What Makes Goldbach's Conjecture Correct

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Abstract

After a brief review of Goldbach's conjecture, we give speculations about why Goldbach's conjecture is true.

Introduction

Certainly Goldbach's conjecture is the ultimate in easily expressed and understood difficult number theory problems. What could be simpler that every even number is the sum of two primes?

Apostol spends some time in the beginning and end of his *Introduction to Analytic Number Theory* book to give contemporary research results. Chen's result is mentioned: allow the second number to have just two (not one) prime factor and the result is proven. It's a two page proof, not easy. The last chapter of Apostol's book is on *partitions* which he evolves, one can sense, from the mathematical frustration at getting no where with Goldbach's conjecture. I sense Waring's problem and the like are a kind of sour grape story. If we can't get anything concrete with the easiest sum of two primes, Goldbach what can be done with arbitrary sums of numbers to various powers?

Perhaps, like many open number theory problems, what drives researchers to write programs that test results on hard numbers (into the trillions) must be the biting, irritating sense that there is some easy explanation that we just can't yet see. For me the primordial example of mathematical puzzles resolved with solutions that eventually turn out to be thought of as simple, obvious and beyond reproach are at least two: Cantor's work on set theory and the positional number system.

The former has counter-intuitive elements. Consider that the limit of $(-1/n, 1/n)$ as n goes to infinity is the empty set. This despite the fact that for each $n > 0$ the number of points in these open intervals is uncountable. So how can something that's uncountably infinite go to something that has no elements, zero, nada without passing through countable infinite and just plain finite first? Yet, these symbols, our minds do understand it, believe it, accept this counter-intuitive truth and we use it to build lots of great mathematics. The continuum hypotheses is really not an hypotheses anymore – there is not in between this limit goes through.

The other wonder of mathematics, the other great success story is the positional number system and the use, great of late, of various number bases. We can add, subtract, multiple, and divide with relative alacrity. We can, using the binary number base, get machines to do these operations for us in blinding speed. Who invented the positional number system and the idea of various number bases? After much research, reading Dickson and exploring Jstor it isn't particularly clear that one person or that some school of thought came up with the idea. It may be that the idea is so fundamental, so basic historians, mathematicians themselves don't feel inclined to give credit to anyone or anything – its just too obviously the right way to conceive of all numbers – all natural numbers anyway. Reals, decimals are another story.

These two success story I claim relate to a take on Goldbach's problem. There is a squeeze action of Cantor involved with bases and the positional number system that yields some clarity and reliefs some frustration.

The Take

One can express Goldbach's conjecture as an existence of a number bases problem. What is a prime number base less than $2n$ such that when $2n - \{primes\}$ is divided by $\{primes\}$ their quotient is 1 with a remainder of 0? The set of $\{primes\}$ are all the primes between 3 and $2n$. Take the even number 108. Are there primes such that $(108 - p_1)/p_2$ is 1? Yes: 103 and 5.

Why or how could this always be true? Per Bertrand's postulate there are primes between 54 and 108 and there are also primes between 27 and 54, but also primes between, using the first set, between 27 and 54, using the second set 13 and 27, and so forth. Using this squeeze within the positional number system, one is forced to find a base and thence to two primes.

References

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