The 5 equations that generate all composite numbers

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Abstract:

This paper provides a study on composite numbers and delivers 5 equations that, when put together, generate all existing composite numbers. This is done by sorting all natural numbers in 6 groups and following the mathematical reasoning that explains the generation of all composite numbers for each one of these groups. Derived from this, two different ways to obtain prime numbers by iteration are provided, although slower in computational speed than the ones existing today. An iteration method to find twin prime numbers is also described.

Introduction:

For all natural numbers N, where N = {1, 2, 3, 4, 5…} each number is either a prime or a composite number, with the exception of number 1. A prime number is a natural number that is only divisible by itself and 1. A composite number is a natural number that is divisible by itself, 1 and at least a third number.

The first records of prime numbers date from the Rhind mathematical papyrus, around 1550 BC and already in ancient Greece prime numbers were studied in detail, with the finding of Mersenne prime numbers (prime numbers M that follow $M_n = 2^n-1$) and the Sieve of Eratosthenes (a method where the multiples of each prime are eliminated from the next natural numbers). Some of the learning from that time are still used today or are the foundation to develop similar methods (i.e. Sieve of Sieve of Pritchard, Sieve of Atkin, Sieve of Sundaram…)[1,2].

The most important use of prime numbers nowadays lies in computing and cybersecurity, in fields such as bank transactions, cryptocurrency or computer safety. [3] It is in this field where the obtention of particularly large prime numbers becomes important, with public contests awarding important prizes for teams delivering primes numbers of millions of cyphers, e.g. the EFF Cooperative Computer Awards.

One of the most common ways to test if a number is or not a prime number is by using a primality test. Primality tests are algorithm to determine if an input number is prime or not and they can range from rather simple ones like the trial by division test or the 6k±1 method (other than 2 and 3, all prime numbers are known to be of the form 6k±1). The use of computers and algorithms helped to speed the process but this simpler methods become impractical for large numbers, and more sophisticated primality tests exist, such as the Fermat primality test, the Miller-Rabin and Solovay-Strassen primality test, the Frobenius primality test or the Baillie-PSW primality, to name a few. Some methods to find prime number, as the Miller Rabin primality test, are not 100% accurate as a price to their faster speed. Other methods pay a price on speed of calculation, for example the AKS primality test.

With the goal of finding new large prime numbers faster, the search of specific families of prime numbers is an used method. For example, the above mentioned Mersenne numbers. Other methods that are restricted to specific number forms include Pépin's test for Fermat numbers (1877), [4] Proth's theorem (c. 1878),[5] the Lucas–Lehmer primality test (originated 1856), and the generalized Lucas primality test.[2,6] Bernstein (2004) summarized 14 of methods to prove that an integer is prime, three additional methods to prove that an integer is prime if certain conjectures are true, and four methods to prove that an integer is composite. [7]

It is not the aim of this paper to go deeper into the algorithms to find and prove prime numbers, but to understand, by means of mathematical reasoning, the factors that generate all composite numbers and provide an explanation to the apparent randomness of their gaps. Prime gaps (the differences between consecutive primes) are still seen as to some degree arbitrarily occurrences and other questions remain unsolved. [8]

On the last years, more patterns on the apparent randomness of prime numbers have been found. [9] Dan Goldston, János Pintz, and Cem Yıldırım proved that there are infinitely many primes for which the gap to the next prime is as small as we want compared to the average gap between consecutive primes. [10], a phenomenon further stuied by Soundararajan. [11] A phenomenon of interdependency between the structure of positive integers and the form of their prime factors was discovered by Karatsube in 2011. [12] In 2015 Granville developed further the findings of Zhang on twin prime numbers (2 prime numbers separated by only one composite number). [13, 14] Prime numbers near to each other tend to avoid repeating their last digits, and most importantly, as observed by Lemke and Soundararajan in 2016, all primes have a remainder of 1 or 5 when divided by 6 (otherwise, they would be divisible by 2 or 3) and the two remainders are on average equally represented among all primes. [15]. This occurrence in groups of 6, together with it happening in Euler's theorem (where 6n+1 are analyzed) and the primality test, with almost all prime numbers are of the form 6n+1 or 6n-1 is from where the author starts his analysis.

Detailed analysis of the generation of composite numbers:

The study presented in this paper starts from the following premise:

If we could find a set of equations that generate all existing and only composite numbers, then we can prove that the occurrence of composite numbers is not random.

A way to be able to prove that a set of equations generates all and only composite numbers is to sort all natural numbers in groups and then find the equation/s that generate all composite numbers for each one of these groups. In 2016, Lemke and Soundararajan found that all prime numbers are never divisible 6, an observation well aligned with the simpler primality tests. In this work all natural numbers N are sorted in 6 groups, A to F:

A contains all natural numbers generated by N = 1+6n where n = {0, 1, 2, 3, 4…}

B contains all natural numbers generated by N = 2+6n where n = {0, 1, 2, 3, 4…}

C contains all natural numbers generated by N = 3+6n where n = {0, 1, 2, 3, 4…}

D contains all natural numbers generated by N = 4+6n where n = {0, 1, 2, 3, 4…}

E contains all natural numbers generated by N = 5+6n where n = {0, 1, 2, 3, 4…}

F contains all natural numbers generated by N = 6+6n where n = {0, 1, 2, 3, 4…}

To help visualize the calculations and reasoning of this paper, the first 48 natural numbers are represented in Table 1, where each column is one of the groups described above. Number 1, not being considered a prime number, is already marked in red in Table 1:

Table 1 Natural numbers sorted in rows, with 6 columns each. Number one not being a prime is marked as red.

1+6n Group A	2+6n Group B	3+6n Group C	4+6n Group D	5+6n Group E	6+6n Group F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
…	…	…	…	…	…

The first equation that generates only composite numbers, c1:

The first prime number is 2, and therefore by the composite definition, we know that any multiple of 2 will be a composite number, because it will be divisible by 2. Composite numbers that are multiples of 2 (c1) are generated by equation 1:

Equation 1:

$$c1 = 2(n + 2)$$

where n = {0, 1, 2, 3, 4…}

so if n=0 then c1=4, leaving this way the prime number 2 out of the composite generating equation.

Groups B , D and F contain only numbers which are 2 or multiples of 2, while groups A, C and E contain only odd numbers. This is easily demonstrated by dividing the equation that generates each group by 2 and checking if the result is a natural number or not.

Group A: (1+6n)/2 = 0.5+3n will never be a natural number, none of these numbers are divisible by 2.

Group B: (2+6n)/2 = 1+3n will always be a natural number, all these numbers are divisible by 2.

Group C: (3+6n)/2 = 1.5+3n will never be a natural number, none of these numbers are divisible by 2.

Group D: (4+6n)/2 = 2+3n will always be a natural number, all these numbers are divisible by 2.

Group E: (5+6n)/2 = 2.5+3n will never be a natural number, none of these numbers are divisible by 2.

Group F: (6+6n)/2 = 3+3n will always be a natural number, all these numbers are divisible by 2.

From above, only groups B, D and F are divisible by 2 and therefore means that groups B, D and F are generated by equation 1 and hence composite numbers, with exception of the number 2.

Table 2 shows all composites multiple of 2 marked in red for ease of visualization.

Table 2 All the composite numbers divisible by 2 have been now marked in red too.

1+6n Group A	2+6n Group B	3+6n Group C	4+6n Group D	5+6n Group E	6+6n Group F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
…	…	…	…	…	…

From above, we can affirm that groups B, D and F contain only and all of the composite numbers that are a multiple of 2 with the exception of number 2 itself, which is prime. This forms the group of all even natural composite numbers, c1. Equation 1 is the first equation that generates only composite numbers, but not all composite numbers. More equations are missing as more groups have not been analyzed.

<u>Second equation that generates only composite numbers, c2:</u>

The second prime number is 3, and therefore by the composite definition, we know that any multiple of 3 will be a composite number, because it will be divisible by 3. Composite numbers multiple of 3 (c2) are generated by equation 2:

Equation 2:

$$c2 = 3(n + 2)$$

where n = {0, 1, 2, 3, 4…}

so if n=0 then c2=6, leaving the prime number 3 out of this composite generating equation.

Groups C and F contain only numbers which are 3 or multiples of 3, while groups A, B, C and E contain only numbers that are not multiple of 3. This is easily demonstrated by dividing the equation that generates each group by 3 and checking if the result is a natural number or not.

Group A: (1+6n)/3 = 1/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group B: (2+6n)/3 = 2/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group C:(3+6n)/3 = 1+2n will always be a natural number, all these numbers are divisible by 3.

Group D: (4+6n)/3 = 4/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group E: (5+6n)/3 = 5/3+2n will never be a natural number, none of these numbers are divisible by 3.

Group F: (6+6n)/3 = 2+2n will always be a natural number, all these numbers are divisible by 3.

From above, only groups C and F are divisible by 3. They are generated by equation 2, with the exception of the number 3.

Table 3 shows all composites multiple of 2 and 3 marked in red for means of visualization.

Table 3: All composite numbers multiple of 3 are now marked in red too.

1+6n Group A	2+6n Group B	3+6n Group C	4+6n Group D	5+6n Group E	6+6n Group F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
…	…	…	…	…	…

From above, we can affirm that groups C and F contain only and all of the composite numbers that are a multiple of 3 with the exception of number 3 itself which is prime. This forms the group of all natural composite numbers c2, the multiples of 3. Equation 2 is the second equation that generates only composite numbers, but not all composite numbers. More equations are missing as more groups have not been analyzed.

With the above, we have demonstrated that, besides numbers 2 and 3, all remaining prime numbers must belong to either group A (1+6n) or group E (5+6n). This is in good agreement with the primality test and previous findings where all prime numbers besides 2 and 3 are of the type 6n+1 and 6n-1, because 6n+1 equals a group A prime number and 6n-1 a group E prime number.

Composite numbers will also be present in A and E together with the prime numbers. Now we must understand what rules define the nature of the composite numbers in groups A and E.

<u>Defining how the composite numbers in groups A and E are generated:</u>

By definition, any composite number can be obtained by multiplying 2 natural numbers (or the same natural number twice), being those numbers other than itself and 1.

Any multiplication of 2 natural numbers where at least one of them is even, will always result in a composite number which also is even. For example, being:

2n an even number

where n = {1, 2, 3, 4…}

and

m = {1, 2, 3, 4…}

then by equation 3, a multiplication of any m by any 2n, will always be divisible by 2, and therefore, an even number.

Equation 3:

$$\frac{2n \times m}{2} = n \times m$$

We have already established when defining c1 composites, that all even composite numbers are only found in groups B, D and F. Therefore, any composite numbers in groups A and E is not even. Therefore any composite number in groups A and E cannot result from multiplying a number from B, D or F with any other given number, because the result would be even, according to equation 3, but we have already established that even numbers cannot be found in groups A and E.

In short: composite numbers in groups A and B are not divisible by any number found in groups B, D and F.

In a similar way, any multiplication of 2 natural numbers where at least one of them is 3 or a multiple of 3, will always result in composite number which also is multiple of 3. For example, being:

3n a multiple of 3

where n = {1, 2, 3, 4…}

and

m = {1, 2, 3, 4…}

then by equation 4, the multiplication of any 3n by any m will be always divisible by 3, and therefore a multiple of 3 itself.

Equation 4:

$$\frac{3n \times m}{3} = n \times m$$

We have already established when defining c2 composites, that all composite numbers multiple of 3 are only found in groups C and F. Therefore, any composite number in groups A and E cannot be a multiple of 3. Therefore any composite number in groups A and E cannot be a result of multiplying a number from C, or F with any other given number, because the result would be a multiple of 3 according to equation 4, but we have already established that numbers multiple of 3 cannot be found in groups A and E.

In short: composite numbers in groups A and E are not divisible by any number in groups C and F. Not by any number in groups B, D and F, as said earlier.

If numbers in groups A and E are not divisible by any number found in B, C, D or F as already established, we have demonstrated that any composite number found in groups A and E is a result of multiplying 2 natural numbers also found in groups A and/or E.

<u>The 3 remaining equations that generate only composites, c3, c4 and c5:</u>

There are three ways to generate composite numbers using 2 numbers from groups A and/or E.

1.- By multiplying 2 numbers from group A, generating a composite number c3:

Equation 5:

$$c3 = (1 + 6n) \times (1 + 6m)$$

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}. This restriction is very important, neither n or m can be 0, otherwise the number resulting would be 1 for that one and c3 could then be a composite (e.g. n=0 and m=4), a primer (e.g. n=0 and m=1) or even 1 (if both n and m were 0). For all composite, there must be a solution other than itself multiplied by 1, so restricting the equation to not be able to generate 1 in any of the two numbers, all primers are ruled while still allowing to generate all composite.

2.- By multiplying two numbers from group E generating a composite number c4:

Equation 6:

$$c4 = (5 + 6n) \times (5 + 6m)$$

where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

3.- By multiplying one number from group A and one number from group E, generating a composite number c5:

Equation 7:

$$c5 = (1 + 6n) \times (5 + 6m)$$

where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}. n cannot be 0 for the same reason stated above, it would generate number 1 on that factor and a prime number could be generated too.

We have established the 3 equations that can generate all the composite numbers in columns A and E.

The next step is to see if the 3 of them are present in both groups A and E or not and to prove that all primes in groups A and E are not a valid solution of any of these 3 equations. We will start by analyzing in detail group A and afterwards group E.

Finding what composites appear in group A:

Checking the appearance of composites c3 in group A:

If a composite number in A of the form c3=1+6k where k = {1, 2, 3, 4…} can be formed by multiplying two other numbers from group A (1+6n) and (1+6m), then equation 8 must have at least one valid solution, otherwise a composite number in group A would never be obtained by multiplying two other numbers from group A:

Equation 8:

$$\frac{1 + 6k}{(1 + 6n) \times (1 + 6m)} = 1$$

where n = {1, 2, 3, 4…}, m = {1, 2, 3, 4…} and k = {1, 2, 3, 4…}

$$1 + 6k = 1 + 6(n + m) + 36nm$$

$$6k = 6(n + m) + 36nm$$

$$k = n + m + 6nm$$

Because it is possible to obtain valid values of k for equation 8 using valid values for n and m, it will be possible to find c3 composite numbers in group A. For example, 91 is a group A number (1+6x15) and is a c3 composite originated by multiplying 7 and 13 (two other group A numbers). In this example, k=15 and n=1 and m=2.

Checking the appearance of composites c4 in group A:

If a composite number in A of the form c_4=1+6k where k = {1, 2, 3, 4…} can be formed by multiplying two numbers from group E (5+6n) and (5+6m), then equation 9 must have at least one valid solution, otherwise a composite number in group A would never be obtained by multiplying two other numbers from group E:

Equation 9:

$$\frac{1+6k}{(5+6n)\times(5+6m)}=1$$

where n = {0, 1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {1, 2, 3, 4…}

$$1+6k=25+30(n+m)+36nm$$

$$6k=24+30(n+m)+36nm$$

$$k=4+5(n+m)+6nm$$

Because it is possible to obtain valid values of k for equation 9 using valid values for n and m, it will be possible to find c4 composite numbers in group A. For example, 55 is a group A number (1+6x9) and is a composite c4 originated by multiplying 5 and 11 (two numbers from group E) with k=9 and where n=0 and m=1.

Checking the appearance of composites c5 in group A:

If a composite number in A of the form c5=1+6k where k = {1, 2, 3, 4…} can be formed by multiplying one number from group A (1+6n) by one number from group E (5+6m), then equation 10 must have at least one valid solution, otherwise a composite number in group A would never be obtained by multiplying one number from group A by one number from group E:

Equation 10:

$$\frac{1+6k}{(1+6n)\times(5+6m)}=1$$

where n = {1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {1, 2, 3, 4…}

$$1+6k=5+6m+30n+36nm$$

$$6k=4+6m+30n+36nm$$

$$k=\frac{4}{6}+m+5n+6nm$$

Equation 10 cannot result in a valid solution, because when using valid values for n and m, a number which is not natural is always obtained. Since k must be a natural number too, c5 composites can never found in group A.

In short: Group A is formed by number 1, prime numbers, composite numbers obtained by equation 8 (c3) and composite number obtained by equation 9 (c4). This can be easier visualized in Table 4, where all numbers in group A are either prime, result of multiplying A_nxA_m or result of multiplying E_nxE_m, and of course, number 1.

Table 4 (now extended) shows now the composite numbers for columns A marked in red as well:

Table 4: All composite numbers except those in group E are marked in red, also 1.

1+6n Group A	2+6n Group B	3+6n Group C	4+6n Group D	5+6n Group E	6+6n Group F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
…	…	…	…	…	…

Finding what composites appear in group E:

Checking the appearance of composites c3 in group E:

If a composite number in E of the form c3=5+6k where k = {0, 1, 2, 3, 4…} can be formed by multiplying two numbers from group A (1+6n) and (1+6m), then equation 11 must have at least one valid solution, otherwise a composite number in group E would never be obtained by multiplying two numbers from group A:

Equation 11:

$$\frac{5+6k}{(1+6n)\times(1+6m)} = 1$$

where n = {1, 2, 3, 4…}, m = {1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

$$5+6k = 1+6(n+m)+36nm$$

$$6k = -4+6(n+m)+36nm$$

$$k = -\frac{4}{6}+(n+m)+6nm$$

Equation 11 cannot result in a valid solution, because when using natural numbers for n and m, there will be always decimals in the solution, hence a no valid k. Therefore c3 composites can never found in group E.

Checking the appearance of composites c4 in group E:

If a composite number in E of the form c4=5+6k where k = {0, 1, 2, 3, 4…} can be formed by multiplying two other numbers from group E (5+6n) and (5+6m), then equation 12 must have at least one valid solution otherwise a composite number in group E would never be obtained by multiplying two numbers from group E:

Equation 12:

$$\frac{5+6k}{(5+6n)\times(5+6m)}=1$$

where n = {0, 1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

$$5+6k=25+30(n+m)+36nm$$

$$6k=20+30(n+m)+36nm$$

$$k=\frac{10}{3}+5(n+m)+6nm$$

Equation 12 cannot result in a valid solution, because a valid value for k cannot be obtained using valid values of n and m: when using natural numbers or 0 for n and m, a number which is not natural is always obtained for k. Since k must be a natural number too, c4 composites can never found in group E.

Checking the appearance of composites c5 in group E:

If a composite number in group E c5=5+6k where k = {0, 1, 2, 3, 4…} can be formed by multiplying one number from group A (1+6n) by one number from group E (5+6m), then equation 13 must have at least one valid solution, otherwise a composite number in group E would never be obtained by multiplying one number from group A by one number from group E::

Equation 13:

$$\frac{5+6k}{(1+6n)\times(5+6m)}=1$$

where n = {1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

$$5+6k=5+6m+30n+36nm$$

$$6k=6m+30n+36nm$$

$$k=m+5n+6nm$$

Equation 13 can obtain valid values of k using valid values for n and m, therefore it will be possible to find c5 composite numbers in group E. For example, 65 is a group E number (5+6x10) and is a composite c5 originated by multiplying 13 (group A) and 5 (group E) with k=10 and where n=2 and m=0.

In short: Group E is formed by prime numbers and by composite numbers obtained by equation 13 (c5). This can be easier visualized in Table 5, where all numbers in group E are either prime or the result of multiplying A_n*E_m.

Table 5: All composite numbers are marked in red, also 1.

1+6n Group A	2+6n Group B	3+6n Group C	4+6n Group D	5+6n Group E	6+6n Group F
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
…	…	…	…	…	…

<u>The set of 5 equations that calculate all composites numbers:</u>

We have now a set of 5 equations that when put together, generate all existing composite numbers:

Equation 1 for all composites in groups B, D and F:

$c1 = 2(n+2)$ where n = {0, 1, 2, 3, 4…}

Equation 2 for all composites in groups C and F:

$c2 = 3(n+2)$ where n = {0, 1, 2, 3, 4…}

Equation 5 for part of the composites in group A:

$c3 = (1+6n) \times (1+6m)$ where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

Equation 6 for part of the composites in group A:

$c4 = (5+6n) \times (5+6m)$ where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

Equation 7 for all composites in group E:

$c5 = (1+6n) \times (5+6m)$ where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

These 5 equations were programmed and tested up to the first 10.000 natural numbers, and the results obtained compared with available recorded results on primes and composites. We can, for means of demonstration, check the first 100 natural numbers using this method in Table 6.

Table 6: The first 100 natural numbers classified

Number	Prime or Composite Family
1	not a prime number by definition
2	prime number
3	prime number
4	c_1 composite number
5	prime number
6	c_1 and c_2 composite number
7	prime number
8	c_1 composite number
9	c_2 composite number
10	c_1 composite number
11	prime number
12	c_1 and c_2 composite number
13	prime number
14	c_1 composite number
15	c_2 composite number
16	c_1 composite number
17	prime number
18	c_1 and c_2 composite number
19	prime number
20	c_1 composite number
21	c_2 composite number
22	c_1 composite number
23	prime number
24	c_1 and c_2 composite number
25	c_3 composite number
26	c_1 composite number
27	c_2 composite number
28	c_1 composite number
29	prime number
30	c_1 and c_2 composite number
31	prime number
32	c_1 composite number
33	c_2 composite number
34	c_1 composite number
35	c_5 composite number
36	c_1 and c_2 composite number
37	prime number
38	c_1 composite number
39	c_2 composite number
40	c_1 composite number
41	prime number
42	c_1 and c_2 composite number
43	prime number
44	c_1 composite number
45	c_2 composite number
46	c_1 composite number
47	prime number

48	c_1 and c_2 composite number
49	prime number
50	c_1 composite number
51	c_2 composite number
52	c_1 composite number
53	prime number
54	c_1 and c_2 composite number
55	c_4 composite number
56	c_1 composite number
57	c_2 composite number
58	c_1 composite number
59	prime number
60	c_1 and c_2 composite number
61	prime number
62	c_1 composite number
63	c_2 composite number
64	c_1 composite number
65	c_5 composite number
66	c_1 and c_2 composite number
67	prime number
68	c_1 composite number
69	c_2 composite number
70	c_1 composite number
71	prime number
72	c_1 and c_2 composite number
73	prime number
74	c_1 composite number
75	c_2 composite number
76	c_1 composite number
77	c_5 composite number
78	c_1 and c_2 composite number
79	prime number
80	c_1 composite number
81	c_2 composite number
82	c_1 composite number
83	prime number
84	c_1 and c_2 composite number
85	c_4 composite number
86	c_1 composite number
87	c_2 composite number
88	c_1 composite number
89	prime number
90	c_1 and c_2 composite number
91	c_3 composite number
92	c_1 composite number
93	c_2 composite number
94	c_1 composite number
95	c_5 composite number
96	c_1 and c_2 composite number
97	prime number

98	c_1 composite number
99	c_2 composite number
100	c_1 composite number

Additionally this shows that for any number x, the next composite number c>N will be the smallest possible solution cy from any of the 5 equations of the set being cy>x. Any numbers between x and cy are prime numbers, as long as cy is the next possible valid solution of the set larger than x.

Obtaining prime numbers by iteration:

For any composite number of group A (1+6k) where k = {1, 2, 3, 4…} using equations 5 and 6 we have seen that k can either be:

k=n+m+6nm (from equation 8)

where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

or

k=4+5(n+m)+6nm (from equation 9)

where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

Therefore, for any N value of k that cannot be obtained using valid solutions of n and m in any of these two equations, then 1+6k will be a prime number.

For example, 7 is a prime number from group A with k=1 that cannot be obtained by valid values of n and m in equations 5 and 6.

On a similar way, for group E, any number (5+6k) where k = {0, 1, 2, 3, 4…} that cannot be generated using equation 13:

k=m+5n+6nm

where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

will be a prime number.

For example, 29 is a prime number from group E with k=4 that cannot be obtained using 0 or natural numbers for m and n.

Therefore, for a given number x, and after confirming that x belongs either to group A or to group E, (by means of checking if it can be turned into 1+6k or by 5+6k. Then if it belongs to A or to E, we can iterate equation 7 (if is from E) or equations 5 and 6 (if it is from A).

The iteration method was tested up to the first 10.000 natural numbers and compared with one of the existing simpler primality test. The results were found correct but the speed of iteration slower than the already established primality test.

Twin prime numbers:

A twin prime is a prime number that is separated just by one composite from another prime number—for example, either member of the twin prime pair (41, 43). If we divide all natural numbers N following the method described in this paper, the only way to obtain prime numbers would be that one of them belongs to group E and the other to group A, being the prime number from group E p_E 5+6k the smaller one of the pair and the primer number form column A p_A = 1+6(k+1) the larger one of the pair.

Following the reasoning above, in order to find 2 twin prime numbers, one would have to iterate:

k=m+5n+6nm (from equation 13) where k = {0, 1, 2, 3, 4…} until a k is found where there is not a single combination of n and m where n = {1, 2, 3, 4…} and where m = {0, 1, 2, 3, 4…} that can originate a valid result. Followed by iterating equations 5 and 6, for k+1 instead of k:

(k+1)=n+m+6nm (from equation 8)

where n = {1, 2, 3, 4…} and where m = {1, 2, 3, 4…}

and

(k+1)=4+5(n+m)+6nm (from equation 9)

where n = {0, 1, 2, 3, 4…} and where m = {0, 1, 2, 3, 4…}

If none of the two can obtain the value of k using valid values for n and for m, then these conditions are met, and: 5+6k and 1+6(k+1) are twin prime numbers.

We can look at 41 and 43, two twin prime numbers an as example.

41 = 5+(6x6) group E, while 43 = 1+(6x7) from group A. In this case, k=6.

6=m+5n+6nm

where n = {1, 2, 3, 4…} and here m = {0, 1, 2, 3, 4…}

A valid solution cannot be obtained. n=1 m=0 gives a value of 5 for k. n=m=1 gives a value for k of 12, and of course, increasing n or m would only increase the value of k. So n or m should be no natural numbers, something not possible. 41 is our lowest prime number of the twin set and has a k of 6.

Now we continue by replacing k by 6 in the two remaining equations:

(6+1)=n+m+6nm (from equation 8)

where n = {1, 2, 3, 4…} and where m = {1, 2, 3, 4…}

A valid solution cannot be found. n=m=1 gives a value for k of 7, and of course, increasing n or m would only increase it.

and

(6+1)=4+5(n+m)+6nm (from equation 9)

where n = {0, 1, 2, 3, 4…} and where m = {0, 1, 2, 3, 4…}

A valid solution cannot be found. n=m=0 would be a k of 3. n=1 and m=0 would be for a k of 8. n=0 and m=1 would be for a k of 8, and of course increasing them more would only increase k.

So a k of 6 does not find a valid set of values of m and n for any of the 2 equations. Therefore is a prime of the form 1+6(k+1), 43. 41 and 43 are two prime numbers, with k 6 for the former and 7 for the later.

With enough time and computational power, large twin prime numbers can be found using the same method. It is not in the scope of this paper to go deeper into iterations. To check if by using this method twin prime numbers can be obtained faster compared with the methods prior to this, remains as work for another study.

Results:

Equations leading to all composite numbers:

By dividing all natural numbers in 6 groups, A to F, and following the reasoning behind the generation of all composite numbers in each group, we have generated a set a 5 equations. These 5 equations define that a composite number c, must fulfill at least one of the following conditions:

$c1 = 2(n + 2)$ where n = {0, 1, 2, 3, 4…}

$c2 = 3(n + 2)$ where n = {0, 1, 2, 3, 4…}

$c3 = (1 + 6n) \times (1 + 6m)$ where n = {1, 2, 3, 4…} and m = {1, 2, 3, 4…}

$c4 = (5 + 6n) \times (5 + 6m)$ where n = {0, 1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

$c5 = (1 + 6n) \times (5 + 6m)$ where n = {1, 2, 3, 4…} and m = {0, 1, 2, 3, 4…}

Any number not fulfilling at least one of the equations will be a prime number.

If the search of prime number is desired, one method can be to generate composite numbers using the above equations and those not fulfilling any of them will be prime numbers, but it is also possible to look for prime numbers by iteration, looking for values of k in group A or in group E that cannot be obtained valid solutions of n and m.

For group A, from equations 8 and 9 we have seen that k can either be:

k=n+m+6nm (for equation 8)

where n = {1, 2, 3, 4…}, m = {1, 2, 3, 4…} and k = {1, 2, 3, 4…}

or

k=4+5(n+m)+6nm (for equation 9)

where n = {0, 1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {1, 2, 3, 4…}

Therefore, for a valid value of k that cannot be obtained in one of the two equations using valid solutions for n and m, then 1+6k will be a prime number.

One can as well obtain a prime number from group E using equation 13 leading to:

k=m+5n+6nm

where n = {1, 2, 3, 4…}, m = {0, 1, 2, 3, 4…} and k = {0, 1, 2, 3, 4…}

For a valid of k that cannot be obtained using valid solutions for n and m, then 5+6k will be a prime.

Finally, the method to obtain prime numbers by iteration presented in this paper can be used to obtain twin prime numbers. First by finding a value of k that generates a prime number in the group E 5+6k. Then by checking by iteration in equations 5 and 6 if 1+6(k+1) is also a prime number of group A.

Conclusion:

In this paper we have showed an elegant and simple way to obtain a set of equations that can generate all composite numbers, proving that the appearance of composite numbers is not random. We have provided 2 different routes to find prime numbers by iteration, although not optimal on computational speed, and one to find twin prime numbers by iteration.

Contributions:

Segura J. J.: Mathematical reasoning on the finding of the 5 equations.

Barchino R.: Programming the equations and testing the equations against primality test.

Annex:

The equations provided in this paper were tested by programming the algorithm in Java

```
public class CompositeNumbers {

   public boolean isComposite(long testNumber, long n, long m) {
      return isC1(testNumber, n) ||
            isC2(testNumber, n) ||
            isC3(testNumber, n, m) ||
            isC4(testNumber, n, m) ||
            isC5(testNumber, n, m);
   }

   private boolean isC1(long testNumber, long n) {
      return testNumber == c1(n);
   }
   public long c1(long n) {
      return 2 * (n + 2);
   }
   private boolean isC2(long testNumber, long n) {
      return testNumber == c2(n);
   }
   public long c2(long n) {
      return 3 * (n + 2);
   }
   private boolean isC3(long testNumber, long n, long m) {
      if (n == 0 || m == 0) return false;
      return testNumber == c3(n, m);
   }
   public long c3(long n, long m) {
      return (1 + 6 * n) * (1 + 6 * m);
   }
   private boolean isC4(long testNumber, long n, long m) {
      return testNumber == c4(n, m);
   }
   public long c4(long n, long m) {
      return (5 + 6 * n) * (5 + 6 * m);
   }
   private boolean isC5(long testNumber, long n, long m) {
      if (n == 0) return false;
      return testNumber == c5(n, m);
   }
   public long c5(long n, long m) {
      return (1 + 6 * n) * (5 + 6 * m);
```

```
    }
}
```

The values obtained were compared to those obtained by well-established primality test:

```
public class PrimeNumbers {
    private CompositeNumbers compositeNumbers = new CompositeNumbers();
    public boolean isPrime(long testNumber) {
        if (testNumber == 1) return false;
        if (testNumber < 3) return true;
        long top = testNumber / 2;
        for (long n = 0; n <= top; n++) {
            if (compositeNumbers.c1(n) > testNumber) {
                return true;
            }
            for (int m = 0; m <= top; m++) {
                if (compositeNumbers.isComposite(testNumber, n, m)) {
                    return false;
                }
                if (compositeNumbers.c3(n, m) > testNumber) {
                    break;
                }
            }
        }
        return true;
    }
}
```

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