

Calculation of the arbitrarily complex shape optical receiver's aperture averaging function by the two-dimensional matrix mapping method

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Abstract: A method for calculating the arbitrarily complex shape optical receiver's aperture averaging function is proposed in this paper. In this method, the mathematical shape of the optical receiver is mapped into a two-dimensional $n \times n$ matrix. The calculation of the aperture averaging function is transformed into the mathematical operation between two two-dimensional matrices. To verify this method, the Cassegrain telescope's aperture averaging function is calculated by mapping to a 300×300 matrix and compared with the theoretical results. The comparison results show that the error of the aperture averaging function calculated by this method is very small. This method can be trusted to calculate the arbitrarily complex shape optical receiver's aperture averaging function.

1. Introduction

Laser propagation through the turbulent atmosphere has been a research topic for a long time. This research is mainly aimed at some applications in recent decades, such as atmospheric optical imaging and free-space optical communication [1,2]. In some cases, free-space optical communication is also called atmospheric optical communication. The light intensity of the laser that transmitted in a turbulent atmosphere will fluctuate. This fluctuation is called atmospheric optical scintillation.

The existence of atmospheric optical scintillation will not only deteriorate the imaging quality but also increase the bit-error rate of free-space optical communication. To solve the influence of scintillation, many technologies, including atmospheric optical compensation, partially polarized light, and multi-beam transmission, have been studied thoroughly [3-5]. This paper is the preparatory work of trying to study this problem from another angle.

In long-term research, human beings have found that there is an aperture averaging phenomenon in laser atmospheric transmission [6]. The light intensity fluctuations of each part of the wavefront of the optical signal transmitted over a long distance are different. Some of which are large and some are small. After entering the receiving telescope, the light intensity fluctuations obtain an ensemble averaging effect. This is the aperture averaging phenomenon. It is actually an effect of smoothing fluctuations. This smoothing effect suppresses the undulating sharp burr peaks. It relates to the parameters of the turbulent atmosphere. When the size of the receiving telescope is less than the correlation scale of light intensity fluctuation, the receiving telescope at this time can be regarded as a point receiver. There is no aperture averaging effect at this time. With the increasing size of the receiving telescope, the aperture averaging effect becomes more and more significant, and finally reaches saturation. Aperture averaging saturation means that when the size of the receiving telescope is larger than a certain size, even if the size of the optical telescope at the receiving end is continuously increased, more aperture averaging effects to suppress light intensity scintillation can not be obtained.

In the earliest days, the receiving telescope used by humans was very small. Human beings had the desire to increase the receiving telescope's size. Finally, one day many years ago, human beings realized from the experimental phenomenon that this aperture averaging effect exists in nature to suppress the fluctuation of light intensity. Therefore, the aperture averaging effect can be considered as a reward of nature for human exploration of atmospheric optics. When human beings expect to continue to increase the size of telescopes to obtain more rewards, nature flatly refuses this request with the aperture averaging saturation. This is also a natural result. Otherwise, this demand will lead to the out-of-control situation of excessive use of resources.

For a long time, optical receivers used by human beings included lenses, Cassegrain telescope, Gregory telescope, Newton reflection telescope, and the corresponding array, etc. There are only limited mathematical shapes of their aperture functions: circle, ring, and the corresponding array. Copy the mathematical shape of a receiving aperture. Move the copy a distance. Calculate the overlapping area between the mathematical shape of the receiving aperture and the replica. This is the aperture averaging function corresponding to this centroid spacing vector. When the overlapping areas corresponding to all centroid spacing vectors are obtained, the complete picture of the receiving aperture's aperture averaging function can be known. This is the calculation method of the aperture averaging function. The analytical solution of the circular aperture's averaging function can be obtained by using the mathematical knowledge of junior middle school [6]. The process of obtaining the analytical solution of the circular ring and circular array's aperture averaging functions is a little cumbersome.

When it is necessary to calculate an arbitrarily complex shape optical receiver's aperture averaging function, there was no method before. This paper proposes a method for calculating an arbitrarily complex shape optical receiver's aperture averaging function.

In the second part of this paper, the author introduces this method by taking a simple optical receiving aperture as an example. In the third part of this paper, to prove the correctness of this method, the author uses this method to calculate the Cassegrain telescope's aperture averaging function and compares it with the theoretical calculation results.

2. Taking a simple optical receiver shape as an example

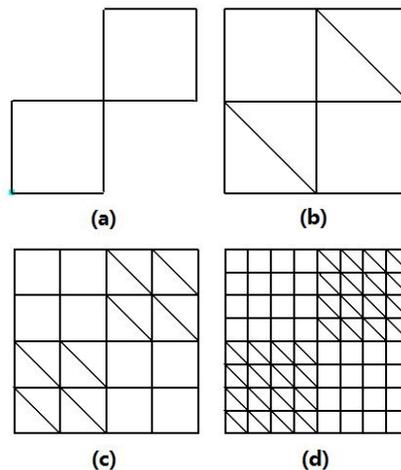


Fig. 1. Add a square grid frame to an arbitrary optical receiver shape.

Suppose there is a receiver with non-circular symmetry, and the mathematical shape of its aperture is shown in figure 1 (a).

Set parameter n to 2. It means that a square grid frame with 2 rows and 2 columns is used to cover the receiver. As is shown in figure 1 (b), the slashed area is the real aperture of the optical receiver. In figure 1 (b), the area without a slash is the filled blank area. The outermost

square in figure 1 (b) is the square grid frame. Figure 1 (b) corresponds to a 2*2 matrix. Where there is a real aperture, the corresponding matrix element is 1. The filled area corresponds to a matrix element of 0. Therefore, the matrix corresponding to figure 1 (b) can be expressed as below:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (1)$$

If the parameter n is set to 4, it means that a square grid frame with 4 rows and 4 columns is used to cover the receiver. As is shown in figure 1 (c), the slashed area is the real aperture of the optical receiver. In figure 1 (c), the area without a slash is the filled blank area. Figure 1 (c) corresponds to a 4*4 matrix. Where there is a real aperture, the corresponding matrix element is 1. The filled area corresponds to a matrix element of 0. Therefore, the matrix corresponding to figure 1 (c) can be expressed as below:

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}. \quad (2)$$

If the parameter n is set to 8, it means that a square grid frame with 8 rows and 8 columns is used to cover the receiver. Similarly, corresponding aperture information is shown in figure 1 (d). The matrix corresponding to figure 1 (d) can be expressed as below:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

Next, the author will give an application example to introduce this method of calculating the aperture averaging function. It is assumed that the shape of the optical receiver is shown in figure 1 (a).

Step 1: We will extend it to the form of figure 1 (b). Therefore, the optical receiver shape of figure 1 (a) can be mapped to the matrix of Eq. (1).

Figure 1 (a) is called the original aperture. Figure 1 (b) is called the basic aperture. The process of mapping figure 1 (b) to Eq. (1) is very important. For any other complex optical receiver shape, this is the process of digitizing the cold optical receiver into a live matrix. After digitizing the matrix, the matrix can be subjected to MATLAB data operation.

Step 2: Establish the moving range. Copy 8 copies of the 2*2 square frame in figure 1 (b). They are closely arranged around the square frame in figure 1 (b). The results are shown in figure 2. The BEFG region of figure 2 is the square frame of figure 1 (b). The HLPT area in figure 2 is the moving range.

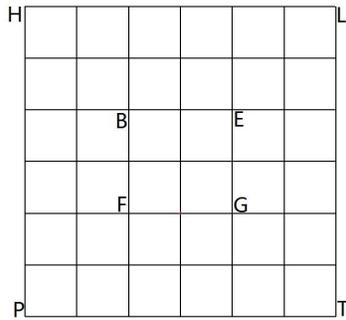


Fig. 2. Schematic diagram of the formation of the moving range.

Step 3: Place the square frame of the basic aperture in the BEFG area of the moving range in figure 2, as is shown in figure 3. Figure 3 is mapped to a matrix. The matrix mapped by the basic aperture and its square frame is called the basic matrix. This step can be realized by the matrix expansion of MATLAB.

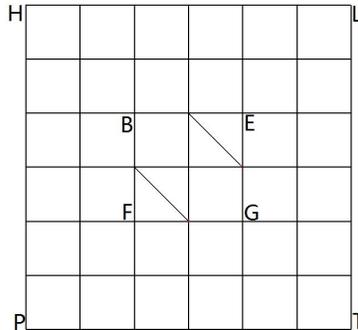


Fig. 3. Schematic diagram of a square frame with the basic aperture.

The results of matrix mapping are as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (4)$$

Step 4: Copy the basic aperture and its square frame. Place it in the upper leftmost corner area UVWX of the moving range of figure 2, as is shown in figure 4. The copy of the basic aperture is called the shifting aperture. The copy of the basic aperture and its square frame is called the shifting aperture and its square frame. Figure 4 is mapped to a matrix. The matrix mapped by the shifting aperture and its square frame is called the shifting matrix.

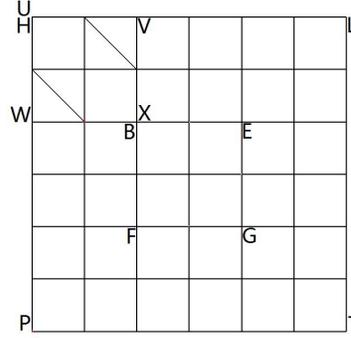


Fig. 4. Schematic diagram of the formation and placement of shifting aperture and its square frame.
Figure 4 can be mapped to a matrix as follows:

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (5)$$

Step 5: Calculate the overlapping area of the two apertures at this position. The overlapping area can be expressed by the number of square grids. It is a positive integer.

The specific method is to mathematically multiply the basic matrix and the shifting matrix by bit and assign them to the newly established matrix “temp”. Figure 3 and figure 4 have no aperture overlapping area. Accordingly, Eq. (4) and Eq. (5) perform matrix bitwise multiplication, and all elements of the matrix of the operation result are also 0. It can be shown as below:

$$\begin{cases} temp_{i,j} = A_{i,j} \cdot B_{i,j} \\ i = 1, 2, \dots, 6 \\ j = 1, 2, \dots, 6 \end{cases} \Rightarrow \begin{matrix} temp = \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}. \quad (6)$$

The subscripts i and j in Eq. (6) represent the sequence numbers of rows and columns of the matrix, respectively. If one element in the temp matrix is 1, it means that there is a square grid overlapping between the basic aperture and the shifting aperture in this centroid spacing vector. If n elements in the temp matrix are 1, it means that there are n square grids overlapping between the basic aperture and the shifting aperture in this centroid spacing vector.

Calculate the number of 1 of all elements of matrix temp. All elements of the temp matrix are accumulated and assigned to the first element of the newly established matrix C. Namely:

$$C(1,1) = \sum_{j=1}^6 \sum_{i=1}^6 temp_{i,j} . \quad (7)$$

Step 6: Calculate the centroid spacing vector between the basic aperture and the shifting aperture. The data points of the aperture averaging function at this centroid spacing vector are obtained. When the square frame of the basic aperture is in figure 3, its starting position is point B. Point B is called the datum point. When the square frame of the shifting aperture is in figure 4, its starting position is point U. Introduce a ray from point B to point U, which is the centroid spacing vector between the basic aperture and the shifting aperture.

In the horizontal direction, the distance between point U and point B is -2 square grids. The datum is recorded in the first element of the newly established matrix D. Namely:

$$D(1,1) = -2 . \quad (8)$$

In the vertical direction, the distance between point U and point B is +2 square grids. The datum is recorded in the first element of the newly established matrix E. Namely:

$$E(1,1) = 2 . \quad (9)$$

Take matrix D as X-axis data. Take matrix E as Y-axis data. Take matrix C as Z-axis data. The first set of data points (-2, 2, 0) in the X-Y-Z coordinate system can be obtained, which is the first set of data points of the aperture averaging function of this optical receiver in figure 1 (b).

Step 7: Move the shifting aperture and its square frame UVWX to the right by a square grid. When moving to the right will cause it to overflow the moving range, move the shifting aperture and its square frame to the far left of the next row. After the shift is performed, the position of the shifting aperture and its square frame is shown in figure 5.

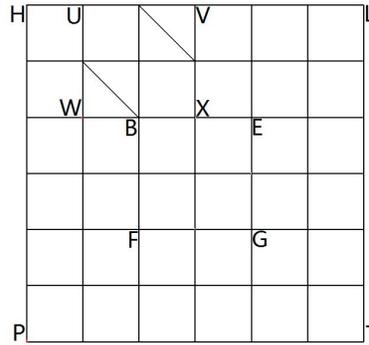


Fig. 5. Move the shifting aperture and its square frame UVWX to the right by a square grid.

Thus, the mapped matrix in figure 5 can be obtained as follows:

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \quad (10)$$

The 7th step includes shift and mapping. These can be realized by MATLAB programming.

Step 8: Jump to step 5. Repeat steps 5–7. That is to calculate the overlapping area of the two apertures at this position. Calculate the square frame centroid spacing vector of two apertures at this position. Thus, the next set of data points is obtained.

When moving the square frame of the shifting aperture to the right causes it to overflow the T point of the moving range, the cycle ends.

The flow chart of the above steps is shown in figure 6.

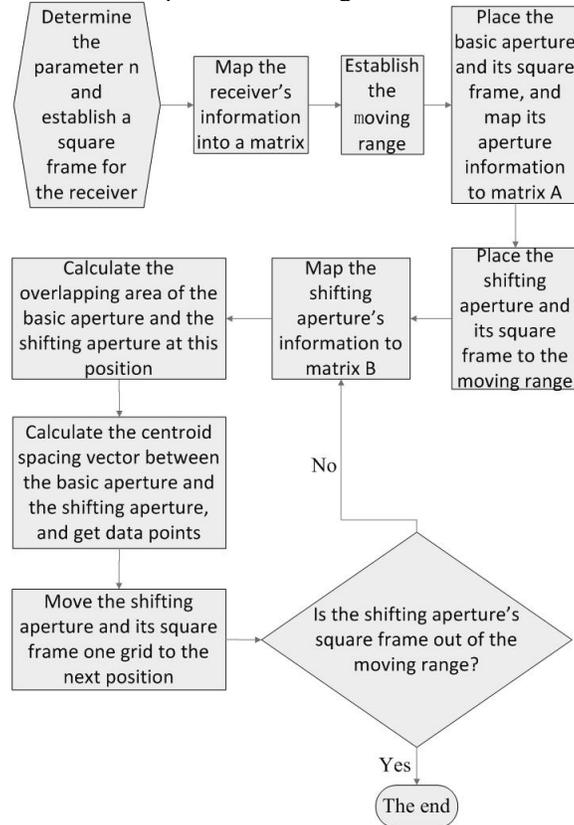


Fig. 6. Flow chart.

3. Calculating the aperture averaging function of the Cassegrain optical telescope

In this section, we will calculate the aperture averaging function of the Cassegrain telescope. Aperture function is a mathematical description of the two-dimensional projection of the telescope shape. Therefore, the Gregory optical telescope and the Cassegrain optical telescope with the same projection have the same aperture averaging function. The method in this section is also applicable to the Gregory optical telescope.

The case of the Cassegrain optical telescope is different from that of the circular lens. A circular lens has only one parameter, that is, its diameter. The Cassegrain optical telescope has not only the primary mirror but also the secondary mirror. The relevant parameter is called the occlusion ratio, that is, the ratio of the diameters of the secondary mirror to the primary mirror. Therefore, when calculating the aperture averaging function of the Cassegrain optical telescope, it is necessary to set the occlusion ratio before computer calculation. In this section, it is assumed that the occlusion ratio of the Cassegrain optical telescope is $d/D = 1:5$. Where d is the diameter of the secondary mirror and D is the diameter of the primary mirror. Figure 7 shows the shape of the two-dimensional projection of the Cassegrain telescope.

A square frame is used to cover the two-dimensional shape of a Cassegrain telescope with an occlusion ratio of $d/D = 1:5$. Take the parameter $n = 100$. So this square frame has 100×100 square grids as is shown in figure 7.

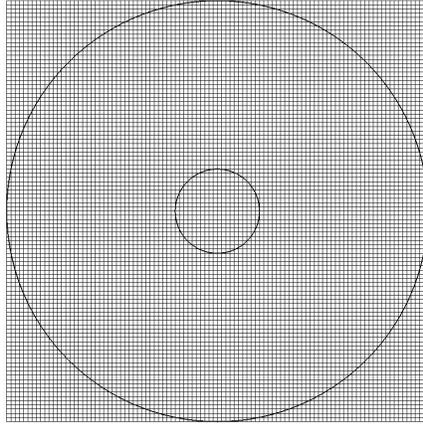


Fig. 7. Construct a square frame with 100*100 square grids for the Cassegrain telescope. $d/D=1:5$.

The outer circle of figure 7 corresponds to the projection of the primary mirror of the Cassegrain optical telescope. The diameter of the outer circle is 100 square grids. The inner circle of figure 7 corresponds to the projection of the secondary mirror of the Cassegrain optical telescope. The diameter of the inner circle is 20 square grids. Map figure 7 to a 100*100 matrix. The square grid of the ring area in figure 7 is mapped to 1. Map the square grid of the non-annular region of figure 7 to 0. Figure 7 contains 10,000 square grids. In other words, figure 7 contains 10,000 matrix elements. If the square grids in figure 7 are mapped manually, it is undesirable. The mapping process is as follows: Each square grid in figure 7 corresponds to a matrix element, which has row and column numbers; find the four square grids in the center of figure 7 and record their row and column numbers; the four square grids correspond to other square grids in the four quadrants respectively; for any other square grid to be considered, calculate the distance between it and the central grid; the square grid whose distance is greater than the radius of the inner ring and less than the radius of the outer ring is mapped to matrix element 1, otherwise it is mapped to matrix element 0.

Establish the moving range. In figure 7, the basic aperture and its square frame have 100 * 100 square grids. Therefore, the moving range should have 300 * 300 square grids.

Place the basic aperture and its square frame of figure 7 in the center of the moving range. The effect is shown in figure 8.

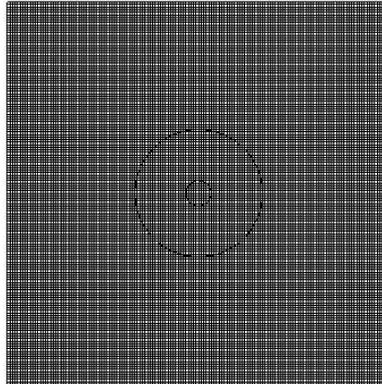


Fig. 8. Place the basic aperture and its square frame of figure 7 in the center of the moving range.

Map figure 8 to a matrix. The square grids of the annular region are mapped to 1. The square grids of the rest region are mapped to 0. Thus, the basic matrix A can be obtained.

Make a copy of the basic aperture and its square frame in figure 7 and place it at the top left corner of the moving range. The effect is shown in figure 9.

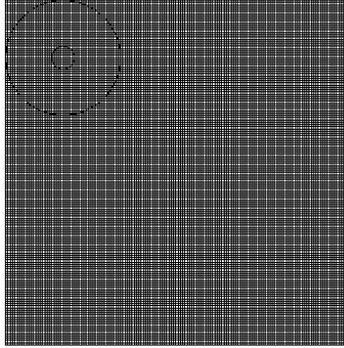


Fig. 9. Place the shifting aperture and its square frame at the top left corner of the moving range.

Map figure 9 to a matrix. The square grids of the annular region are mapped to 1. The square grids of the rest region are mapped to 0. Thus, the shifting matrix B can be obtained.

Calculate the overlapping area of the two apertures at this position. That is, multiply the elements of matrix A and matrix B of each address. The result of the multiplication is the matrix temp. Calculate the number of 1 in the temp matrix. If the elements of the same address of matrix A and matrix B are all 1, it means that there is a square grid overlapping. As many elements of temp matrix are 1, so many square grids overlap.

Calculate the distance between the shifting aperture of figure 9 and the basic aperture of figure 8 in the X and Y directions. The distance in the X and Y directions is represented by the number of square grids. The result of the calculation is the projection component of the centroid spacing vector in the X and Y directions.

Thus, a set of aperture averaging function data points when the shifting aperture is at this position are obtained, which has three components: (the distance along the X direction, the distance along the Y direction, and the number of square grid overlaps). A new matrix F is established. Record this set of data in the first column of matrix F. The format of matrix F is as follows:

$$F = \begin{bmatrix} \text{Distance along the X direction} & \dots & \dots \\ \text{Distance along the Y direction} & \dots & \dots \\ \text{Number of square grid overlaps} & \dots & \dots \end{bmatrix}. \quad (11)$$

Then move the shifting aperture of figure 9 to the right as a whole by a square grid. The translation result is shown in figure 10.

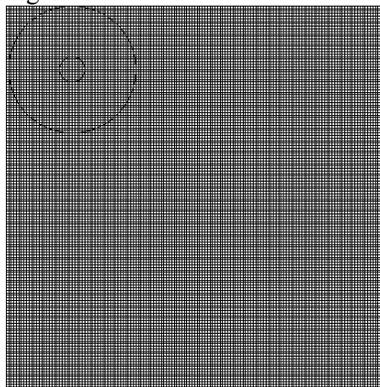


Fig. 10. Move the shifting aperture of figure 9 as a whole to the right by a square grid.

Because the parameter n is very large, the moving range has $300 * 300 = 90,000$ square grids. Therefore, although figure 10 has a grid offset relative to figure 9, it is still difficult to find the difference in figure 10.

Map figure 10 to matrix B. In MATLAB, the rightmost column of matrix B corresponding to figure 9 is removed, and a leftmost column 0 is added. Thus, the mapping of figure 10 to matrix B can be realized.

Calculate the number of square grid overlaps of matrix B and matrix A. Calculate the centroid spacing vector of the shifting aperture of figure 10 and the basic aperture of figure 8. Thus, the second set of data points is obtained and stored in the second column of matrix F.

Then, the shifting aperture and its square frame in figure 10 are shifted to the right as a whole by a square grid. If the translation will cause the shifting aperture and its square frame to exceed the moving range, translate the shifting aperture and its square frame down one row and place it on the leftmost side of the next row. Repeat the previous steps. If the shifting aperture and its square frame overflow from the bottom right corner of the moving range during the displacement, the repeated process ends.

Finally, the complete data of matrix F is obtained. Matrix F corresponds to the complete picture of the aperture averaging function of the receiver. Take the first row of matrix F as X, the second row of matrix F as Y, and the third row of matrix F as Z. By drawing with MATLAB, the three-dimensional diagram of the aperture averaging function of the receiver can be seen. Figure 11 shows the three-dimensional surface of the aperture averaging function of the Cassegrain telescope. Matrix F can also be stored in the memory of the FPGA chip for calling from other control systems.

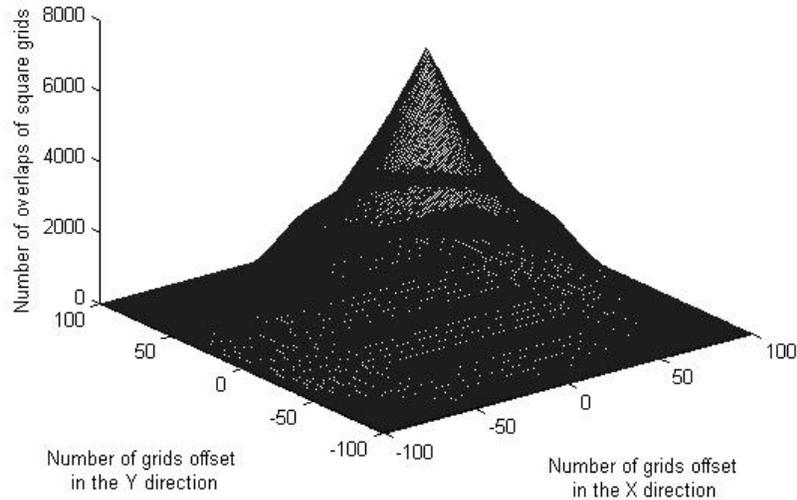


Fig. 11. Three-dimensional surface of the aperture averaging function of the Cassegrain telescope.

When the shifting aperture moves to completely coincide with the basic aperture, the maximum value of the aperture averaging function is obtained. At this time, 7664 square grids overlap. This corresponds to the Z-axis peak in figure 11. When $n = 100$ is selected, the maximum number of overlaps of the square grid is 7664. When n is selected as other values, there are other maximum values. Therefore, when studying the aperture averaging function, we generally study its normalization. Along the X-axis, the maximum number of square grid offsets is 100. Along the Y-axis, the maximum number of square grid offsets is 100. Along the Z-axis, the maximum number of square grid overlaps is 7664. By normalizing the data of each row of matrix F, the normalized three-dimensional surface of the aperture averaging function of the Cassegrain telescope can be obtained.

Figure 12 shows the normalized three-dimensional surface of the aperture averaging function of the Cassegrain telescope.

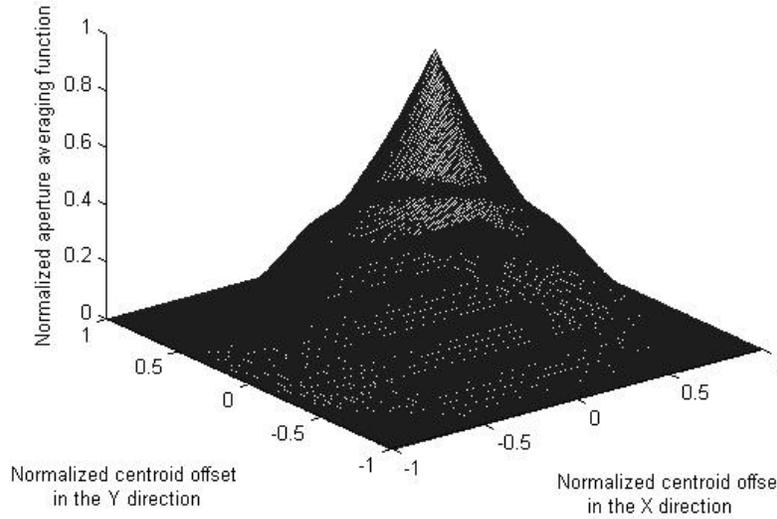


Fig. 12. The normalized three-dimensional surface of the aperture averaging function of the Cassegrain telescope.

It can be seen that the normalized three-dimensional surface of the aperture averaging function of the Cassegrain telescope is rotationally symmetric. The three-dimensional surface can be obtained by rotating a two-dimensional curve 360 degrees. Next, we study the two-dimensional curve of the aperture averaging function of the Cassegrain telescope. Cut a knife down from the top view direction of figure 12 and take only half to obtain the curve.

Figure 13 shows the normalized two-dimensional curve of the aperture averaging function of the Cassegrain telescope. The occlusion ratio is $d/D = 1:5$.

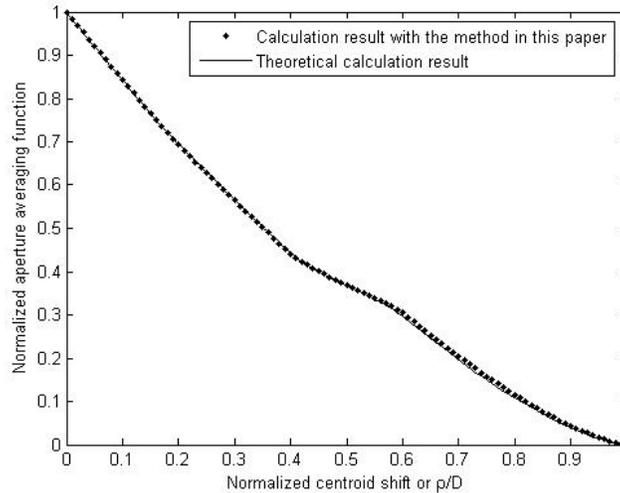


Fig. 13 The normalized two-dimensional curve of the aperture averaging function of the Cassegrain telescope. $d/D=1:5$.

The broken line in figure 13 is the result calculated according to the theoretical formula [7]. The dotted line in figure 13 is the result calculated according to the method in this paper. When the parameter $n = 100$, the difference between the two curves is surprisingly small. It is difficult to identify the error with the naked eye. The largest absolute error occurs at the place $\rho/D = 0.62$. At this place, the percentage error is 3.21%. It can be considered that the calculation method in this paper can well calculate the aperture averaging function of the Cassegrain telescope.

4. Discussions

The author attempts to study how to suppress atmospheric optical scintillation from a different angle. This paper is the preparatory work for this attempt. In this paper, the calculation of aperture averaging function in the field of atmospheric optics is studied.

A method for calculating the aperture averaging function of optical receivers with arbitrarily complex shapes is established in this paper. By mapping the mathematical shape of the receiver into a two-dimensional matrix, the basic aperture, the shifting aperture, and the moving range are constructed. By moving the shifting aperture step by step, a variable shifting matrix can be obtained. By calculating the same elements of the shifting matrix and the basic matrix, the aperture overlapping area of different centroid spacing vectors can be calculated, and the complete picture of the aperture averaging function of the Cassegrain telescope can be obtained. Through the mapping of two-dimensional matrices, cold physical concepts such as receiver, aperture, moving range, and overlapping area are transformed into something with fresh meaning.

In this paper, the three-dimensional surface and the two-dimensional curve of the Cassegrain telescope's aperture averaging function are obtained. The results are compared with the theoretical results. The error is very small and almost negligible. When the parameter n is 100, the moving range includes 300×300 square grids. It means that the particle size reaches 90,000 square grids. This calculation accuracy and error are acceptable.

From now on, there will be no problem in calculating the optical receiver's aperture averaging function with any shape.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the author upon reasonable request.

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