Formula for number of primes less than a given number.

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0- Abstract.

In this paper I want to expose the possibility of making a formula which counts the exact number of primes less than a given number, using some tools in analysis of functions, some tools in series of functions and some new tools.

1- Introduction.

We should first take a look to my development in the serial tool named divisory: the serial operator of divisions.

Theorem 1.0: The division operation can be translated into a serial operator with delta notation, it has some different variations of notation, we can see two of them:

$$(1) \quad \underset{n=a}{\overset{b}{\Delta}} f(n) = f(a) \div f(a+1) \div f(a+2) \div \ldots \div f(b-2) \div f(b-1) \div f(b)$$

$$(2) \quad \underset{n=i}{\overset{k}{\Delta}} a_i = a_1 \div a_2 \div a_3 \div \ldots \div a_{(k-1)} \div a_k$$

Remark 1.1: For example, if we want to do the serial divisions between the interval (2,5) we can start in the smaller number and apply the serial division:

$$(3) \quad \underset{n=2}{\overset{5}{\Delta}} a_i = 2/(3/(4/5)) = 1/30$$

2- The divisory of sets.

Theorem 2.0: We can use the tool of divisory but instead a variable we introduce a constant, we should apply one-to-one division operations to obtain a set as result:

$$(4) \quad \underset{n=i}{\overset{k}{\{\Delta\}}} a_i = \{a/1, a/2, a/3, \ldots, a/(k-1), a/k\}$$

As you can see, we do not obtain a single number, we obtain a set of numbers.

Remark 2.1: For example, if you do the set divisory of 4 at all range, you will get the following set:

$$(5) \quad \underset{n=1}{\overset{4}{\{\Delta\}}} 4 = \{4/1, 4/2, 4/3, 4/4\} = \{4, 2, 4/3, 1\}$$

3- Definition of our f(x) function.

Theorem 3.0: We define our f(x) function as 1 if in the process of analyze the set we have, s=2 and we define f(x) function a 0 if we have s < 2 or s > 2. Where s is the number of Natural numbers.

$$(6) \quad f(x) := \begin{cases} 0 \; if \; \underset{n=i}{\overset{k}{\{\Delta\}}} has \; s < 2 \\ 1 \; if \; \underset{n=i}{\overset{k}{\{\Delta\}}} has \; s = 2 \\ 0 \; if \; \underset{n=i}{\overset{k}{\{\Delta\}}} has \; s > 2 \end{cases}$$

Where s= #n for $n \in \mathbb{N}$

With the option of s < 2 we exclude the option of number 1 as prime later on.

Remark 3.1: For example f(x) is equal to 1 for the number 3:

$$\{\Delta\}_{n=1}^{3} 3=\{3/1,3/2,3/3\}=\{3,3/2,1\} \quad (7)$$

f(x)=1 because in set {3,3/2, 1} there are 2 natural numbers.

Remark 3.2: For example f(x) is equal to 1 for the number 7:

$$\{\Delta\}_{n=1}^{7} 7=\{7/1,7/2,7/3,7/4,7/5,7/6,7/7\}=\{7,7/2,7/3,7/4,7/5,7/6,1\} \quad (8)$$

f(x) =1 because s=2.

Remark 3.3: For example f(x) is equal to 0 for the number 4:

$$\{\Delta\}_{n=1}^{4} 4=\{4/1,4/2,4/3,4/4\}=\{4,2,4/3,1\} \quad (9)$$

f(x)=0 because s=3>2.

Remark 3.4: For example f(x) is equal to 0 for the number 1:

$$\{\Delta\}_{n=1}^{1} 1=\{1/1\}=\{1\} \quad (10)$$

f(x)=0 because s=1<2

Lemma 3.5: A number prime always give an f(x)=1 and a composite number always give an f(x)=0.

4- A formula for the number of prime numbers less than a given number.

Theorem 4.0: The formula for the exact number of prime less than a given number implies a summation for the accumulation of the results and a set serial division which gives us the partial results, and it is the next formula:

$$(11) \quad \sum_{m=j}^{t} a_j \{\Delta\}_{n=i}^{k} a_i = \#\, primes\, less\, than\, a_i$$

Where $f(x) := \{\Delta\}_{n=i}^{k} a_i$ and a_j ranges from 1 to k.

Note: the variable t is a hypothetical result as maximum of the summation, the result of number of primes will always less than t.

Remark 4.1: For example the number of primes less than 4 are given by:

$$(12) \quad \sum_{m=1}^{4} a_j \{\Delta\}_{n=1}^{4} a_i = \sum_{m=1}^{4} f(x_1)+f(x_2)+f(x_3)+f(x_4)=$$

$$= \sum_{m=1}^{4} (\{\Delta\}_{n=1}^{1} 1)+(\{\Delta\}_{n=1}^{2} 2)+(\{\Delta\}_{n=1}^{3} 3)+(\{\Delta\}_{n=1}^{4} 4)=0+1+1+0=2$$

5- Number primes in an interval.

Finally, I will exemplify the use of this formula as an interval, in example if you want to get the number of primes between 10 and 15 (there are 2 it is obvious), you can put this numbers on the formula:

$$(13)\quad \sum_{m=10}^{15} a_j \{\Delta\}_{n=1}^{15} a_i = \sum_{m=10}^{4} f(x_{10})+f(x_{11})+f(x_{12})+f(x_{13})+f(x_{14})+f(x_{15})=$$

$$=\sum_{m=10}^{15} \left(\{\Delta\}_{n=1}^{10} 10\right)+\left(\{\Delta\}_{n=1}^{11} 11\right)+\left(\{\Delta\}_{n=1}^{12} 12\right)+\left(\{\Delta\}_{n=1}^{13} 13\right)+\left(\{\Delta\}_{n=1}^{14} 14\right)+\left(\{\Delta\}_{n=1}^{15} 15\right)=$$

$$=0+1+0+1+0+0=2$$

6- Conclusions:

This is my perception of a truly formula for this old problem, I imagine that it is not the most efficient way to compute in a machine a result of this formula (it will take a long process to compute this with large numbers), but in my opinion this is an exact method and number theory needs the two ways of solving the problems: the approximation way methods and the methods of accuracy.