gone through so far and proposed by many people were all far from the objectives we have defined in the Surielle mill. If you don't go through this step, you can't get a trivial cycle that reaches 1 for any known sequence in this genre. This will be a waste of time.

Now, you can have any cycle with the Surielle mill and test as many as it has been tested with the Collatz cycle, you will have infinite sequences that will always wait for 1. All you need is the patience to see that your goal will always be reached. We will discover that each integer has its own duration in each universe before reaching 1. In any case, one thing is sure, it will always reach 1.

That's when I realized that any sequence ends up reaching 1, so I had to explain how and why all this was possible. And you will have the answer by tomorrow, in a month, in a year, but the resolution is there.

To relieve you a little bit and to prove what we have said, we are going to take for example the case of some numbers and to determine their duration in given universes. We will give you the duration of flight. Let us also not give the whole cycle for all the sequences. This would allow us to move faster and save pages. I am an environmentalist and I don't like to abuse the Word environment too much.

1- **Duration of Flight of the base integer 2 in different trivial cycles.**

In the trivial cycle 1 2 4, the duration of 2 is 2

In the trivial cycle 1 3 9, the duration of 2 is 561

In the trivial cycle 1 7 49, the duration of 2 is 231

In the trivial cycle 1 9 81, the duration of 2 is 195

In the trivial cycle 1 11 121, the duration of 2 is 172

In the trivial cycle 1 13 169, the duration of 2 is 156

In trivial cycle 1 15 225, the duration of 2 is 151

2- **Duration of Vol of the base integer 3 in different trivial cycles.**

In the trivial cycle 1 2 4, the duration of 3 is 8

In the trivial cycle 1 3 9, the duration of 3 is 2

In the trivial cycle 1 7 49, the duration of 3 is 230

In the trivial cycle 1 9 81, the duration of 3 is 201

In the trivial cycle 1 11 121, the duration of 3 is 178

In the trivial cycle 1 13 169, the duration of 3 is 161

In the trivial cycle 1 15 225, the duration of 3 is 141

3- **Duration of Flight of the base integer 7 in different trivial cycles.**

In the trivial cycle 1 2 4, the duration of 7 is 17

In the trivial cycle 1 3 9, the duration of 7 is 565

In the trivial cycle 1 7 49, the duration of 7 is 2

In the trivial cycle 1 9 81, the duration of 7 is 195

In the trivial cycle 1 11 121, the duration of 7 is 166

In the trivial cycle 1 13 169, the duration of 7 is 152

In the trivial cycle 1 15 225, the duration of 7 is 159

4- Duration of Vol of the base 11 integer in different trivial cycles.

In the trivial cycle 1 2 4, the duration of 11 is 15

In the trivial cycle 1 3 9, the duration of 11 is 585

In the trivial cycle 1 7 49, the duration of 11 is 232

In the trivial cycle 1 9 81, the duration of 11 is 193

In the trivial cycle 1 11 121, the duration of 11 is 2

In the trivial cycle 1 13 169, the duration of 11 is 156

In the trivial cycle 1 15 225, the duration of 11 is 158

5- Duration of Vol of the base integer 13 in different trivial cycles.

In the trivial cycle 1 2 4, the duration of 13 is 10

In the trivial cycle 1 3 9, the duration of 13 is 525

In the trivial cycle 1 7 49, the duration of 13 is 237

In the trivial cycle 1 9 81, the duration of 13 is 188

In the trivial cycle 1 11 121, the duration of 13 is 166

In the trivial cycle 1 13 169, the duration of 13 is 2

In the trivial cycle 1 15 225, the duration of 13 is 152

6- Duration of Vol of the basic integer 1217 in different trivial cycles.

In the trivial cycle 1 2 4, the duration of 1217 is 133

In the trivial cycle 1 3 9, the duration of 1217 is 511

In the trivial cycle 1 7 49, the duration of 1217 is 246

In the trivial cycle 1 9 81, the duration of 1217 is 181

In the trivial cycle 1 11 121, the duration of 1217 is 163

In the trivial cycle 1 13 169, the duration of 1217 is 152

In the trivial cycle 1 15 225, the duration of 1217 is 136

7- Duration of Vol of the basic integer 6113 in different trivial cycles.

In the trivial cycle 1 2 4, the duration of 6113 is 156

In the trivial cycle 1 3 9, the duration of 6113 is 528

In the trivial cycle 1 7 49, the duration of 6113 is 226

In the trivial cycle 1 9 81, the duration of 6113 is 187

In the trivial cycle 1 11 121, the duration of 6113 is 170

In the trivial cycle 1 13 169, the duration of 6113 is 149

In the trivial cycle 1 15 225, the duration of 6113 is 154

Note: in most algorithms, the number 1 is given a duration of 0, while in our algorithm, the number 1 has a duration of 1. This means that our basic integers have a duration of 2. This is what we have underlined in blue. If you have algorithms that admit a duration of 0 for 1, then we will have some differences. So each time, subtract 1 in our flight times.

We will not say more than what we just said here. The purpose of the document has been achieved. It's up to you to see if you will consider all this or if you will use it as you know how. We have said a new heaven for Collatz, we have offered it to you, you go in or you stay out.

Thank you

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