Similarity of a Ramanujan Formula for π with Plouffe's Formulae, and Use of this for Searching of Physical Background for some Guessed Formula for the Elementary Physical Constants

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Abstract. The paper is comprised of two parts. In the first part, it discusses the similarity between one of Ramanujan's formulae for π and Plouffe's formulae where he uses the Bernoulli numbers. This similarity is help for further determining either that the similarity is only accidental, or that we can derive the Ramanujan formula in this way. This is also the help for setting up a calculation system where we would estimate the probabilities with which we can obtain guessed formulae for π that are very accurate and very simple. (We only consider formulae that are not the approximations of the exact formulae for π.) In the second part, it discusses various guessed formulae for the fine structure constant and for the other physical constants, and how the above probability calculation would help estimate whether these formulae have a physical basis or are just random.

Keywords: pi, approximations for pi, the exact formulae for pi, Ramanujan, Ramanujan machine, Plouffe, the Bernoulli numbers, mnemonics, the dimensionless physical constants, the fine structure constant, the gravitational coupling constants of the elementary particles, the dimensionless masses of the elementary particles, neutrino mass, guessing of formulae, repeating of the guessed formulae, mathematics, statistics, quantum gravity

1. Introduction

People guess formulae for the fine structure constant (α), or for other physical constants. Some formulae are quite simple and quite accurate. (Some examples also follow in the paper and in the references.) But if we see such a formula, we still wonder either it is just a coincidence or it has a physical background. In addition to many ways to check this, (I will describe some of them) it would be well to have some statistical calculation of the probability that such an accurate formula is just a coincidence when combining various numbers and operators. At this we should have some numerical estimates of which formulae are more and less simple. This is difficult, there are some estimates, but it is good to check and improve these estimates a bit.

In the Section 2, I present the similarity of the Ramanujan formula[1, 2] with Plouffe's formulae.[3, 4] This is the essence of this paper, but then in Section 3, I analyse how some accidental formulae for π are either approximations for the exact formulae for π or they are not. I try to build a system which can measure occurrence of the later ones. This procedure can be used also as an estimation for probability how guessed formulae for the physical constants have a physical basis, as evident in Section 6.

2. Partial explanation of the Ramanujan approximation for π

By guessing I obtained rather an accurate formula for π and later found out that Ramanujan had already calculated this:[1, 2]

$$\pi_{\mathrm{R}} = \pi_{r=\pi} = \left(97.5 - \frac{1}{11}\right)^{1/4} = \mathbf{3.141592652582}646.. \tag{1}$$

where this approximation for π was signed with $\pi_{r=\pi}$ in Ref.[1].‡ The discrepancy with the true value for π can be deciphered here

$$\pi = \pi_{\mathrm{R}} + 1.007147254483698 \times 10^{-9} = \mathbf{3.141592653589}793.. \tag{2}$$

There are many formulae that calculate π arbitrarily accurately.[5] I investigated the similarity of the formula for π_{R} with these formulae. Of these, only the following formulae were appropriate:[6][3, 4, Pgs. 1-2]

$$\pi^n \approx \frac{2n!}{B_n 2^n} \tag{3}$$

$$\pi^n \approx \frac{2n!}{B_n 2^n (1-2^{-n})(1-3^{-n})(1-5^{-n})(1-7^{-n})..} \tag{4}$$

Let us see how Eq. (1) can be compared with Eqs. (3) and (4). Let us choose $n = 4$, the value of B_4 is $-1/30$,[10] use absolute value, $|B_4|$, let us sign so calculated

‡ Assignment π_{R} will be used in this paper, and it means Ramanujan's result of calculation, or it can also means that R equals π. (The inaccuracy is that the former R is a romanized letter, but the latter R should be italicized.)

approximations of π with π_{B}, and insert numbers for successive corrections:§

$$\pi_{\mathrm{B}}^4 = 90 + 6 + 1.2 + 0.155769.. + 0.040565.. + .. \tag{5}$$

The first approximation gives what we obtain with Eq. (3), and subsequent corrections are due to the added factors in the denominator of Eq. (4) from left to right. And π_{B}^4 is slowly but certainly converging toward π^4. Equation (5) can also be written as

$$\pi_{\mathrm{B}}^4 = 90 \times \left(1 + \frac{1}{15}\right) \times \left(1 + \frac{1}{80}\right) \times \left(1 + \frac{1}{624}\right) \times \left(1 + \frac{1}{2400}\right) \times .. \tag{6}$$

π_{R} and π can be so written in comparison with Eq. (5) as

$$\pi_{\mathrm{R}}^4 + \frac{1}{11} = 97.5 \tag{7}$$

$$\pi^4 + \frac{1}{11} = 97.500000124911523.. = \pi_{\mathrm{R}}^4 + \frac{1}{11} + 0.000000124911523.. \tag{8}$$

Now let us write Eq. (7) so that it will be similar to the first term of the right-hand side of Eq. (5), respectively to the first two factors of Eq. (6)

$$\pi_{\mathrm{R}}^4 + \frac{1}{11} = 90 \times \left(1 + \frac{\mathbf{1}}{\mathbf{12}}\right) = 97.5 \tag{9}$$

Thus, the denominator is modified from 3×5 to 3×4, which is still ever simple, and the other terms in Eq. (5) are erased. As second, let us make Eq. (7) similar to the first two terms of Eq. (5)

$$\pi_{\mathrm{R}}^4 + \frac{1}{11} = 90 + 6 \times \left(1 + \frac{\mathbf{1}}{\mathbf{4}}\right) = 97.5 \tag{10}$$

The correction is quite simple. As third, let us make Eq. (7) similar to the first three terms of Eq. (5)

$$\pi_{\mathrm{R}}^4 + \frac{1}{11} = 90 + 6 + 1.2 \times \left(1 + \frac{\mathbf{1}}{\mathbf{4}}\right) = 97.5 \tag{11}$$

The correction is quite simple, the same as in Eq. (10).

As further, let us make Eq. (7) similar to the first four terms of Eq. (5)

$$\pi_{\mathrm{R}}^4 + \frac{1}{11} = 90 + 6 + 1.2 + 0.155769.. \times \left(1 + \frac{\mathbf{925}}{\mathbf{999}}\right) = 97.5 \tag{12}$$

The correction is less simple than before. Now, let us make Eq. (7) similar to the first five terms of Eq. (5)

$$\pi_{\mathrm{R}}^4 + \frac{1}{11} = 90 + 6 + 1.2 + 0.155769.. + 0.040565.. \times \left(1 + \frac{\mathbf{23}}{\mathbf{9}}\right) = 97.5 \tag{13}$$

Both corrections in Eqs. (12) and (13) are already less simple and they grow by size. This growth is because of 1/11, which is added to the left-hand side. The most probably, the continuation will not give any simple correction, neither will give a complete identity

§ Let us name $\pi_{\mathrm{B0}}^4 = 90$, $\pi_{\mathrm{B1}}^4 = 90 + 6$, etc, but here π_{B} is a substitute name for π_{B0}, π_{B1}, and for all the next approximations.

with Eq. (7). But some simplicity remains in these corrections, and it is essential that simplicity of corrections exists in Eqs. (9) to (11).

Let us look at the result of Eq. (3) at $n = 8$, where $B_8 = -1/30$.

$$\frac{2 \times 8!}{|B_8| 2^8} = 9450 = 42 \times 15^2 \tag{14}$$

But if we modify the right-hand side to $(42 + 1/4) \times 15^2$, this would be exactly equal to 97.5^2, thus we could obtain the essential number from Eq. (7). Here only $-1/11$ and absolute value of $|B_8|$ should be explained, thus the same failed what failed also in Eq. (5). Therefore, similarity with π_{R} is evident also at $n = 8$.

An ideal continuation of this calculation would be either to find some modified version of the formula for π_{B} of which some approximation is exactly the same as π_{R}, or to confirm that this connection does not exist. Maybe some other explanation for the accuracy of π_{R} exists. A few steps are missed here to find out this for sure.

As a hint, it can be expected that a better approximation exists than those described with Eq. (4). For instance, the factor in Eq. (9) means a stronger approximation than the first factor in Eq. (6).

At this, graphical presentation of these formulae and numbers is important, because maybe some additional essence can be found, and graphical derivation is more mnemonical.[7][8, Pg. 43, Fig. 1, Fig. 2]

It is even possible that it is a known answer about my question above, but I have not found such an answer. For instance, references where something can be found about Eq. (1) are Refs.[7, 8, 9].

3. A system for searching of accurate formulae which are simple, are approximations for π, and are not approximations of the exact formulae for π

Let us say that we have some formulae that are approximations for π and are simple. However, let us denote by $\pi_{\not\infty}$ those that are not approximations of any exact formulae for π. As contrary, let us denote by π_{∞} those that are approximations of any exact formula for π. For instance, the record $\pi_{\mathrm{B}} \in \{\pi_{\infty}\}$ means that the formulae for π_{B} are exact, or are approximations for the exact formulae.

Let us say that we have one formula for π and we know for n formulae that are equally or more simple and equally or more accurate ($\geq SA$),‖ and they belong to $\in \{\pi_{\not\infty}\}$. We denote this number with $n_{\not\infty}$. With $p_{\not\infty}$ we denote the probability that a formula is $\in \{\pi_{\not\infty}\}$. With $p_{1\not\infty}$ we denote the probability that we obtain at least one formula $\in \{\pi_{\not\infty}\}$, which is also $\geq SA$ according to the formula we are looking at.

Thus, such an analysis of the similarity of π_{R} and π_{B}, in the section before, is one step towards calculating or estimating such probabilities $p_{1\not\infty}$ and $p_{\not\infty}$.

‖ In true, if we are precise, we need a common measure of simplicity and accuracy, for instance, one formula can be less precise but much more simple. But, we can disregard such cases here, and can use them for the next approximation.

The question is how many such random formulae exist that are $\geq SA$ according to Eq. (1), and that it is valid $\in \{\pi_{\not\infty}\}$. At this, it is difficult to accurately estimate simplicity. And also such a number of these formulae and therefore such probabilities are also difficult to be estimated. By rule of thumb, we can say that Eq. (1) is quite simple and precise.

(i) If it turns out that Eq. (1) is only an approximation of some modification of π_{B} or an approximation of some other exact formula for π, thus $\pi_{\mathrm{R}} \in \{\pi_{\infty}\}$, then this will also reduce $p_{1\not\infty}$ for other formulae that are $\in \{\pi_{\not\infty}\}$.

(ii) Since I unintentionally repeated Ramanujan's formula,[1] this reduces $p_{\not\infty}$ and $p_{1\not\infty}$. Namely, if there were enough simple formulae $\in \{\pi_{\not\infty}\}$ that were accurate enough, it would rarely happen that someone would guess and repeat such a formula. If it turns out that $\pi_{\mathrm{R}} \in \{\pi_{\infty}\}$ is valid, together with the repetition Ref.[1, 2] this will have a synergic effect on $p_{1\not\infty}$.

(iii) It would be useful to estimate a probability that someone unintentionally repeats a derivation of a formula which obeys $\in \{\pi_{\not\infty}\}$.

(iv) A measure of simplicity is also how effective mnemonics can be used. Some examples are in the next section and in Appendix.

(v) One way of estimating is also through a number of digits we use for the formula and a number of first successive digits that are accurate in a result.[11]¶ And what is $p_{1\not\infty}$ when 7 digits of formula give 9 digits of accuracy, as is at π_{R}? So let us suppose that a number of digits is more important than a number of operators, and more than a simplicity of these digits. Of course, this is only a rough approximation in such an estimation.

(vi) I will show some of random formulae in Section 5.

(vii) An artificial intelligence approach could help with such analyses.[12, 13]

(viii) Disagreements of random guessed formulae with $\in \{\pi_{\infty}\}$ can be tested quite simply in principle, only all known approximations of $\in \{\pi_{\infty}\}$ (below some accuracy) should be written with a lot of digits, and the same for a random formula which we wish to check.+ For instance, if we make such a database, we will check additionally whether it is valid $\pi_{\mathrm{R}} \in \{\pi_{\infty}\}$, thus we could use many other known formulae for which it is valid $\in \{\pi_{\infty}\}$. In Section 2 it is tested only for π_{B}, maybe there are also other formulae.

(ix) Similar analyses should be done for other basic mathematical numbers, such as e.

4. Examples of simplicity analysis for formula for π_{R}

A good mnemonics means also a simplicity of a formula. As an example, here I try to compare the mnemonics of two different records of Eq. (1), of the main part of this

¶ Such quick assessments are recurrent, but more precise procedure should be found. Therefore, it is necessary to develop it.

+ Of course, it is easier to look the difference with π.

formula.

I wrote this main part as 97.5-1/11, Eq. (1), and Pickover (Plouffe) wrote it as 2143/22.[14] The record 97.5-1/11 is easier to remember than 2143/22. The number of digits is more important than of operations. It can be easier to remember 11 than 22. It is easier to remember 97.5 than 97.4 and easier than 96.5. Namely, $100 - 5/2 = 100 - 10/4$, the number of digits is bigger, but these digits are more basic. 97.5 can be expressed also as $97.5 = 13 \times 15/2 = (14 - \mathbf{1}) \times (14 + \mathbf{1})/2$. Number of digits here is increased, but this is a good mnemonic, also because it is valid $1.4^2 \approx 2$. The further numbers from Eq. (4) can be expressed as $97.2 = (14 - 1.6^{0.5}/2) \times (14 + 1.6^{0.5}/2)/2$, $96 = (14 - \mathbf{2}) \times (14 + \mathbf{2})/2$, and $90 = (14 - \mathbf{4}) \times (14 + \mathbf{4})/2$. Therefore, these numbers 1, 2, and 4 are simple, $1.6^{0.5}/2$ is not.

But even for 2143 there are mnemonics, say like 1234, or $43 - 21 = 22$ or $43\pi^4 - 21\pi^4 = 2143$.[15] A graphical mnemonics is also a derivation of the original Ramanujan formula.[8, Pg. 43]

Let us look Eq. (9). I estimate that it has 6 digits, others are operators. I assess all three single 1's as operators.

These are all just examples of how to evaluate the simplicity of formulae.

Mnemonicity is an important criterion for estimation of the simplicity of formulae, and simplicity is important for estimating the probability that such a formula is only random without a deeper mathematical basis.

5. Examples of some formulae $\in \{\pi_{\not\infty}\}$

Many formulae can be found in Ref.[5]. One example is Ref.[16], which is also easy to remember:

$$\pi_{\mathrm{P}}^4 + \pi_{\mathrm{P}}^5 = \mathrm{e}^6 \tag{15}$$

$$\pi_{\mathrm{P}} = \pi + 2.892211.. \times 10^{-8} = \mathbf{3.14159262}467 \tag{16}$$

We can also easily remember the following formula:[17]

$$\left(1 + \frac{1}{\pi_{\mathrm{L}}}\right)^{1+\pi_{\mathrm{L}}} = \pi_{\mathrm{L}} \tag{17}$$

$$\pi_{\mathrm{L}} = \pi - 5.5112817.. \times 10^{-4} = \mathbf{3.141}04152541 \tag{18}$$

This formula is otherwise very similar to the known limit:

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n} = \mathrm{e} \tag{19}$$

Maybe there is some explanation for this connection?

One of Plouffe's formulae is also[18]

$$\mathrm{e}^{\pi} - \pi = 19.999099979.. \tag{20}$$

Another formula mentioned by Pickover is[19]

$$\mathrm{e}^{\pi\sqrt{163}} = 262537412640768743.999999999999.. \tag{21}$$

Because of a lot of digits and decimal places, it is interesting for estimating simplicity and accuracy. I suppose that its $p_{1\not\infty}$ and $p_{\not\infty}$ are large. The next formula is interesting because of simplicity:[20]

$$\pi_\mathrm{S} = \frac{9 - \mathrm{e}}{2} = \mathbf{3.14}0859.. \tag{22}$$

The following formula for e is similar to Ramanujan's formula:[17]

$$\mathrm{e} \approx 3 - \sqrt{\frac{5}{63}} = \mathbf{2.718281}915.. \tag{23}$$

$$\mathrm{e} = \mathbf{2.718281}828.. \tag{24}$$

Equation (23) is quite simple. How Eq. (1) was obtained is described in Ref.[1], and here we would calculate similarly, i.e. $3 - e$, then root calculation, and finally the search for the closest simple fraction.

More can be found in Refs.[5, 8, 9].

Here it would be possible to do analyses on both accuracy and simplicity, as well checking whether it is valid $\in \{\pi_{\not\infty}\}$.

6. Motivation for checking guessed formulae for physical constants

People have guessed formulae for α a lot, as well as for other physical constants.[21, 22, 23, 24, 25, 26, 27] The simplicity of these formulae together to their accuracy could be estimated in the same way as the formula for π_R, Eq. (1), and the experience of guessing formulae for π could be used to check the physical basis of these formulae. However, we must be aware of the distinction that these constants are measured to a finite number of digits, so we do not have as many orientational digits as, say, for π. Here the formulae should have a physical form on some way, but the exact formulae for π have fewer restrictions and are of various forms.

Some scientists argue that the true physical formula for α is overcomplicated anyway:[28, 29] "*It would be like trying to derive a fundamental formula for the average temperature in Los Angeles.*"[28] It can either be really overcomplicated, or it cannot be, it is not sure. In any case, it would be good to have a mathematical apparatus that evaluates for a formula whether its AS are high enough. Thus, we will be able to estimate whether it has a physical basis or not. This is also the further purpose of the above analysis for π_R.

I, too, have guessed such formulae, roughly speaking. More precisely said, I was actually looking for a connection between α and gravitational coupling constants.[30, 31] One of the formulae is:[30, Eq. (7)]

$$\frac{4}{3}\mu_e^2 = \exp\left(\frac{-3}{4\alpha}\right) \times (1 + 0.009042(23)) \tag{25}$$

In this case, μ_e^2 means $m_e^2 G/(\hbar c)$,[*] where m_e is the mass of the electron, G is the gravitational constant, $\hbar$ is the reduced Planck constant and c is the speed of light. The

[*] This is the gravitational coupling constant.

latest input data can be found in Ref.[32]. After developing the formulae, I realized that something similar already existed.[26, Eq. (5)][30, Eq. (9)] Thus, Landau developed a similar formula, as I did:

$$\sqrt{2}\alpha\mu_e^2 = \exp\left(\frac{-\pi}{4\alpha}\right) \times (1 - 0.001527(22)) \tag{26}$$

It is essential that $\pi/(4\alpha)$ is very similar to $3/(4\alpha)$.

This similarity is another hint that guessing does not give us many simple formulae that are also very accurate.

One of my formulae is also:[30, Eq. (8)]

$$\frac{4}{3}\mu_e^2 = \left(1 + \frac{4}{3}\alpha\right) \exp\left(\frac{-3}{4\alpha}\right) \times (1 - 0.000681(22)) \tag{27}$$

In 2021, however, it also happened that Straser[33, Eq. (15)] repeated one of my formulae:[34]

$$G = e_0^2/m_e^2 \left[\alpha \left(1 + \frac{1}{\alpha}\right) \mathrm{e}^{-\alpha}\right] \times (1 - 0.000681(22)) \tag{28}$$

e_0 is the elementary charge in Gaussian units. Straser used a different definition for α, he redefined $\frac{3}{4\alpha} \to \alpha$. However with the same input, we obtain the same result and the same deviation. Straser described the theory of the late Fernand Léon Van Rutten. (The most probably, Straser and Van Rutten repeated my formula not knowing for my papers.[30, 31])

In the paper here, this is the third example of repetition. The goal of this analysis in the paper here is also to determine whether it is likely that some random formula will be repeated again.

With such formulae, it is important that they give some physical predictions. I predicted neutrino mass.[35] It may be possible to verify this as early as in 2024. Probably the improved G measurements would also say something more about the physical background of the above formulae, but the measurements are only progressing slowly. Here I am waiting for the completion of the MEGANTE project.[36, 37]

In the case of a guessed physical formula, it is also important whether it is similar to the known physical formulae, otherwise it may be impossible to find a model for it. In my opinion, it is difficult to describe a guessed formula physically. Both I and Van Rutten had these problems. But, I think that I have some arguments of physical background. I hope that someone will find some more clear model. Thus things will become more clear about physical background for Eqs. (25) and (27).

7. Conclusion

I showed similarities between π_R and π_B. Although it is not yet sure whether it is valid $\pi_\mathrm{R} \in \{\pi_\infty\}$, this is a way either to confirm or refute this. If this is confirmed, then $n_{\not\infty}$ for sufficiently accurate and simple formulae for π will not be as large as it seems. Since I repeated the formula for π_R without knowing Ramanujan's calculation,[1] this also gives us a suspicion that this $n_{\not\infty}$ is not very large. This may also confirm that it is valid $\pi_{\mathrm{R}} \in \{\pi_\infty\}$.

However, more needs to be done to improve the probability calculation of $p_{1\not\infty}$. Some of what to do, I described in the paper. In addition, a database of exact and simple formulae should be made for π, for which $\in \{\pi_{\not\infty}\}$ is valid. And a lot of values of $\in \{\pi_\infty\}$ close to Eq. (1) can be written in another database, etc.

For the guessed formulae for α and for the other dimensionless physical constants, the important question is whether they are all random or some of them have some physical background. $n_{\not\infty}$ for sufficiently accurate and simple formulae for π would give an estimate for α formulae as well.

Equation (1) is one example of an unintentional repetition, not knowing that Ramanujan already wrote this formula. Straser's and Van Rutten's Eq. (28) is also unintentional repetition of Eq. (27) of me; and I found Eq. (25), which is similar to Eq. (26), and before I did not know for it. Thus these repetitions increase the probability for the physical background of these formulae.

8. Appendix: One independent example of mnemonics for π

π value with 26 digits is:[12]

$$\pi = 3.14159265358979323846264 33.. \tag{29}$$

The first 8 digits can be remembered with the help of time, π day is 3.14,$\sharp$ and we take time 15:9:26. Date and time can be easier to remember than pure digits.

I am going to show two options to present digits from 9th place on. The first is that 53:58 is a surrogate for minutes and seconds, and is close to 59:59. 97:93 cannot be expressed with time, but they are close to 99:99.

The second option is: **535**, 8, **979**, **323**, 84, **626**, 4, 3, 3, thus there are four triplets with similar symmetry. The first digits of triples go as $5 + 4 + 4 + 3$. The second digits of triplets go as $3 + 4 + 5 + 0$. The rest digits are 884433, or [8][84][433] according to how they are grouped. They are one, two and three in groups. The first digit 8 is after one triplet, 84 is after the middle two triples, and 433 is after the last triplet. All these relations are useful mnemonics.

The third example of mnemonics is Ref.[38], etc.

Even the contests in memorizing π exist. But, instead of time used for memorizing π it is better to memorize formulae for π and only the first digits of π. The existence of these contests was also my motivation for studying formulae for π.

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$\sharp$ This is the American format of date.

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[37] There is a problem due to a bug of *href* function in Latex, because so a link can be written only in full. Thus you should click Ref.[36], not the link below, but in the link below it is written what is hidden above on the right:
"https://indico.cern.ch/event/830432/contributions/3497155/attachments/1883896/3104657/2019.07.22
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