Division by zero

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An algebraic theory of extended complex numbers.

$$\frac{x^x}{x} = x^{x-1} \Rightarrow \frac{0^0}{0} = 0^{-1} = \frac{1}{0} = \infty \Leftrightarrow \frac{0}{0} = 0^0 = \frac{1}{0} \times 0 = \infty \times 0 = 1 \tag{1}$$

$$\frac{x}{\infty} = \frac{1}{\infty} \times x = 0 \times x = 0 \Leftrightarrow \frac{\infty}{x} = \frac{1}{0 \times x} = \frac{1}{0} = \infty, \qquad \begin{array}{l} x \neq 0^{n \neq 0}, \\ -\infty < x < \infty \end{array} \tag{2}$$

$$\ln\left(1^{\infty}\right) = \infty \times \ln(1) = \infty \times 0 = 1 \Leftrightarrow 1^{\infty} = e \tag{3}$$

$$\ln\left((-1)^{\infty}\right) = \infty \times \ln(-1) = \infty \times \pi i \Leftrightarrow (-1)^{\infty} = e^{\pi i \infty} \tag{4}$$

An extended number is in the form $\tau = x\infty^n + y$ where the infinite part ∞^n ($n > 0$) is a number with the property $(\pm\infty)^n \times 0^n = (\pm 1)^n$ (1), and a result of the indivisibility of infinite elements (2) is $f(x) = x\infty \Rightarrow f|_{\mathbb{Q}} : \mathbb{Q} \to \mathbb{Z}_{\infty}$; the sign is located in the dividend.

The theory is noncommutative to keep compatibility with multiplication by zero: $x\infty^n 0^n = x(\infty 0)^n = x$ and (4) are only true with the retention of finite elements; i.e. with multiplication from the right if both sides are raised to the power of 0 as only then Euler's identity holds, this also solves the problem
"which y is the solution to $y \times 0 = x$?": $y = x\infty$. It is also nondistributive in this case:
$1 = \infty \times (0 \pm 0) \neq \infty \times 0 + \infty \times 0 = 2 \neq \infty \times 0 - \infty \times 0 = 0$; zeros are cancelled out inside parentheses first.

"The Seven Spirits of God": $\infty - \infty$, $0 \times \infty$, $0 \div 0$, $\infty \div \infty$, 0^0, ∞^0 and 1^∞ which equals 1, 1, 1, 1, 1, 1 and e (3). Fallacies are corrected by first multiplying by 0 if also dividing by 0 in this case: $0 \times a \div 0 = 0 \times b \div 0 \Rightarrow (0 \times a) \div 0 = (0 \times b) \div 0 = 0 \div 0 = 1$.