INTEGER PART E FRACTIONAL OF FUNCTION

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This study aims to bring to the knowledge of the scientific-mathematical community of mathematical formulas that calculate the integer and fractional part of a positive or negative function.

What do the formulas calculate?

The integer and fractional part of a function or simply of a fraction, logarithm, trigonometric formula etc. such that, the function does not cancel the mathematical formula.

Trigonometric functions admitted: Cosh, Sinh, etc.

Trigonometric functions not allowed: Cos, Sin, Tan, etc. as they assume positive and negative results.

In general when the function is continuous negative:

To the integer part is added (+ 1)

To the fractional part is added (- 1)

Examples with some functions

Definition**:**

***f(v)*=** Function for which we have to find the integer and fractional part

f(x) = Integer part

f(y) = Fractional Part

therefore:

f(v) = f(x) + f(y)

Legend for all schemes:

N = Number

f(v) = Function

f(x) = Function for the integer part

f(y) **= Function for the fractional part**

Integer Part $f(x) = -\frac{1}{2} + f(v) - \frac{i\text{Lo}\ \left((-1)*e^{-2i\pi f(v)}\right)}{2\pi}$

Fractional Part $f(y) = \frac{\pi + i\ \text{Log}\left((-1)*e^{-2i\pi f(v)}\right)}{2\pi}$

Example with logarithm

f(v)= Log(k)

f(x) = $\mathbf{Log}(\boldsymbol{k}) - \frac{\boldsymbol{\pi}+\boldsymbol{i}\mathbf{Log}(-\boldsymbol{k}^{-2\boldsymbol{i}\boldsymbol{\pi}})}{2\boldsymbol{\pi}}$

f(y) = $\frac{\boldsymbol{\pi}+\boldsymbol{i}\mathbf{Log}(-\boldsymbol{k}^{-2\boldsymbol{i}\boldsymbol{\pi}})}{2\boldsymbol{\pi}}$

N	*f(v)*	*f(x)*	*Int.* [f(v)]	*f(y)*	*Fraction [f(v)]*
2	Log[2]	0	0	Log[2]	Log[2]
3	Log[3]	1	1	-1+Log[3]	-1+Log[3]
4	Log[4]	1	1	-1+Log[4]	-1+Log[4]
5	Log[5]	1	1	-1+Log[5]	-1+Log[5]
6	Log[6]	1	1	-1+Log[6]	-1+Log[6]
7	Log[7]	1	1	-1+Log[7]	-1+Log[7]
8	Log[8]	2	2	-2+Log[8]	-2+Log[8]
9	Log[9]	2	2	-2+Log[9]	-2+Log[9]
10	Log[10]	2	2	-2+Log[10]	-2+Log[10]
11	Log[11]	2	2	-2+Log[11]	-2+Log[11]
12	Log[12]	2	2	-2+Log[12]	-2+Log[12]
13	Log[13]	2	2	-2+Log[13]	-2+Log[13]
14	Log[14]	2	2	-2+Log[14]	-2+Log[14]

Example with a fraction

19 = Random number

f(v)= $\frac{\mathbf{19}}{\mathbf{k}}$

f(x) = $-\frac{1}{2}+\frac{19}{k}-\frac{i\text{Log}[-e^{-\frac{38i\pi}{k}}]}{2\pi}$

f(y) = $\frac{\pi+i\text{Log}[-e^{-\frac{38i\pi}{k}}]}{2\pi}$

N f(v) f(x) Int. **[f(v)]** ***f(y) Fraction*** **[f(v)]**

1	19	19	19	0	0
2	19/2	9	9	1/2	1/2
3	19/3	6	6	1/3	1/3
4	19/4	4	4	3/4	3/4
5	19/5	3	3	4/5	4/5
6	19/6	3	3	1/6	1/6
7	19/7	2	2	5/7	5/7
8	19/8	2	2	3/8	3/8
9	19/9	2	2	1/9	1/9
10	19/10	1	1	9/10	9/10
11	19/11	1	1	8/11	8/11
12	19/12	1	1	7/12	7/12
13	19/13	1	1	6/13	6/13
14	19/14	1	1	5/14	5/14
15	19/15	1	1	4/15	4/15

Example with the Stirling formula for the approximation of the Gamma function.

f(v)= $\sqrt{2\pi k}\left(\frac{k}{e}\right)^{k}$

f(x) = $-\frac{1}{2} + e^{-k}k^{\frac{1}{2}+k}\sqrt{2\pi} - \frac{i\text{Lo}\ \left[-e^{-2i\sqrt{2}e^{-k}k^{\frac{1}{2}+k}\pi^{3/2}}\right]}{2\pi}$

f(y) = $\frac{\pi + i\text{Log}\left[-e^{-2i\sqrt{2}e^{-k}k^{\frac{1}{2}+k}\pi^{3/2}}\right]}{2\pi}$

N	*f(v)*	*f(x)*	*Int.* [f(v)]	*f(y)*	*Fraction [f(v)*
2	$(8\sqrt{\pi})/e^2$	1	1	$-1+(8\sqrt{\pi})/e^2$	$-1+(8\sqrt{\pi})/e^2$
3	$(27\sqrt{6\pi})/e^3$	5	5	$-5+(27\sqrt{6\pi})/e^3$	$-5+(27\sqrt{6\pi})/e^3$
4	$(512\sqrt{2\pi})/e^4$	23	23	$-23+(512\sqrt{2\pi})/e^4$	$-23+(512\sqrt{2\pi})/e^4$
5	$(3125\sqrt{10\pi})/e^5$	118	118	$-118+(3125\sqrt{10\pi})/e^5$	$-118+(3125\sqrt{10\pi})/e^5$
6	$(93312\sqrt{3\pi})/e^6$	710	710	$-710+(93312\sqrt{3\pi})/e^6$	$-710+(93312\sqrt{3\pi})/e^6$
7	$(823543\sqrt{14\pi})/e^7$	4980	4980	$-4980+(823543\sqrt{14\pi})/e^7$	$-4980+(823543\sqrt{14\pi})/e^7$
8	$(67108864\sqrt{\pi})/e^8$	9902	39902	$-39902+(67108864\sqrt{\pi})/e^8$	$-39902+(67108864\sqrt{\pi})/e^8$
9	$(1162261467\sqrt{2\pi})/e^9$	359536	359536	$-359536+(1162261467\sqrt{2\pi})/e^9$	$-359536+(1162261467\sqrt{2\pi})/e^9$

Example with a continuous function that returns a negative number.

To the integer part is added (+ 1)

To the fractional part we add (- 1)

f(v)= $-\text{Log}[k]$

f(x) $= \frac{1}{2} - \text{Log}[k] - \frac{i\text{Log}[-k^{2i\pi}]}{2\pi} + 1$

f(y) $= -\frac{1}{2} + \frac{i\text{Log}[-k^{2i\pi}]}{2\pi} - 1$

N	*f(v)*	*f(x)*	**[f(v)]**	*f(y)*	***Fraction [f(v)]***
2	-Log[2]	0	0	-Log[2]	-Log[2]
3	-Log[3]	-1	-1	1-Log[3]	1-Log[3]
4	-Log[4]	-1	-1	1-Log[4]	1-Log[4]
5	-Log[5]	-1	-1	1-Log[5]	1-Log[5]
6	-Log[6]	-1	-1	1-Log[6]	1-Log[6]
7	-Log[7]	-1	-1	1-Log[7]	1-Log[7]
8	-Log[8]	-2	-2	2-Log[8]	2-Log[8]
9	-Log[9]	-2	-2	2-Log[9]	2-Log[9]
10	-Log[10]	-2	-2	2-Log[10]	2-Log[10]
11	-Log[11]	-2	-2	2-Log[11]	2-Log[11]
12	-Log[12]	-2	-2	2-Log[12]	2-Log[12]
13	-Log[13]	-2	-2	2-Log[13]	2-Log[13]
14	-Log[14]	-2	-2	2-Log[14]	2-Log[14]
15	-Log[15]	-2	-2	2-Log[15]	2-Log[15]

Simplifications of the formulas in particular of the general formula.

$$-\frac{1}{2}+\boldsymbol{f}(\boldsymbol{v})-\frac{i\text{Log}[-e^{-2i\boldsymbol{f}(\boldsymbol{v})\pi}]}{2\pi}$$

Negative Logarithm($\text{Log}[-e^{-2i\boldsymbol{f}(\boldsymbol{v})\pi}]$) can be written in various forms:

$$\text{Log}\left[-e^{-2if(v)\pi}\right] = \text{Log}[-\text{Cos}[2\pi\boldsymbol{f}(\boldsymbol{v})]+i\text{Sin}[2\pi\boldsymbol{f}(\boldsymbol{v})]]$$

$$\text{Log}\left[-e^{-2\,i\,f(v)\pi}\right] = \text{Log}[(-1)(-1)^{-2\,f(v)}]$$

Reference texts.:

[1] Rademacher H. (Springer 1973). “Topics in Analytic Number Theory

Reference sites:

[1] https://oeis.org/?language=italian (The on-line Encyclopedia of Integer Sequence)

Publications

[1] https://www.matematicamente.it/staticfiles/approfondimenti/Palmioli-number-theory.pdf (Sum of exponents of consecutive integers)

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