A Result of Even & Prime

Shan Jian Wang

wangjianshan@oakmon.cn

Abstract

Objective:
Any even number greater than 2 can be written as the sum of two prime numbers:
Does the prime pair exist universally? If does, is the prime pair unique relatively? If not, how many prime pairs sum to one particular even?
Method:
H(a)=Z(a)/Y(a)
Result:
Any even a greater than 2 can be written as the sum of two prime numbers, there are T(a) forms of the two prime numbers.

Keywords

Goldbach, Euler, even, prime.

1. Structure

1.1. Concept

Set of natural numbers is denoted as N, N={n}.

If one variable belongs to N, then it is denoted as n.

If two variables belong to N, then they are denoted as n1 and n2.

Set of even numbers is denoted as A, A={a|a=2*n}.

If one variable belongs to A, then it is denoted as a.

If two variables belong to A, then they are denoted as a1 and a2.

Set of odd numbers is denoted as B, B={b|b=2*n+1}.

If one variable belongs to B, then it is denoted as b.

If two variables belong to B, then they are denoted as b1 and b2.

Set of odd composite numbers is denoted as C,

C={c|c=(2*n1+1)*(2*n2+1), n1 is not 0 and n2 is not 0.}.

If one variable belongs to C, then it is denoted as c.

If two variables belong to C, then they are denoted as c1 and c2.

Set of prime numbers is denoted as D:

If {1 is also a prime number} is true,

then D={d|d belongs to B and d does not belong to C};
If {1 is also a prime number} is false,
then D={d|d belongs to B and d does not belong to C, d is not 1.}.
If one variable belongs to D, then it is denoted as d.
If two variables belong to D, then they are denoted as d1 and d2.

1.2. N(a)~a/4

a=a/2+a/2, a>0.
If a/2 belongs to A, define a=[(a/2-1)-2n)]+[(a/2+1)+2n].
(a/2+1)-2n is denoted as bL, (a/2+1)+2n is denoted as bR.
n<(a-2)/4, Card(n)=a/4.
If a/2 belongs to B, define a=(a/2-2n)+(a/2+2n).
a/2-2n is denoted as bL, a/2+2n is denoted as bR.
n<a/4, Card(n)=(a+2)/4.
Three piecewise functions: bL, bR; N(a).
bL=(a/2+1)-2n, a/2 belongs to A; bL=a/2-2n, a/2 belongs to B.
bR=(a/2+1)+2n, a/2 belongs to A; bR=a/2+2n, a/2 belongs to B.
Card(n) is denoted as N(a):
N(a)=a/4, a/2 belongs to A; N(a)=(a+2)/4, a/2 belongs to B.
N(a)~a/4, a>a0. Error is denoted as O(a), O(a)~0 when a>a0.

1.3. e=bR-bL

Increasing positive even sequence corresponds to {a|a>0}, set e with the sequence incrementally.

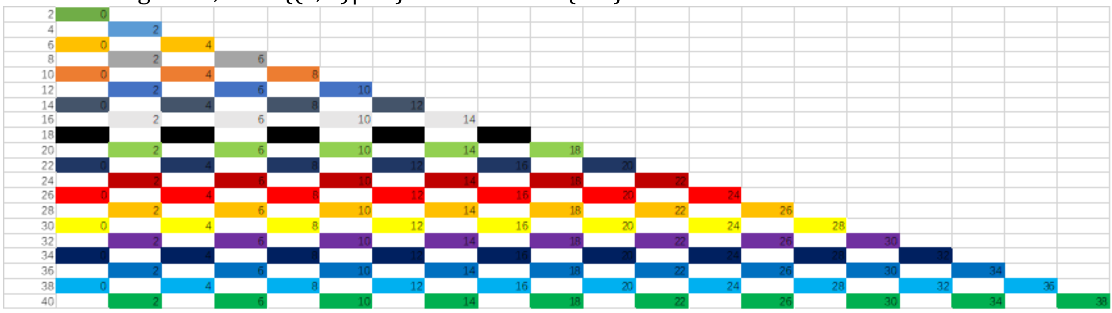
2	0																						
4		2																					
6	0		4																				
8		2		6																			
10	0		4		8																		
12		2		6		10																	
14	0		4		8		12																
16		2		6		10		14															
18	0		4		8		12		16														
20		2		6		10		14		18													
22	0		4		8		12		16		20												
24		2		6		10		14		18		22											
26	0		4		8		12		16		20		24										
28		2		6		10		14		18		22		26									
30	0		4		8		12		16		20		24		28								
32		2		6		10		14		18		22		26		30							
34	0		4		8		12		16		20		24		28		32						
36		2		6		10		14		18		22		26		30		34					
38	0		4		8		12		16		20		24		28		32		36				
40		2		6		10		14		18		22		26		30		34		38			
42	0		4		8		12		16		20		24		28		32		36		40		
44		2		6		10		14		18		22		26		30		34		38		42

Triangular lattice, any cell corresponds to (a, e) and (bL, bR).

	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42
2	1,1																						
4		1,3																					
6	3,3		1,5																				
8		3,5		1,7																			
10	5,5		3,7		1,9																		
12		5,7		3,9		1,11																	
14	7,7		5,9		3,11		1,13																
16		7,9		5,11		3,13		1,15															
18	9,9		7,11		5,13		3,15		1,17														
20		9,11		7,13		5,15		3,17		1,19													
22	11,11		9,13		7,15		5,17		3,19		1,21												
24		11,13		9,15		7,17		5,19		3,21		1,23											
26	13,13		11,15		9,17		7,19		5,21		3,23		1,25										
28		13,15		11,17		9,19		7,21		5,23		3,25		1,27									
30	15,15		13,17		11,19		9,21		7,23		5,25		3,27		1,29								
32		15,17		13,19		11,21		9,23		7,25		5,27		3,29		1,31							
34	17,17		15,19		13,21		11,23		9,25		7,27		5,29		3,31		1,33						
36		17,19		15,21		13,23		11,25		9,27		7,29		5,31		3,33		1,35					
38	19,19		17,21		15,23		13,25		11,27		9,29		7,31		5,33		3,35		1,37				
40		19,21		17,23		15,25		13,27		11,29		9,31		7,33		5,35		3,37		1,39			
42	21,21		19,23		17,25		15,27		13,29		11,31		9,33		7,35		5,37		3,39		1,41		
44		21,23		19,25		17,27		15,29		13,31		11,33		9,35		7,37		5,39		3,41		1,43

1.4. e=|(a-g)-g|, a>g.

If f belongs to A, then {(a, e)|a=f} is denoted as {L=f}.

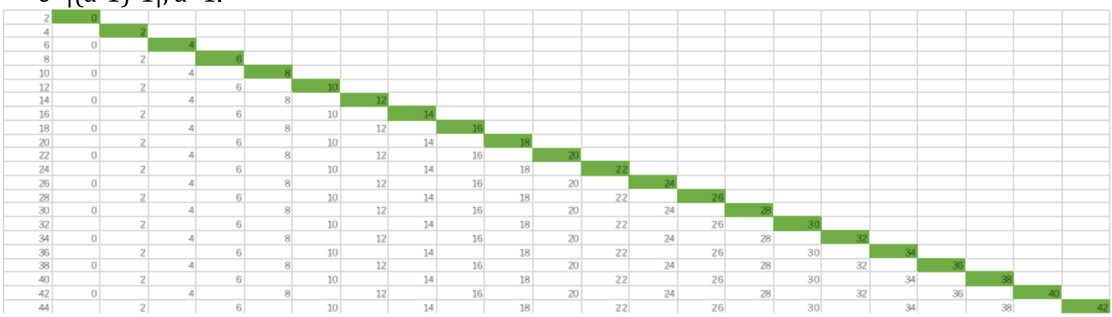


If g belongs to B, then G={(bL, bR)|bL=g or bR=g} is denoted as {R=g}.

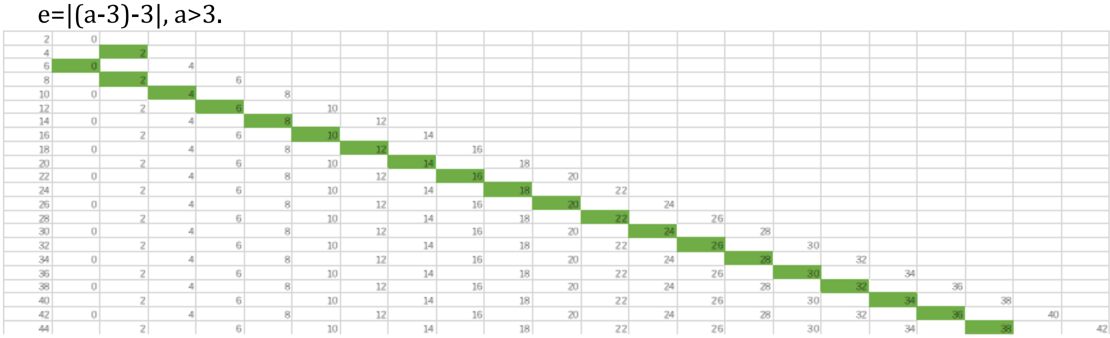
e is one function of a when g is invariable, any odd composite number belongs to (0, a) corresponds to one cell in {L=a}.

Equation is e=|(a-g)-g|, a>g.

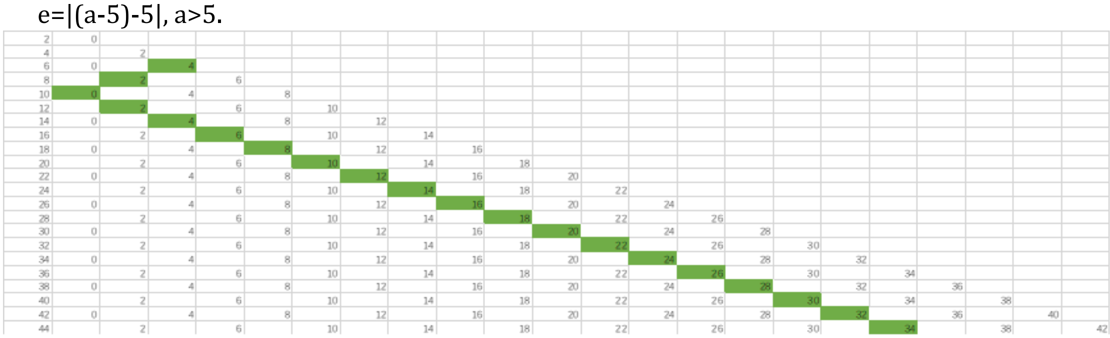
e=|(a-1)-1|, a>1.



e=|(a-3)-3|, a>3.



e=|(a-5)-5|, a>5.



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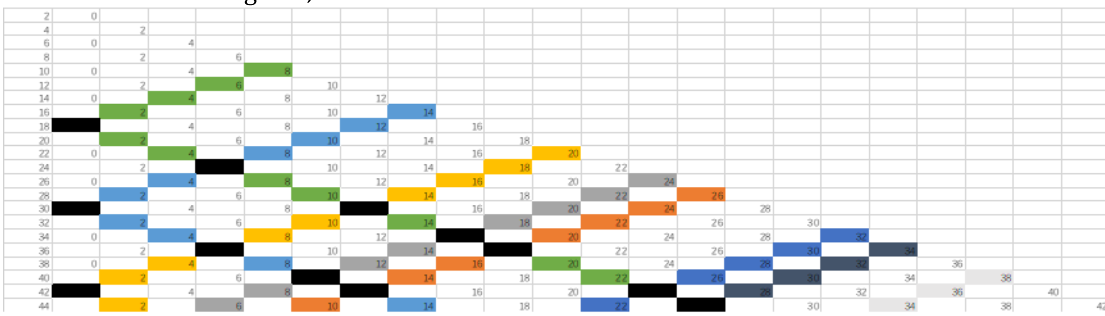
1.5. U(a)-T(a)=S(a)-N(a)

If bL or bR belongs to C, then color the white cell.

If bL and bR belong to C, then black the white cell.



The number of prime numbers in (0, a] is denoted as I(a),

The number of odd composite numbers in (0, a] is denoted as S(a).

The number of black cells in {L=a} is denoted as U(a),

The number of colored non-black cells in {L=a} is denoted as V(a);

The number of colorless cells in {L=a} is denoted as T(a).

V(a)+T(a)+U(a)=N(a), V(a)=S(a)-2*U(a).

1.6. Algebra

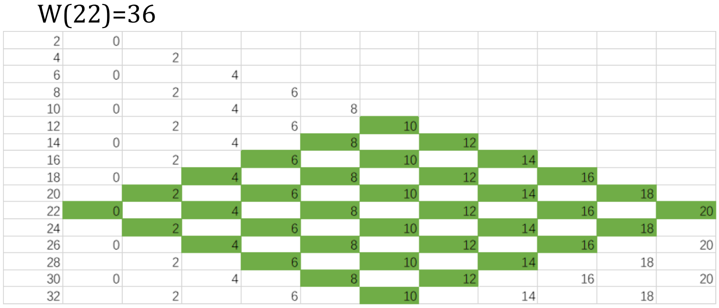
{Any even number greater than 2 can be written as the sum of two prime numbers} can be denoted as {Any T(a)>1, a>4.}

2. Analysis

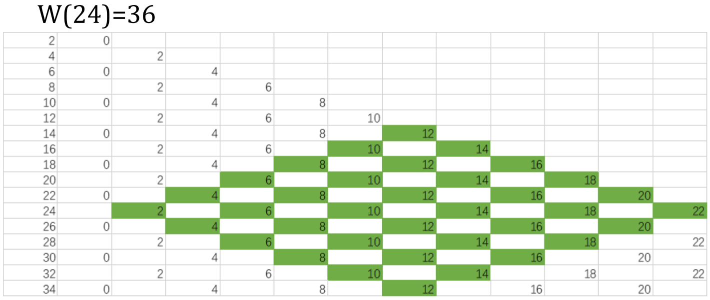
W={(bL, bR)|bL belongs to(0, a/2], bR belongs to [a/2, a).}

Card(bL, bR) is denoted as W(a), W(a)=N(a)^2.

W(22)=36



W(24)=36



If bL and bR belong to C, then the cell is denoted as (cL, cR).
X={(cL, cR)|cL and cR belong to (0, a/2-1].};
Card(cL, cR) is denoted as X(a), X(a)=S(a/2-1)*(S(a/2-1)+1)/2.
Y={(cL, cR)|cL belongs to (0, a/2] and cR belongs to [a/2, a-1].};
Card(cL, cR) is denoted as Y(a), Y(a)=S(a/2)*(S(a-1)-S(a/2-1)).
If bL and bR belong to Y, then the cell is denoted as (yL, yR).
Z={(yL, yR)|yL+yR belongs to (0, a].};
Card(yL, yR) is denoted as Z(a), Z(a)=H(a)*Y(a).

2.1. H(a)~H(a-2)

Maximum error is denoted as Or(a), Or(a)~0 when a>a0.
M={(cL, cR)|cL+cR belongs to (0, a]}, Card(cL, cR) is denoted as M(a).
M(a)=X(a)+Y(a), U(a)=M(a)-M(a-2).
Let T(a)=0, U(a)=S(a)-N(a).
H(a)~(S(a)-N(a)-X(a)+X(a-2))/(Y(a)-Y(a-2)),
H(a)~(a/4-a/ln(a)-((a/2-1)/2-(a/2-1)/ln(a/2-1))*((a/2-1)/2-(a/2-1)/ln(a/2-1)+1)/2+((a/2-2)/2-(a/2-2)/ln(a/2-2))*((a/2-2)/2-(a/2-2)/ln(a/2-2)+1)/2)/(((a/2)/2-(a/2)/ln(a/2))*(((a-1)/2-(a-1)/ln(a-1))-((a/2-1)/2-(a/2-1)/ln(a/2-1)))-((a/2-1)/2-(a/2-1)/ln(a/2-1))*(((a-3)/2-(a-3)/ln(a-3))-((a/2-2)/2-(a/2-2)/ln(a/2-2)))).
Let T(a)=1, U(a)=S(a)-N(a)+1.
H(a)~(S(a)-N(a)+1-X(a)+X(a-2))/(Y(a)-Y(a-2)),
H(a)~(a/4-a/ln(a)+1-((a/2-1)/2-(a/2-1)/ln(a/2-1))*((a/2-1)/2-(a/2-1)/ln(a/2-1)+1)/2+((a/2-2)/2-(a/2-2)/ln(a/2-2))*((a/2-2)/2-(a/2-2)/ln(a/2-2)+1)/2)/(((a/2)/2-(a/2)/ln(a/2))*(((a-1)/2-(a-1)/ln(a-1))-((a/2-1)/2-(a/2-1)/ln(a/2-1)))-((a/2-1)/2-(a/2-1)/ln(a/2-1))*(((a-3)/2-(a-3)/ln(a-3))-((a/2-2)/2-(a/2-2)/ln(a/2-2)))).

2.2. H(a)~J(a)/(J(a)+K(a)), a>0.

Maximum error is denoted as O0(a), O0(a)~(W(a)-(J(a)+K(a)))/W(a)~1/2.
J={(cL, cR)|cL belongs to (0, a/4] and cR belongs to (a/2, 3*a/4]};
Card(cL, cR) is denoted as J(a), J(a)=S(a/4)*(S(3*a/4)-S(a/2)).
K={(cL, cR)|cL belongs to (a/4, a/2] and cR belongs to (3*a/4, a]};
Card(cL, cR) is denoted as K(a), K(a)=(S(a/2)-S(a/4))*(S(a)-S(3*a/4)).
J(a)/(J(a)+K(a))~(((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))/((((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))+((((a/2)/2-(a/2)/ln(a/2))-((a/4)/2-(a/4)/ln(a/4)))*((a/2-a/ln(a))-((3*a/4)/2-(3*a/4)/ln(3*a/4))))).

2.2.1 H(a)~(J(a)+p1+p2)/(J(a)+K(a)+p1+p2+q1+q2), a>8.
Maximum error is denoted as O2(a), O2(a)~1/4.

P1={(cL, cR)|cL belongs to (0, a/8] and cR belongs to (3*a/4, 7*a/8]};
Card(cL, cR) is denoted as p1, p1=S(a/8)*(S(7*a/8)-S(3*a/4)).
P2={(cL, cR)|cL belongs to (a/4, 3*a/8] and cR belongs to (a/2, 5*a/8]};
Card(cL, cR) is denoted as p2, p2=(S(3*a/8)-S(a/4))*(S(5*a/8)-S(a/2)).
Q1={(cL, cR)|cL belongs to (a/8, a/4] and cR belongs to (7*a/8, a]};
Card(cL, cR) is denoted as q1, q1=(S(a/4)-S(a/8))*(S(a)-S(7*a/8)).
Q2={(cL, cR)|cL belongs to (3*a/8, a/2] and cR belongs to (5*a/8, 3*a/4]};
Card(cL, cR) is denoted as q2, q2=(S(a/2)-S(3*a/8))*(S(3*a/4)-S(5*a/8)).

2.2.2 H(a)~(J(a)+p1+...+p6)/(J(a)+K(a)+p1+...+p6+q1+...+q6), a>24.
Maximum error is denoted as O6(a), O6(a)~1/8.
P3={(cL, cR)|cL belongs to (0, a/16] and cR belongs to (7*a/8, 15*a/16]};
Card(cL, cR) is denoted as p3, p3=S(a/16)*(S(15*a/16)-S(7*a/8)).
P4={(cL, cR)|cL belongs to (a/8, 3*a/16] and cR belongs to (3*a/4, 13*a/16]};
Card(cL, cR) is denoted as p4, p4=(S(3*a/16)-S(a/8))*(S(13*a/16)-S(3*a/4)).
P5={(cL, cR)|cL belongs to (a/4, 5*a/16] and cR belongs to (5*a/8, 11*a/16]};
Card(cL, cR) is denoted as p5, p5=(S(5*a/16)-S(a/4))*(S(11*a/16)-S(5*a/8)).
P6={(cL, cR)|cL belongs to (3*a/8, 7*a/16] and cR belongs to (a/2, 9*a/16]};
Card(cL, cR) is denoted as p6, p6=(S(7*a/16)-S(3*a/8))*(S(9*a/16)-S(a/2)).
Q3={(cL, cR)|cL belongs to (a/16, a/8] and cR belongs to (15*a/16, a]};
Card(cL, cR) is denoted as q3, q3=(S(a/8)-S(a/16))*(S(a)-S(15*a/16)).
Q4={(cL, cR)|cL belongs to (3*a/16, a/4] and cR belongs to (13*a/16, 7*a/8]};
Card(cL, cR) is denoted as q4, q4=(S(a/4)-S(3*a/16))*(S(7*a/8)-S(13*a/16)).
Q5={(cL, cR)|cL belongs to (5*a/16, 3*a/8] and cR belongs to (11*a/16, 3*a/4]};
Card(cL, cR) is denoted as q5, q5=(S(3*a/8)-S(5*a/16))*(S(3*a/4)-S(11*a/16)).
Q6={(cL, cR)|cL belongs to (7**a/16, a/2] and cR belongs to (9*a/16, 5*a/8]};
Card(cL, cR) is denoted as q6, q6=(S(a/2)-S(7*a/16))*(S(5*a/8)-S(9*a/16)).

2.2.3 H(a)~(J(a)+p1+...+pα)/(J(a)+K(a)+p1+...+pα+q1+...+qα), α < N(a).
Maximum error is denoted as Oα(a), Oα(a)~1/(α+2).
α =2^β-2, β belongs to N and β>0.
Let β=[ln(a/4)/ln(2)], Oα(a)~0 when a>a0.
H(a)~(J(a)+p1+...+p14)/(J(a)+K(a)+p1+...+p14+q1+...+q14)
Maximum error is denoted as O14(a), O14(a)~1/16.
H(a)~(J(a)+p1+...+p30)/(J(a)+K(a)+p1+...+p30+q1+...+q30)
Maximum error is denoted as O30(a), O30(a)~1/32.
H(a)~(J(a)+p1+...+p62)/(J(a)+K(a)+p1+...+p62+q1+...+q62)
Maximum error is denoted as O62(a), O62(a)~1/64.
H(a)~(J(a)+p1+...+p126)/(J(a)+K(a)+p1+...+p126+q1+...+q126)
Maximum error is denoted as O126(a), O126(a)~1/128.
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2.3. Conclusion

S(a)=Ch*(a/2-a/ln(a)), Ch~1 when a>a0.

Error of S(a)~a/2-a/ln(a) is denoted as Oe(a), Oe(a)~0 when a>a0.

(1) H(a)~(J(a)+p1+...+pα)/(J(a)+K(a)+p1+...+pα+q1+...+qα), α=2^[ln(a/4)/ln(2)]-2.

(2) If T(a)=0, then H(a)~(S(a)-N(a)-X(a)+X(a-2))/(Y(a)-Y(a-2))

But,

(J(a)+p1+...+pα)/(J(a)+K(a)+p1+...+pα+q1+...+qα)>(a/4-a/ln(a)-((a/2-1)/2-(a/2-1)/ln(a/2-1))*((a/2-1)/2-(a/2-1)/ln(a/2-1)+1)/2+((a/2-2)/2-(a/2-2)/ln(a/2-2))*((a/2-2)/2-(a/2-2)/ln(a/2-2)+1)/2)/(((a/2)/2-(a/2)/ln(a/2))*(((a-1)/2-(a-1)/ln(a-1))-((a/2-1)/2-(a/2-1)/ln(a/2-1)))-((a/2-1)/2-(a/2-1)/ln(a/2-1))*(((a-3)/2-(a-3)/ln(a-3))-((a/2-2)/2-(a/2-2)/ln(a/2-2)))).

H(a)>(H(a) when T(a)=0), which is contradictory.

So, T(a)>0 when a>a1.

Error analysis endorse a1(minimum)=0, appendix.

Any T(a)>0, a>0.

(3) If T(a)=1, then H(a)~(S(a)-N(a)+1-X(a)+X(a-2))/(Y(a)-Y(a-2))

But,

(J(a)+p1+...+pα)/(J(a)+K(a)+p1+...+pα+q1+...+qα)>(a/4-a/ln(a)+1-((a/2-1)/2-(a/2-1)/ln(a/2-1))*((a/2-1)/2-(a/2-1)/ln(a/2-1)+1)/2+((a/2-2)/2-(a/2-2)/ln(a/2-2))*((a/2-2)/2-(a/2-2)/ln(a/2-2)+1)/2)/(((a/2)/2-(a/2)/ln(a/2))*(((a-1)/2-(a-1)/ln(a-1))-((a/2-1)/2-(a/2-1)/ln(a/2-1)))-((a/2-1)/2-(a/2-1)/ln(a/2-1))*(((a-3)/2-(a-3)/ln(a-3))-((a/2-2)/2-(a/2-2)/ln(a/2-2)))).

H(a)>(H(a) when T(a)=1), which is contradictory.

So, T(a)>1 when a>a2.

Error analysis endorse a2(minimum)=4, appendix.

Any T(a)>1, a>4.

Conclusion: Any even number greater than 2 can be written as the sum of two prime numbers.

2.4. T(a)

The number of prime pair an even a can be written as is denoted as T(a),

T(a)~(Y(a)-Y(a-2))*J(a)/(J(a)+K(a))+X(a)-X(a-2)-S(a)+N(a);

T(a)~(((a/2)/2-(a/2)/ln(a/2))*(((a-1)/2-(a-1)/ln(a-1))-((a/2-1)/2-(a/2-1)/ln(a/2-1)))-((a/2-1)/2-(a/2-1)/ln(a/2-1))*(((a-3)/2-(a-3)/ln(a-3))-((a/2-2)/2-(a/2-2)/ln(a/2-2))))*(((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))/((((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))+((((a/2)/2-(a/2)/ln(a/2))-((a/4)/2-(a/4)/ln(a/4)))*((a/2-a/ln(a))-((3*a/4)/2-(3*a/4)/ln(3*a/4)))))+((a/2-1)/2-(a/2-1)/ln(a/2-1))*((a/2-1)/2-(a/2-1)/ln(a/2-1)+1)/2-((a/2-2)/2-(a/2-2)/ln(a/2-2))*((a/2-2)/2-(a/2-2)/ln(a/2-2)+1)/2+a/ln(a)-a/4.

2.5. New Conjecture

J(a)/(J(a)+K(a))~(J(a)+p1+…+pα)/(J(a)+K(a)+p1+…+pα+q1+…+qα).

APPENDIX

Table 1. APPENDIX

Function	If	Error	
N(a)=a/4, a/2 belongs to A; N(a)=(a+2)/4, a/2 belongs to B. (a>0)	N(a)~a/4	O(a)	
H(a)=Z(a)/Y(a)	H(a)~H(a-2)	Or(a)	
H(a)=Z(a)/Y(a)	H(a)~(J(a)+p1+…+pα)/(J(a)+K(a)+p1+…+pα+q1+…+qα)	Oα(a)	
S(a)=a/2-I(a)	S(a)~a/2-a/ln(a)	Oe(a)	
I(a)=a/ln(a)+O(a*e^(ln(a)^(-1/c))), c~1.08366.	Oo(a)~O(a*e^(ln(a)^(-1/c))), c=1.08366.	Oo(a)	Big O notation

Appendix shows that the paper only covers five errors, and all five errors have their own limits for any H(a) form, after a finite expansion of H(a)~(J(a)+p1+…+pα)/(J(a)+K(a)+p1+…+pα+q1+…+qα), Oα(a) becomes smaller, and the direction of inequality can be determined.

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