Unprovability of First Maxwell's Equation in Light of EPR's Completeness Condition

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Maxwell's verbal statement of Coulomb's experimental verification of his hypothesis, concerning force between two electrified bodies, is suggestive of a modification of the respective computable expression on logical grounds. This modification is in tandem with the completeness condition for a physical theory, that was stated by Einstein, Podolsky and Rosen in their seminal work. Working with such a modification, I show that the first Maxwell's equation, symbolically identifiable as "$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$" from the standard literature, is *unprovable*. This renders Poynting's theorem to be *unprovable* as well. Therefore, the explanation of 'light' as 'propagation of electromagnetic energy' comes into question on theoretical grounds.

I. INTRODUCTION

Nobody has seen light, I believe. Rather light lets us 'see'. Thus, propagation of light is a hypothesized phenomenon which is theoretically established by the equations which were written by Maxwell[1], and what we know today as Maxwell's equations[2, 3]. Today we have the understanding of light propagation as "propagation of electromagnetic wave", which forms the basis of our explanations of reflection, refraction, interference, diffraction, polarization and other optical phenomena[4]. That the propagation of light is indeed "propagation of electromagnetic energy" is theoretically established by a theorem which was proved by Poynting[5], and what we know today as Poynting's theorem[2, 3]. However, Einstein's special theory of relativity suggests that the one way propagation speed of light is not knowable within the theory because the theory itself it founded on a choice of clock synchronization convention that is based on the assumption of a two-way velocity of light, which Einstein elaborated before stating the two postulates[6]. Such a choice can be safely considered as an independent axiom of special relativity and it has continued to remain a matter of investigation and debate since the advent of special relativity[7]. Therefore, if there is any claim that Maxwell's equations are consistent with the special relativity, then the two way velocity of light is also a necessary assumption for the relativistic Maxwell's equations, which indeed was the case for Einstein's own work[6]. This leaves room for skepticism regarding the mathematical structure that provides us the foundation, and hence the logic, behind our confidence on the theoretical propagation model given by Maxwell's equations. Such skepticism seems justified from Poincare's bold declaration from the stand point of mathematical reasoning, on page no. 6 of ref.[8], that "*it is precisely in the proofs of the most elementary theorems that the authors of classic treatises have displayed the least precision and rigour.*" It was such precision and rigour, from the mathematical logical perspective, that Hilbert searched for through his sixth problem [9].

The motto of the present work is to investigate the proof of the first Maxwell's equation, symbolically identifiable as "$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$" from standard modern textbooks[2, 3] and also known as the differential form of Gauss' law, by considering the steps of reasoning in propositional form so that mathematical logic becomes vividly applicable[10, 11]. This is a particular, nevertheless a significant, attempt to implement Hilbert's philosophy that is manifested in his sixth problem, namely, "*Mathematical Treatment of the Axioms of Physics*"[9] or "axiomatization of physics" in modern terminology[16]. As I expose, a propositional truth analytic (formal) proof of the first Maxwell's equation halts at a decision problem[1] and therefore, the first Maxwell's equation is formally unprovable; it can only be written by *choice*. Since the standard "proof" of Poynting's theorem[5], as available in the modern standard textbooks[2, 3], is dependent on the first

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[1] In this context I find it intriguing to mention that Turing demonstrated the halting problem[26], as an application of the decision problem[10] by considering only "automatic machines" and excluded "choice machines" from the context. In case of "choice machines" an external operator can make an arbitrary choice when the machine halts at a decision problem. The role of choices that I discuss here, only brings to light how humans actually make choices to write the first Maxwell's equation, which otherwise is impossible/illegal in a strictly logical computation.

Maxwell's equation, therefore Poynting's theorem becomes formally unprovable as a consequence. The preciseness of the computable expressions[2] corresponding to the verbal statements of Coulomb's hypothesis ('law') stated by Maxwell in his book[12], in light of the completeness condition of a physical theory stated by Einstein, Podolsky and Rosen[13], plays the pivotal role in this analysis.

To proceed through the logical analysis of the concerned problem, in section(II), at first I revisit Coulomb's hypothesis as was stated by Maxwell in his book *The Treatise of Electricity and Magnetism: Vol. 1*[12] and modify the respective computable expression to seek a more precise and truthful conversion of the verbal expressions. I explain how such modification is a necessity in light of the completeness condition of a physical theory, as was stated by Einstein, Podolsky and Rosen[13]. In section (III), I rewrite the definition of electric field due to a source point charge in symbolic terms, keeping track of the associated logical conditions, considering the modified computable expression of Coulomb's hypothesis as the premise. Then, I present the formal computation of the divergence of electric field. In section (IV), I generalize the analysis for a source configuration of non-overlapping point charges. In section (V), I analyze the case of a continuous distribution of source charge and consequently expose the formal unprovability of the first Maxwell's equation and the Poynting's theorem. In section (VI), I conclude with a few remarks.

In accord with the standard practice in the field theory literature, I use Dirac delta function[25] (henceforth, to be called as delta function) to present my analysis[3]. Also, I mean "classical logic" while using the word "logic" in this work, following the standard practice in mathematical logic e.g. see ref.[10, 11].

II. MAXWELL'S STATEMENT OF COULOMB'S HYPOTHESIS AND EPR COMPLETENESS CONDITION

Before analyzing Maxwell's statement of Coulomb's hypothesis, let me provide the motivation to indulge in such an inquiry, which will automatically set the tone of the rest of this work and lay down the attitude with which this work needs to be studied.

While explaining Einstein's operational analysis of concepts that formed the basis of special relativity[6], Bridgman pointed out, on page no. 5 of ref.[14], that a "*concept is synonymous with the corresponding set of operations.*" As concepts are expressed through language, while writing the theories of science, there comes the question of the truthfulness of expressions of experience (operations) in terms of verbal language. Then follows the question of how precisely the verbal language is encoded in the respective computable expressions. So, it boils down to how precise one can make the following interrelations:

$$\text{physical operations} \leftrightarrow \text{expressions through verbal language} \leftrightarrow \text{computable expressions}\ .$$

It is the second interrelation that is under investigation in the present discussion[4].

The importance of language and associated reasoning in physics has been manifested earlier[9, 22] and also it has recently gained emphasis from various independent research investigations[16–21]. Maintaining the attitude of making logico-linguistic, or semantically driven, inquiry into the foundations of physics that has been showcased in ref.[17, 19], in what follows, I demonstrate an important consequence of a truthful conversion of verbal statement to the respective computable expression that concerns Coulomb's hypothesis regarding the force between two charged or electrified bodies.

On p 69 of ref.[12], Maxwell wrote the following:

"*Coulomb showed by experiment that the force between electrified bodies whose dimensions are small compared with the distance between them, varies inversely as the square of the distance. Hence the actual repulsion between two such bodies charged with quantities e and e′ and placed at a distance r is*

$$\frac{ee'}{r^2}\,." \tag{1}$$

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[2] By "computable expression" I mean an expression with which one can perform computations – both mathematical (arithmetical, algebraic, etc.) and logical (truth analysis with logical connectives abiding by the principles of logic).

[3] Unlike Maxwell's original work[12], the modern standard method to prove Maxwell's equations involves the use of the delta function[2, 3] which is a standard mathematical tool in theoretical physics as a whole. This justifies the presentation of this work only through the use of the delta function.

[4] Technically speaking, considering the verbal language, which is English here, as the metalanguage and the computable expressions as the object-language, the present investigation concerns the precision and the rigour with which the object-language may encode the metalanguage e.g. see ref.[27] for a discussion regarding such linguistic issues.

Here, the matter of inquiry is whether the computable expression, that is (1), actually encodes what Maxwell stated verbally in the preceding statements to explicate the set of physical operations that Coulomb needed to perform to experimentally verify his hypothesis or 'law'. The significance of such an inquiry becomes manifest through EPR's Completeness Condition (ECC) for a "physical theory"[13]:

"*every element of the physical reality must have a counterpart in the physical theory*".

I explain as follows. Maxwell's verbal statement about the comparison of dimensions and the distance is an expression of the experience of the experimenter (Coulomb) in the laboratory. This is what I consider as "physical reality", following Einstein[15]. So, according to ECC, if the computable expression (1) is a part of a physical theory, then it should encode the verbal statement about the comparison between "dimensions" and "distance". Now, "dimensions" and "distance" can be compared if and only if "dimensions" means "length dimensions"; otherwise the comparison is meaningless due to mismatch of physical dimensions, which is nevertheless a basic lesson of metrology[24] and, in particular, of dimensional analysis[23]. So, certainly there should be an association of some characteristic lengths with the electrified bodies, say, s and s'. Then, the computable expression (1), being the building block of a physical theory, should be accompanied by the condition "$s, s' < r$". Therefore, in order to satisfy ECC, (1) should be rewritten as follows:

$$\frac{ee'}{r^2} \ni s, s' < r, \tag{2}$$

where the symbol "$\ni$" stands for "such that".

However, the standard practice is to work with the hypothetical notion of "point charge"[2]. Ignoring any hint of doubt regarding whether such a practice is feasible or not from the ECC point of view[5], I consider the following modification of Maxwell's verbal statement in terms of "point charge":

Coulomb showed by experiment that the force between electrified bodies, each considered as a point charge, not overlapping with each other, or equivalently, having a non-vanishing distance between them, varies inversely as the square of the distance.

Such a verbal statement can be encoded in the computable expression by considering the condition "$r \neq 0$" to be mandatory. On such grounds of reasoning, the expression (1) should now be modified, in the standard jargon of "point charge", as follows:

$$\frac{ee'}{r^2} \ni r \neq 0. \tag{3}$$

Now, I write the content of (3) in a more general form by introducing a coordinate system so as to write it in terms of vector notation following the modern textbook standard practice[2, 3].

I consider a test charge q_0 at field point $\vec{r}$ and a source charge q_i at source point $\vec{r}_i$, where I consider both the charges to be positive definite (as per general convention). According to Coulomb's hypothesis, the force on the test charge q_0, due to the source charge q_i situated at $\vec{r}_i$, is written as follows:

$$\vec{F}_{(q_i,\vec{r}_i)}(\vec{r}) = \frac{q_0 q_i}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \quad \ni \vec{r} \neq \vec{r}_i \tag{4}$$

I may note that (4) is the expression of Coulomb's hypothesis, in modern standard notation, that takes into account the ECC. Now, in what follows, I define the notion of electric field based on (4) and do the respective computations.

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[5] One can object that as far as physical reality is concerned nobody has ever experienced a "point charge". It is only to implement the axiom of point from geometry that the notion of "a point" is invoked. While I admit that this is a valid objection that should be taken into account, I may also assert that the consequence would be a drastic change in the computable expressions of electrostatics. I plan to address this issue elsewhere. For the present work, I choose to work with "point charge", following the standard convention that is devoid of s_1, s_2.

III. ELECTRIC FIELD DUE TO A SOURCE POINT CHARGE

Considering (4) as the premise, I define the electric field at the field point $\vec{r}$, due to q_i situated at $\vec{r}_i$, as follows:

$$\begin{aligned}\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) &:= \lim_{q_0\to 0}\frac{\vec{F}_{(q_i,\vec{r}_i)}}{q_0} = \lim_{q_0\to 0}\frac{q_0}{q_0}\cdot\frac{q_i}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3} \ \ni \vec{r}\neq\vec{r}_i \\ &= \frac{q_i}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3} \ \ni \vec{r}\neq\vec{r}_i. \quad [\because \lim_{q_0\to 0}\frac{q_0}{q_0}=1] \end{aligned} \tag{5}$$

This definition is in accord with the standard practice except the use of symbols so as to have a more precise correspondence with the verbal language. Therefore, I may write the divergence of "$\vec{E}_{(q_i,\vec{r}_i)}(\vec{r})$" as follows:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) &= \frac{q_i}{4\pi\epsilon_0}\vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3} \ \ni \vec{r}\neq\vec{r}_i \\ &= \frac{q_i}{\epsilon_0}\delta^3(\vec{r}-\vec{r}_i) \ \ni \vec{r}\neq\vec{r}_i \quad \left[\text{using } \ \vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3} = 4\pi\delta^3(\vec{r}-\vec{r}_i)\right]. \end{aligned} \tag{6}$$

Now, I write the delta function formally, i.e. in logical terms, as follows:

$$\delta^3(\vec{r}-\vec{r}_i) = \ [0 \ni \vec{r}\neq\vec{r}_i] \ \veebar \ [\infty \ni \vec{r}=\vec{r}_i], \tag{7}$$

where the *logical connective* "$\veebar$" stands for "EITHER .... OR .....". Therefore, I may now write

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) &= [0 \ni (\vec{r}\neq\vec{r}_i)\wedge(\vec{r}\neq\vec{r}_i)] \ \veebar \ [\text{undecidable} \ni (\vec{r}=\vec{r}_i)\wedge(\vec{r}\neq\vec{r}_i)] \\ &= [0 \ni \vec{r}\neq\vec{r}_i] \ \veebar \ [\text{undecidable} \ni (\vec{r}=\vec{r}_i)\wedge(\vec{r}\neq\vec{r}_i)]. \end{aligned} \tag{8}$$

The second term is undecidable (i.e. one can not decide what to write) because both "$\vec{r}=\vec{r}_i$" and "$\vec{r}\neq\vec{r}_i$" are true in the same expression. This is a decision problem where, due to the association of two contradictory conditions, I can not decide what mathematical result I can write logically. Therefore, the computation halts. However, to proceed I can make two choices as follows.

- **Choice 1:** I ignore the truth of "$\vec{r}\neq\vec{r}_i$" in the second term and write the following:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) &= [0 \ni \vec{r}\neq\vec{r}_i] \ \veebar \ [\infty \ni (\vec{r}=\vec{r}_i) \ \underbrace{\wedge(\vec{r}\neq\vec{r}_i)}_{\text{ignore by choice}} \ ] \\ &= [0 \ni \vec{r}\neq\vec{r}_i] \ \veebar \ [\infty \ni \vec{r}=\vec{r}_i] \quad \text{(written by choice)}. \end{aligned} \tag{9}$$

  "∞" is a result of a choice of ignorance – a choice by which I ignore a necessary definability condition for "$\vec{E}_{(q_i,\vec{r}_i)}(\vec{r})$" on the left hand side. Thus, defying logic by invoking such a choice, now I can write

$$\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) = \frac{q_i}{\epsilon_0}\delta^3(\vec{r}-\vec{r}_i) \tag{10}$$

  by using (7), where the symbol "$\vec{E}_{(q_i,\vec{r}_i)}(\vec{r})$" is *undefined.* I may emphasize that (10) and (6) should not be considered to be the same expression. It is only (10) that can be found in standard literature and not (6).

- **Choice 2:** I ignore the truth of "$\vec{r}=\vec{r}_i$" in the second term and write the following:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) &= [0 \ni \vec{r}\neq\vec{r}_i] \ \veebar \ [0 \ni \ \underbrace{(\vec{r}=\vec{r}_i)\wedge}_{\text{ignore by choice}} \ (\vec{r}\neq\vec{r}_i)] \\ &= [0 \ni \vec{r}\neq\vec{r}_i] \ \veebar \ [0 \ni (\vec{r}\neq\vec{r}_i)]. \quad \text{(written by choice)} \end{aligned} \tag{11}$$

  Such a choice does not yield a new mathematical result in the second term other than "0" which is the result in the first term. Now, using the fact that $P \veebar P \equiv P$ for any proposition P, I may write

$$\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) = 0 \ \ni \vec{r}\neq\vec{r}_i. \tag{12}$$

Now, I strictly adhere to the rules of logic (without making any of the above choices) and proceed as follows. I note that the decision problem arises from a contradiction in the second term of (8) i.e. "$\vec{r}=\vec{r}_i$" $\wedge$ "$\vec{r}\neq\vec{r}_i$". According to the law of non-contradiction, for any proposition P, $P\wedge\neg P$ is always FALSE (i.e. negated, written as $\neg(P\wedge\neg P)$). Hence, "$\vec{r}=\vec{r}_i$" $\wedge$ "$\vec{r}\neq\vec{r}_i$" is always FALSE. So, the second term of (8) is negated i.e. always FALSE.

Now, I may note that for any proposition P, $P\veebar F\equiv P$, where F stands for logical FALSE-hood. Further, always I can write $P\wedge\neg P\equiv F$. So, from (8), logically I can write only

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r}) &= [0\ \ni\vec{r}\neq\vec{r}_i]\veebar F\\ &= [0\ \ni\vec{r}\neq\vec{r}_i].\end{aligned}\tag{13}$$

I note that **Choice 2** actually leads to the result that I can achieve just by following the usual rules of logic. Hence, two different cases arise from (6). Either I illogically write (10) by **Choice 1**, or I logically write (13) without making any choice. Certainly, which option one accepts is also a choice itself. So, I may conclude that the only logical answer is the following:

$$\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)}(\vec{r})=0\ \ni\vec{r}\neq\vec{r}_i.\tag{14}$$

IV. A SOURCE CONFIGURATION OF POINT CHARGES

Now, I can extrapolate such a logical analysis for the electric field due to a configuration of non-overlapping point charges[6]. Electric field at the field point $\vec{r}$, due to a source configuration of point charges $\{q_i\}$ situated at the points $\{\vec{r}_i\}$ such that $i\in[1,n]$, by the principle of superposition, is written as follows:

$$\begin{aligned}\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r})=\sum_{i=1}^{n}\vec{E}_{(q_i,\vec{r}_i)} &= \sum_{i=1}^{n}\left[\frac{q_i}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}\ \ni\vec{r}\neq\vec{r}_i\right]\\ &= \left[\sum_{i=1}^{n}\frac{q_i}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}\right]\ \ni\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n].\end{aligned}\tag{15}$$

Here, the lower index "$\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)$" stands for the phrase "due to a source configuration of point charges $\{q_i\}$ situated at the points $\{\vec{r}_i\}$ such that $i\in[1,n]$" i.e. "$\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)$" is the shorthand for "$(q_1,\vec{r}_1)\wedge(q_2,\vec{r}_2)\wedge\cdots\wedge(q_n,\vec{r}_n)$" where the symbol "$\wedge$" stands for logical connective "AND". Now, I write

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r})=\sum_{i=1}^{n}\vec{\nabla}\cdot\vec{E}_{(q_i,\vec{r}_i)} &= \left[\sum_{i=1}^{n}\frac{q_i}{4\pi\epsilon_0}\vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}\right]\ \ni\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]\\ &= \left[\sum_{i=1}^{n}\frac{q_i}{\epsilon_0}\delta^3(\vec{r}-\vec{r}_i)\right]\ \ni\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n].\end{aligned}\tag{16}$$

Hence, using the formal structure of the delta function from (7), I can write

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r}) &= [0\ni\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]]\\ &\veebar\ [\text{undecidable}\ni\{\vec{r}=\vec{r}_i\text{ for some }i\in[1,n]\}\wedge\{\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]\}]\end{aligned}\tag{17}$$

Like the previously discussed case of a source point charge, I again face a decision problem and I can make two choices.

- **Choice 1:** I can ignore the truth of "$\{\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]\}$" in the second term and write the following:

$$\begin{aligned}&\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r})\\ &= [0\ni\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]]\ \veebar\ \left[\infty\ni\{\vec{r}=\vec{r}_i\text{ for some }i\in[1,n]\}\underbrace{\wedge\{\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]\}}_{\text{ignore by choice}}\right]\\ &= [0\ni\vec{r}\neq\vec{r}_i\ \forall\ i\in[1,n]]\ \veebar\ [\infty\ni\{\vec{r}=\vec{r}_i\text{ for some }i\in[1,n]\}]\quad\text{(written by choice)}.\end{aligned}\tag{18}$$

[6] Here I may admit that I am unable to take into account in symbolic terms that no two or more of the source point charges overlap. "Non-overlapping" is only a symbolically unexpressed assumption.

"∞" is a result of a choice of ignorance – a choice by which I ignore a necessary definability condition for one of the "$\vec{E}_{(q_i,\vec{r}_i)}(\vec{r})$"-s which is a part of the construction of "$\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r})$" on the left hand side. Thus, defying logic by invoking such a choice, now I can write

$$\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r}) = \sum_{i=1}^{n}\frac{q_i}{\epsilon_0}\delta^3(\vec{r}-\vec{r}_i) \tag{19}$$

by using (7), where the symbol "$\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r})$" is *undefined*. I may emphasize that (19) and (16) should not be considered to be the same expression. It is only (19) that can be found in standard literature and not (16).

- **Choice 2:** I can ignore the truth of "$\{\vec{r}=\vec{r}_i \;\text{ for some }\; i\in[1,n]\}$" in the second term and write the following:

$$\begin{aligned}
&\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r}) \\
&= [0 \ni \vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]] \;\veebar\; \left[0 \ni \underbrace{\{\vec{r}=\vec{r}_i \text{ for some } i\in[1,n]\}}_{\text{ignore by choice}} \wedge \{\vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]\}\right] \\
&= [0 \ni \vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]] \;\veebar\; [0 \ni \{\vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]\}] \qquad \text{(written by choice)}.
\end{aligned} \tag{20}$$

Such a choice does not yield a new mathematical result in the second term other than "0" which is the result in the first term. Now, using the fact that $P \veebar P \equiv P$ for any proposition P, I may write

$$\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r}) = [0 \ni \vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]]\,. \tag{21}$$

Now, I strictly adhere to the rules of logic (without making any of the above choices) and proceed as follows. I note that the decision problem arises from a contradiction in the second term of (17) i.e. "$\vec{r}=\vec{r}_i$ for some $i\in[1,n]$" $\wedge$ "$\vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]$". According to the law of non-contradiction, for any proposition P, $P\wedge\neg P$ is always FALSE i.e. negated (written as $\neg(P\wedge\neg P)$). Hence, "$\vec{r}=\vec{r}_i$ for some $i\in[1,n]$" $\wedge$ "$\vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]$" is always FALSE. So, the second term of (17) is negated i.e. always FALSE.

Now, for any proposition P, $P\veebar F\equiv P$ and also $P\wedge\neg P\equiv F$. So, from (17), logically I can write only

$$\begin{aligned}
\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r}) &= [0 \ni \vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]] \veebar\; F \\
&= [0 \ni \vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]]\,.
\end{aligned} \tag{22}$$

I note that **Choice 2** actually leads to the result that I can achieve just by following the usual rules of logic. Hence, two different cases arise from (16). Either I illogically write (19) by **Choice 1**, or I logically write (22) without making any choice. Certainly, which option one accepts is also a choice itself. However, the only logically valid answer is the following:

$$\vec{\nabla}\cdot\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r}) = 0 \;\ni \vec{r}\neq\vec{r}_i\ \forall\, i\in[1,n]. \tag{23}$$

V. A CONTINUOUS DISTRIBUTION OF SOURCE CHARGE

The calculation of the electric field due to a continuous distribution of source charge consists of the following steps, which can be found in the standard modern textbooks[2, 3], albeit devoid of the symbolic precision that is being discussed here. At first, I consider an infinitesimal volume element $d\tau'$ that contains infinitesimal source charge $dq=\rho(\vec{r}\,')d\tau'$. Then, I write the electric field at the field point $\vec{r}$ due to the infinitesimal source charge dq, situated at $\vec{r}\,'$, as follows:

$$\begin{aligned}
\vec{E}_{(dq,\vec{r}\,')}(\vec{r}) &= \frac{dq}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \quad \ni \vec{r}\neq\vec{r}\,' \quad \text{[by replacing "a point charge" by "an infinitesimal charge" in (5)]} \\
\equiv\; \vec{E}_{(\rho(\vec{r}\,')d\tau',\vec{r}\,')}(\vec{r}) &= \frac{\rho(\vec{r}\,')d\tau\,'}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \quad \ni \vec{r}\neq\vec{r}\,' \quad \text{[using } dq=\rho(\vec{r}\,')d\tau'\text{]}.
\end{aligned} \tag{24}$$

Then, I write the electric field at the field point $\vec{r}$ due to a continuous distribution of source charge $\rho(\vec{r}\,')$ in a volume τ', as follows:

$$\vec{E}_{(\rho(\vec{r}\,'),\tau')}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\tau'}\frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,d\tau' \quad \ni \vec{r}\notin\tau'. \tag{25}$$

The subscript "$(\rho(\vec{r}'), \tau')$" stands for the phrase "due to a continuous distribution of source charge $\rho(\vec{r}\,')$ in a volume τ'". The condition "$\vec{r} \notin \tau'$" means "the field point $\vec{r}$ does not lie within the volume τ'."

Here, I may admit that, unlike the symbolically logical passage from the case of a point source charge "$\vec{E}_{(q_i,\vec{r}_i)}(\vec{r})$" to the case of a collection of non-overlapping point source charges "$\vec{E}_{\bigwedge_{i=1}^{n}(q_i,\vec{r}_i)}(\vec{r})$", I can not find a symbolically logical passage from the case of an infinitesimal source charge "$\vec{E}_{(\rho(\vec{r}')d\tau',\vec{r}')}(\vec{r})$" to the case of a continuous distribution of source charge "$\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r})$". Therefore, the passage from "$\vec{E}_{(\rho(\vec{r}')d\tau',\vec{r}')}(\vec{r})$" to "$\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r})$" is rather symbolically *intuitive*.

Now, using the standard steps of calculation[2, 3], I can write the following:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\int_{\tau'}\rho(\vec{r}\,')\vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,d\tau' \quad \ni \vec{r}\notin\tau' \\ &= \frac{1}{\epsilon_0}\int_{\tau'}\rho(\vec{r}\,')\delta^3(\vec{r}-\vec{r}\,')\,d\tau' \quad \ni \vec{r}\notin\tau' \quad \left[\because \vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} = 4\pi\delta^3(\vec{r}-\vec{r}\,')\right]. \end{aligned} \tag{26}$$

Further, using the properties of the delta function[2, 3, 25], I may write the following formal structure:

$$\int_{\tau'}\rho(\vec{r}\,')\delta^3(\vec{r}-\vec{r}\,')\,d\tau' = [0 \ni \vec{r}\notin\tau'] \veebar [\rho(\vec{r}) \ni \vec{r}\in\tau']. \tag{27}$$

Therefore, using (27), I may recast (26) in the following way:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) &= [0 \ni \{(\vec{r}\notin\tau')\wedge(\vec{r}\notin\tau')\}] \veebar [\text{undecidable} \ni \{(\vec{r}\in\tau')\wedge(\vec{r}\notin\tau')\}] \\ &= [0 \ni \vec{r}\notin\tau'] \veebar [\text{undecidable} \ni \{(\vec{r}\in\tau')\wedge(\vec{r}\notin\tau')\}]. \end{aligned} \tag{28}$$

The second term is undecidable because of the contradictory conditions i.e. I have arrived at a decision problem. Now, I can make two choices like in the previous scenarios.

- **Choice 1:** I can ignore the truth of "$\vec{r}\in\tau'$" in the second term and write the following:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) &= [0 \ni \vec{r}\notin\tau'] \veebar [\rho(\vec{r})/\epsilon_0 \ni \{(\vec{r}\in\tau')\ \underbrace{\wedge(\vec{r}\notin\tau')}_{\text{ignore by choice}}\ \}] \\ &= [0 \ni \vec{r}\notin\tau'] \veebar [\rho(\vec{r})/\epsilon_0 \ni \{(\vec{r}\in\tau')\}] \quad (\text{written by choice}). \end{aligned} \tag{29}$$

  "$\rho(\vec{r})$" is a result of a choice of ignorance − a choice by which I ignore the definability condition of "$\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r})$" on the left hand side. Thus, it is only by defying logic, I am able to write some non-trivial result, namely $\rho(\vec{r})$, other than "0" in the second term.

- **Choice 2:** I can ignore the truth of "$\vec{r}\notin\tau'$" in the second term and write the following:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) &= [0 \ni \vec{r}\notin\tau'] \veebar [0 \ni \{\ \underbrace{(\vec{r}\in\tau')\wedge}_{\text{ignore by choice}}\ (\vec{r}\notin\tau')\}] \\ &= [0 \ni \vec{r}\notin\tau'] \veebar [0 \ni \vec{r}\notin\tau'] \quad (\text{written by choice}). \end{aligned} \tag{30}$$

  Such a choice does not yield a new mathematical result in the second term other than "0" which is the result in the first term. Now, using the fact that $P \veebar P \equiv P$ for any proposition P, I can write

$$\vec{\nabla}\cdot\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) = 0 \ni \vec{r}\notin\tau'. \tag{31}$$

Now, I strictly adhere to the rules of logic (without making any of the above choices) and proceed as follows. I note that the decision problem arises from a contradiction in the second term of (28) i.e. "$\vec{r}\in\tau\,'$" $\wedge$ "$\vec{r}\notin\tau\,'$". According to the law of non-contradiction, for any proposition P, $P\wedge\neg P$ is always FALSE i.e. negated (written as $\neg(P\wedge\neg P)$). Hence, "$\vec{r}\in\tau\,'$" $\wedge$ "$\vec{r}\notin\tau\,'$" is always FALSE. So, the second term of (28) is negated i.e. always FALSE. Further, for any proposition P, $P\veebar F\equiv P$ and also $P\wedge\neg P\equiv F$. So, from (28), logically I can write only

$$\begin{aligned}\vec{\nabla}\cdot\vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) &= [0 \ni \vec{r}\notin\tau\,'] \veebar F \\ &= [0 \ni \vec{r}\notin\tau\,']. \end{aligned} \tag{32}$$

I note that **Choice 2** actually leads to the result that I can achieve just by following the usual rules of logic. Hence, two different cases arise from (28). Either I illogically write (29) by **Choice 1**, or I logically write (32) without making any choice. Certainly, which option one accepts is also a choice itself. However, the only logically valid answer is the following:

$$\vec{\nabla} \cdot \vec{E}_{(\rho(\vec{r}'),\tau')}(\vec{r}) \ = \ 0 \ni \vec{r} \notin \tau'. \tag{33}$$

Thus, I may conclude that the first Maxwell's equation, symbolically identifiable as "$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \rho(\vec{r})/\epsilon_0$" from the standard modern texts[2, 3] is formally unprovable in light of ECC. Since this equation is one of the founding premises of the standard "proof" of Poynting's theorem available in the standard modern textbooks[2, 3], then Poynting's theorem is rendered formally unprovable as well. Consequently, the explanation of 'light' as 'propagation of electromagnetic energy' comes into question on theoretical ground.

VI. CONCLUSION

In this work, following basic rules of logic, I have shown that the first Maxwell's equation, symbolically identifiable as "$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \rho(\vec{r})/\epsilon_0$" from the standard modern texts[2, 3], is formally unprovable in light of EPR Completeness Condition[13]. Although EPR, themselves, only used such completeness condition to analyze certain consequences in quantum mechanics[13], but the present analysis concerning the basics of classical electrodynamics only brings out a broader aspect of such completeness condition that underlies the logic and language of physics in general, irrespective of 'classical' and 'quantum'. To put the emphasis along similar matters of fact, I may further point out that in contrast to the very recent trend of relating decision problems uniquely to quantum physics[28–32], the present work only shows that decision problems have nothing unique to do with quantum physics – rather it is a part of human reasoning and language with which physics is done.

Last but not the least, there is a very different issue that is associated with the conclusion of this work. Due to the unprovability of the Poynting's theorem, now the explanation of 'light' as 'propagation of electromagnetic energy' is jeopardized from the theoretical point of view. This, in turn, provides the room and the motivation to rethink about Tesla's objections to Maxwell-Hertz theory[33]. I intend to report further along such lines of investigation in future.

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