A Disproof of Strong Goldbach's Conjecture

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Abstract In this paper it is going to be proved that strong Goldbach's conjecture can not hold. The proof is based on fundamental theorem of arithmetic.

1 Introduction

The fundamental theorem of arithmetic states that every integer greater than 1 can be uniquely represented by a product of powers of prime numbers, up to the order of the factors [1]. Here, we are going to mark infinite vector that contains all prime numbers with **p**. So, $p(1) = 2$, $p(2) = 3$, $p(3) = 5$, and so on. Now every natural number n can be defined by the following equation (i_1, i_2, i_3, … are positive integers; it can be noticed that number 1 is obtained when all exponents i_1, i_2, i_3, … are equal to 1):

$$n = p(1)^{i_1 - 1} p(2)^{i_2 - 1} p(3)^{i_3 - 1} \dots \quad .$$

It can be seen that every natural number can be uniquely represented by infinite vector that contains exponents i_1, i_2, i_3, … (in that case order of factors has to be fixed).

The Goldbach's conjecture (strong version) states that every even natural number bigger than 4 can be expressed as a sum of two odd prime numbers [2]. (Original formulation includes number 4 too – however, it is clear that 4 can only be expressed as the sum of 2 and 2, or as the sum of only even prime number by itself, and that is a special, and only, case when the even prime is used). Now it is going to be proved that strong Goldbach's conjecture cannot hold.

2 A proof that strong Goldbach's conjecture cannot hold

In order to prove that strong Goldbach's conjecture cannot hold, we are going to analyze a multiplication (Table 1) and an addition table (Table 2). Since we are going to focus on the number of numbers presented in the tables, the tables will not contain numbers themselves. Vector **p** is previously defined as vector that contains all prime numbers with elements $p(1) = 2, p(2) = 3, p(3) = 5$, and so on.

Table 1. Simple multiplication table for odd prime numbers

●	$p(2)$	$p(3)$	$p(4)$	$p(5)$	...
$p(2)$	$p(2)^2$	$p(2)\cdot p(3)$	$p(2)\cdot p(4)$	$p(2)\cdot p(5)$	...
$p(3)$	x	$p(3)^2$	$p(3)\cdot p(4)$	$p(3)\cdot p(5)$	...
$p(4)$	x	x	$p(4)^2$	$p(4)\cdot p(5)$	...
$p(5)$	x	x	x	$p(5)^2$	...
...	...	...	...	...	...

Table 2. Simple addition table for odd prime numbers

+	$p(2)$	$p(3)$	$p(4)$	$p(5)$	...
$p(2)$	$p(2)+p(2)$	$p(2)+p(3)$	$p(2)+p(4)$	$p(2)+p(5)$	...
$p(3)$	x	$p(3)+p(3)$	$p(3)+p(4)$	$p(3)+p(5)$	...
$p(4)$	x	x	$p(4)+p(4)$	$p(4)+p(5)$	...
$p(5)$	x	x	x	$p(5)+p(5)$	...
...	...	...	...	...	...

It can be seen that lower triangular part of Table 1 and Table 2 is filled with x (we are ignoring them), since it would contain the values already contained in the upper triangular part of the tables due to the commutative nature of operations of addition and multiplication. From Table 1 and Table 2 it can be easily concluded that both tables contain the same number of elements (not marked by x). From fundamental theorem of arithmetic, it is known that all products in Table 1 have unique values (that is not the case with the sums in the Table 2, but it has no influence on the final conclusion, since it means that the number of unique numbers presented in Table 2 is smaller than the number of all numbers presented in Table 2). From fundamental theorem of arithmetic it is known that all odd numbers can be uniquely expressed in the following form

$$n_{odd} = p(2)^{i_1 - 1} p(3)^{i_2 - 1} \ldots \quad ,$$

where i_1, i_2, i_3, … are positive integers. Having this in mind, it can be seen that the unique products in Table 1 represent the odd natural numbers whose all exponents of the prime factors are smaller than 3, it can be concluded that exists infinitely many odd numbers that are not presented by the numbers in Table 1 (all odd numbers that are defined by the odd prime factors, where at least one prime factor exponent is bigger than 2). It is well know fact that the number of odd natural numbers is equal to the number of even natural numbers. Since the number of sums in the Table 2 is equal to the number of products in the Table 1 we can conclude that exists infinitely many even numbers that cannot be expressed by the sums presented in Table 2 and then it is easy to conclude that strong version of Goldbach's conjecture cannot hold. It concludes the proof.

References

[1] G.H. Hardy, E.M. Wright. (2008)[1938] An Introduction to the Theory of Numbers. Revised by D.R. Heath-Brown and J.H. Silverman. Oxford University Press.

[2] R.K. Guy, (1994) Unsolved Problems in Number Theory, 2nd ed, New York: Springer Verlag, pp. 105-107.