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Negative proof of Riemann's hypothesis

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Abstract: The Riemann hypothesis asserts that all meaningful solutions to the Riemann zeta function equation ζ(s)=0 lie on a line lying on Re(s)=1/2. This paper proves that for s=1/2+it, where t is any real number, the calculation result of the Riemann zeta function cannot be exactly zero, that is, there is no solution. Therefore, the real number t is any value, and it is not a non-trivial zero point, that is, the Riemann hypothesis is denied.

Keywords: Riemann hypothesis, negative proof, inaccurate number

The Riemann hypothesis was proposed by German mathematician Bernhard Riemann in 1859. Riemann observed that the frequency of prime numbers is closely related to the behavior of a carefully constructed so-called Riemann zeta function ζ(s). The Riemann hypothesis asserts that all meaningful solutions to the equation ζ(s)=0 lie on a line lying on Re(s)=1/2.

1 The calculation method of the non-trivial zeros of the Riemann zeta function ζ(s)

The Riemann zeta function ζ(s) is a series expression [1]

$$\zeta(\mathrm{s})=\sum_{n=1}^{\infty}\frac{1}{n^{s}}$$, where: R(s)>1

Analytical continuation in the complex plane

$$\zeta(s)=\frac{\Gamma(1-s)}{2\pi i}\int_{-\infty}^{+\infty}\frac{(-z)^{s}}{e^{z}-1}\frac{dz}{z}$$

Riemann used the integral expression to prove that the Riemann zeta function satisfies the following algebraic relation

$$\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$$

Riemann introduced an auxiliary function ξ(s)

$$\xi(s)=\Gamma(\frac{s}{2}+1)(s-1)\pi^{-\frac{s}{2}}\zeta(s)$$

Its zeros coincide with the nontrivial zeros of the Riemann zeta function. And then deduced the Riemann-Siegel formula.

Let s=1/2+it, t be a real number, used the definition of ξ(s) , the following is proved [2]

$$\xi(\frac{1}{2}+it)=[e^{\mathrm{Re}\ln\Gamma(\frac{s}{2})}\pi^{-\frac{1}{4}}\frac{(-t^2-\frac{1}{4})}{2}][e^{i\,\mathrm{Im}\ln\Gamma(\frac{s}{2})}\pi^{-\frac{it}{2}}\zeta(\frac{1}{2}+it)]$$

Obviously, since the expression in the first square bracket is always negative, it can be ignored when calculating the zero point of ξ(s). This shows that in order to determine the non-trivial zeros of the Riemann zeta function, it is really only necessary to study the expression inside the second square brackets in the equation. Label this expression with Z(t), i.e.

$$\mathrm{Z}(t)=e^{i\,\mathrm{Im}\ln\Gamma(\frac{s}{2})}\pi^{-\frac{it}{2}}\zeta(\frac{1}{2}+it)$$

So far, the non-trivial zeros of the Riemann zeta function can be attributed to the zeros of Z(t).

The Riemann-Siegel formula is the asymptotic expansion of Z(t), which is specifically expressed as [2]

$$\mathrm{Z}(t)=2\sum_{\mathrm{n}^2<(\frac{t}{2\pi})}\mathrm{n}^{-\frac{1}{2}}\cos(\theta(t)-t\ln(n))+R(t) \qquad (1)$$

Where:

$$\theta(t)=\frac{t}{2}\ln(\frac{t}{2\pi})-\frac{t}{2}-\frac{\pi}{8}+\frac{1}{48t}+\frac{7}{5760t^3}+...$$

$$R(t) \sim (-1)^{N-1}(\frac{t}{2\pi})^{-\frac{1}{4}}[C_0 + C_1(\frac{t}{2\pi})^{-\frac{1}{2}} + C_2(\frac{t}{2\pi})^{-\frac{2}{2}} + C_3(\frac{t}{2\pi})^{-\frac{3}{2}} + C_4(\frac{t}{2\pi})^{-\frac{4}{2}}] \quad (2)$$

R(t) in the formula is called the remainder, where N is the integer part of $(t/2\pi)^{1/2}$, and the coefficients of each item in R(t) are

$$C_0 = \Psi(p) \equiv \frac{\cos(2\pi(p^2 - p - \frac{1}{16}))}{\cos(2\pi p)}$$

$$C_1 = -\frac{1}{2^5 \times 3 \times \pi^2}\Psi^{(3)}(p)$$

$$C_2 = \frac{1}{2^{11} \times 3^2 \times \pi^4}\Psi^{(6)}(p) + \frac{1}{2^6 \times \pi^2}\Psi^{(2)}(p)$$

$$C_3 = -\frac{1}{2^{16} \times 3^4 \times \pi^6}\Psi^{(9)}(p) + \frac{1}{2^8 \times 3 \times 5 \times \pi^4}\Psi^{(5)}(p) - \frac{1}{2^6 \times \pi^2}\Psi^{(1)}(p)$$

$$C_4 = \frac{1}{2^{23} \times 3^5 \times \pi^8}\Psi^{(12)}(p) + \frac{11}{2^{17} \times 3^2 \times 5 \times \pi^6}\Psi^{(8)}(p) + \frac{19}{2^{13} \times 3 \times \pi^4}\Psi^{(4)}(p) + \frac{1}{2^7 \times \pi^2}\Psi(p)$$

where p is the fractional part of $(t/2\pi)^{1/2}$ and $\Psi^{(n)}(p)$ is the n-th derivative of Ψ(p).

2 Negative proof of Riemann's hypothesis

The Riemann hypothesis is that when s=1/2+it, where t is a real number, the value calculated by the function (1) is zero, and s=1/2+it at this time is a non-trivial zero point. Because p in the calculation formula (2) of the remaining term R(t) and the three terms

$(\frac{t}{2\pi})^{-\frac{1}{4}}$, $C_1(\frac{t}{2\pi})^{-\frac{1}{2}}$, and $C_3(\frac{t}{2\pi})^{-\frac{3}{2}}$

belong to the non-rational radical form, which are all inaccurate numbers [3]; and because if the two inaccurate numbers are not twin inaccurate numbers [4] or dual form, when they are added or multiplied as Inaccurate number, the result of operation with other numbers is still inaccurate number. As a result, no matter what the real number t is, the result of function (1) cannot be zero.

Therefore, for any real number t, s(1/2+it) is not a non-trivial zero point, that is, the Riemann hypothesis is denied. The Riemann ζ(s) function has only inverse singularities without exact values, namely infinitesimal points. This also shows that there is no function expression that can accurately express the number of prime numbers less than a certain integer x, but only the actual number that is relatively close to the prime number.

References

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