

# Why the JWST won't see the first galaxies

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This paper explains why the JWST won't see the first galaxies. Among the farthest galaxies it sees will be ones that look much older than expected. The case is made that, rather than assume that galaxies can mature faster than we thought, we should adopt a model of the universe in which space itself doesn't expand. The new model is based on a new, experimentally confirmed metric for Schwarzschild geometry. Unlike the Schwarzschild metric, the new metric obeys the equivalence principle, so it can explain our observations when space itself doesn't expand. In the new model: space expands relatively, so that the expansion depends on the observer; an object thrown upward can accelerate up; black holes aren't predicted, so there's no black hole information paradox; and there's no flatness problem, horizon problem, or need for dark energy. Code is given to numerically integrate the relativistic motion of an object thrown upward. The code shows that equations of special relativity approximate the new metric locally, as required for the metric to obey the equivalence principle.

## 1 An object thrown upward can accelerate up

A formal statement of the Einstein [equivalence principle](#) (EP) is

In any and every locally Lorentz (inertial) frame, the laws of special relativity must hold.

See the equations of special relativity (SR) at [The Relativistic Rocket](#), for a rocket having a constant proper acceleration  $a > 0$ . For example:

$$v = \frac{at}{\sqrt{1 + (at/c)^2}} \quad (1)$$

This equation returns the velocity of a rocket after the time  $t$ , both as measured in the local inertial frame (LIF) in which the rocket launched. The speed of light is  $c$ .

[Equations for a falling body](#) has the velocity of a free-falling object that was dropped in a uniform gravitational field (ignoring air resistance) as

$$v_{\text{old}} = at \quad (2)$$

where  $a$  is the acceleration of gravity and  $t$  is the elapsed time. We call this  $v_{\text{old}}$  since it can be  $\geq c$ .

The EP shows that The Relativistic Rocket's (1) supplants (2):

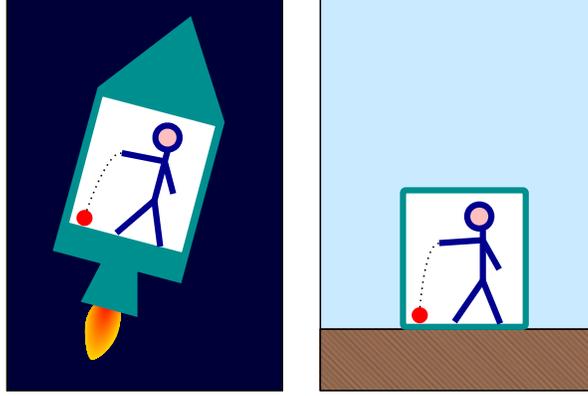


Figure 1: Ball falling to the floor in an accelerating rocket (left) and on Earth (right). By [Pbroks13 / Markus Poessel \(Mapos\), CC BY-SA 3.0](#), via Wikimedia Commons.

According to the EP, the laws of SR hold in both scenarios in Fig. 1. The Relativistic Rocket equations describe the ball's motion relative to the rocket. Then those equations describe the ball's motion relative to the room on Earth as well, where  $a$  is the acceleration of gravity. The time  $t$  in (1) is measured in the ball's LIF, which momentarily co-moves with the rocket or room when the ball is dropped at  $t = 0$  (as if the rocket or room blasts off then).

**Experiment #1:** A ball is thrown upward from the ground to a maximum height  $H = 10$  kilometers in a hypothetical  $1 g$  uniform gravitational field.

Let's plot the ball's rise and fall in the ground's frame, using the following Relativistic Rocket equations:

$$T = \frac{c}{a} \operatorname{acosh}(ad/c^2 + 1) \quad (3)$$

$$\gamma = \cosh \frac{aT}{c} \quad (4)$$

$$d = \frac{c^2}{a} \left( \cosh \frac{aT}{c} - 1 \right) = \frac{c^2}{a} (\gamma - 1) \quad (5)$$

The thrower is like the person in the rocket in Fig. 1. The elapsed time in their frame (i.e. how much they age) is  $T$ . Do a one-time calculation to get the time taken for the ball to fall from its apex to the ground, using (3):

$$T_a = \frac{c}{a} \operatorname{acosh}(aH/c^2 + 1) \quad (6)$$

Use  $c = 299,792,458$  m/s,  $a = 9.80665$  m/s<sup>2</sup>, and  $H = 10,000$  m. The ground accelerates like a rocket to reach the ball after the time  $T_a$ , which is also the time for the ball to rise from the ground to its apex. So you need many values of  $T$  that range from 0 to  $2T_a$ .

Calculate the [gamma factor](#)  $\gamma$  for each value of  $T$ , using (4) with an offset to the time:

$$\gamma = \cosh \frac{a(T_a - T)}{c} \quad (7)$$

Calculate  $d$  for each value of  $T$ , using (5) with an offset to the distance, and the  $\gamma$  from (7):

$$d = H - \frac{c^2}{a} (\gamma - 1) \quad (8)$$

We'll plot  $h$ , the ball's height in the ground's frame, versus  $T$ . The distance  $d$  is measured in the ball's frame; in the ground's/rocket's frame it's length contracted by the gamma factor  $\gamma$ , so

$$h = \frac{d}{\gamma} \quad (9)$$

Now we can make the plot, where  $h$  is in kilometers:

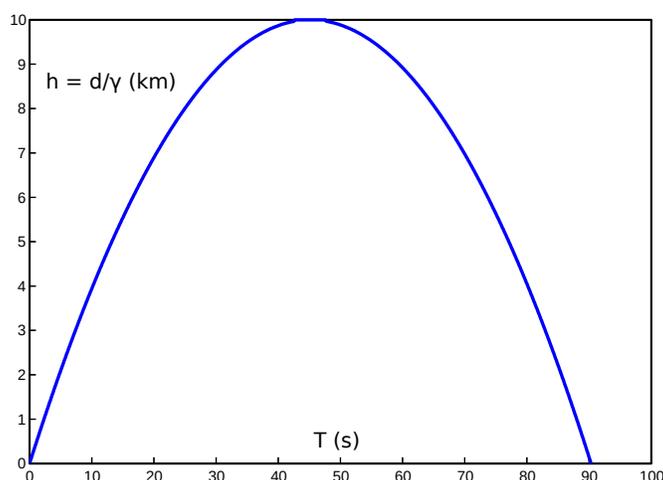


Figure 2: Ball thrown upward from the ground to a maximum height  $H = 10$  km in a  $1 g$  uniform gravitational field.

Compare Fig. 2 to a plot drawn by a [projectile motion calculator](#) that uses Newton's equations for a uniform gravitational field. They look alike, because Newton's equations approximate the Relativistic Rocket equations when the object is thrown upward at a speed less enough than  $c$ .

**Experiment #2:** A ball is thrown upward from the ground to a maximum height  $H = 10$  light years in a hypothetical  $1 g$  uniform gravitational field.

We'll again plot the ball's rise and fall in the ground's frame. Use  $c = 1$  ly/yr,  $a = 1.03$  ly/yr<sup>2</sup>  $\approx g$ , and  $H = 10$  ly.

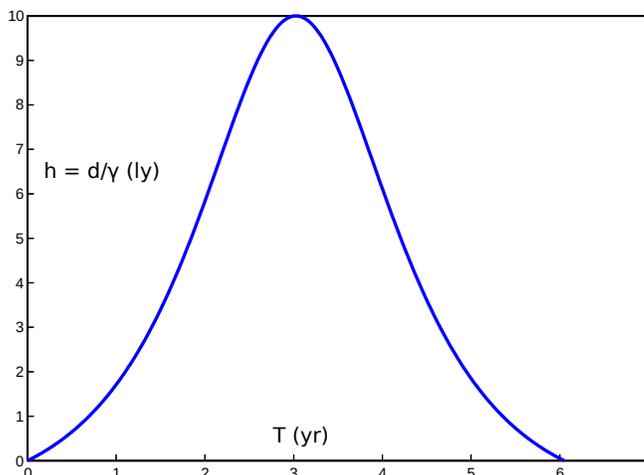


Figure 3: Ball thrown upward from the ground to a maximum height  $H = 10$  ly in a  $1 g$  uniform gravitational field.

See that Fig. 3 looks similar to the plot in the section “The Equivalence Principle and a Stone Thrown Upwards” at The Relativistic Rocket. (The site uses  $a = 1 \text{ ly/yr}^2$  there. It also negates the distance  $d$  since initially the ground is like a rocket decelerating past the ball; we needn’t follow that convention.) A projectile motion calculator that uses Newton’s equations would invalidly plot a parabola.

Fig. 3 shows that the ball accelerates up initially, and decelerates down as it nears the ground.

## 2 Space expands and contracts relatively

The ball accelerates up in Fig. 3 due to the reverse of [length contraction](#) (hereafter called length expansion) of the space between the ground and the ball, that occurs in the ground’s frame. Throw a ball upward, while imagining that a string dangles down from it. The string moves relative to you, so it’s length contracted in your frame, however slightly. The string expands toward its proper length while passing you, so some of the ball’s movement away from you is due to the length expansion of the string. As this length expansion still happens when you remove the string from the picture, it’s simpler seen as an expansion of space that occurs between you and a free-falling object that rises away from you. It’s a relative expansion that causes the object to accelerate up when the expansion is large enough. The reverse happens between you and a free-falling object that falls toward you. In that case the space between you and the object relatively contracts due to length contraction. This causes the ball to decelerate down in Fig. 3.

In Fig. 3 the ball reaches a height of 10 light years in a little over 3 years. So the ball moves at an average speed  $> 3c$  in the ground’s non-inertial frame. In the ball’s LIF the ground’s speed  $v$  is always  $< c$ ; the Relativistic Rocket equations can’t return a value to the contrary. The speed of light is guaranteed to be  $c$  in, and only in, a perfect inertial frame. This means that in our

real, gravity-endowed universe, light's speed is guaranteed to be  $c$  only when measured right next to it. However, we treat LIFs as perfect inertial frames, so the speed of light in a LIF is (treated to be)  $c$ . In a non-inertial frame it can be arbitrarily high in principle, and therefore so can an object's speed. (See also [Is the Speed of Light Everywhere the Same?](#))

The Relativistic Rocket has an equation to predict when an object will accelerate up or decelerate down (in which case its plot of  $h$  vs.  $T$  will be bell-shaped, if only slightly):

$$a > (\sqrt{2} - 1) c^2/H \quad (10)$$

and says “This means that if you want to perform such an experiment, either  $[a]$  or  $H$  will have to be very large.” Note that the minimum  $a$  is inversely proportional to  $H$ .

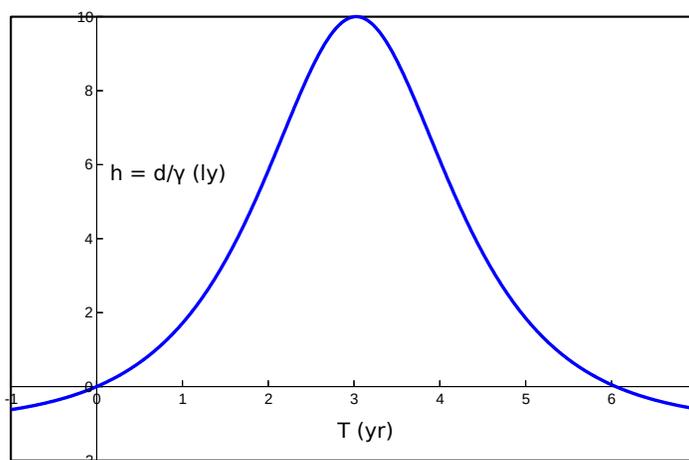


Figure 4: This is Fig. 3 with extended tails.

Shine light upward. Then the maximum height  $H$  in (10) is infinite, so the light accelerates away in your frame (a ball thrown upward at close to  $c$  is initially a proxy for the light that recedes faster). The right tail of Fig. 4 shows that light shone downward decelerates away. (The light asymptotes to your Rindler horizon. See Fig. 1, and the section “Below the rocket, something strange is happening” at [The Relativistic Rocket](#).) Measure the height of a building using a [laser rangefinder](#), from both the top and bottom of the building. This measures the time taken by a laser pulse to reflect off a target and return to the device. Whether the height is measured from the top or bottom, the same set of one-way pulses are used; only their order differs. The height measures taller from the top when the rangefinder is precise enough. We know this because clocks run faster there due to gravitational time dilation, hence more time elapses between the emission and reception of the same set of one-way pulses (see the section “And inside the rocket, something strange is also happening” at [The Relativistic Rocket](#)). Fig. 4 shows what happens to the light. When the height is measured from the bottom, the light accelerates up and decelerates down, its speed averaging  $> c$ . When the height is measured from the top, the light decelerates down and accelerates up, its speed averaging  $< c$ . The proper height of the building is between the two heights reported by the rangefinder.

### 3 Numerical integration for a non-uniform gravitational field

The Relativistic Rocket equations apply only to uniform gravitational fields. [Here is code](#) to numerically integrate the relativistic motion of a ball thrown upward in a real (non-uniform) gravitational field. Click the Run button to get the output:

```
The ball apexes at 1.0000e+01 ly after 3.0230e+00 yr
Its velocity v = 0.3559% of c at the apex
```

This result approximates the expected result for experiment #2, as depicted by Fig. 3. The program stops when the ball apexes. You can change the inputs at the top of the program, as explained therein. The closer the reported velocity  $v$  at the apex is to the expected zero, the better the approximation. To get closer to  $v = 0$  increase the `stepCount`, the number of steps in the numerical integration. At a high enough `stepCount` the program times out when run online. You can install the [Go language](#) to run the program on your own machine. The program outputs data points for  $T$  and  $h$  when `outputCount > 0`.

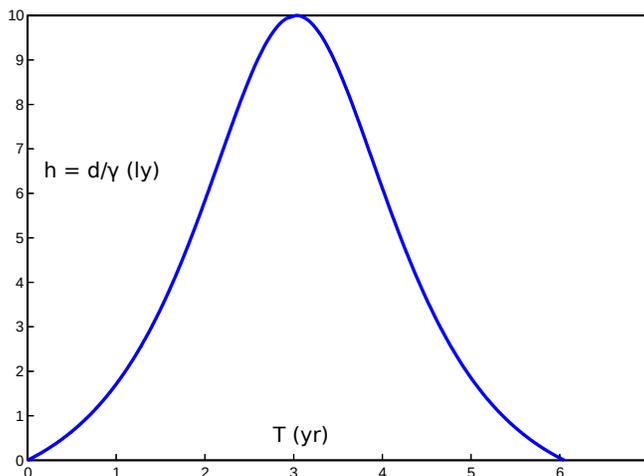


Figure 5: This is Fig. 3, except that the left half of the plot uses the data output by the numerical integration program, for a gravitational field where the acceleration of gravity stays close to  $1g$ .

[Here is code](#) for a helper program to get inputs for the numerical integration program. Click the Run button to get the output:

```
Use these inputs for the numerical integration program:
M = 1.0300e+12 ly
rMin = 1e+06 ly
rDiff = 6.1500e+01 ly
Acceleration of gravity at
rMin = 1.0300e+00 ly/yr^2
rMax = 1.0299e+00 ly/yr^2
The ball apexes at 1.0000e+01 ly after 3.0252e+00 yr
```

This program helps especially for getting inputs for a gravitational field that's close to uniform. The output above is for experiment #2, using [geometric units](#). See that the acceleration of gravity at `rMin` is about the same as at `rMax` ( $= rMin + rDiff$ ). Increase the `uniformityLevel` to make the gravitational field more uniform. Decrease the `uniformityLevel`, even to a negative value, for a less uniform field. But then don't expect the results at "The ball apexes...", which are for a perfectly uniform gravitational field, to approximate those from the numerical integration program.

Let's use the helper program to get inputs for the numerical integration program for experiment #1. Change the inputs at the top to `a = 9.80665`, `H = 10000`, `Mlabel = "kg"`, `dLabel = "m"`, `tLabel = "s"`, `G = 6.67430e-11`, and `c = 299792458`. Set `uniformityLevel = 5`. Run the program. The output includes

```
Use these inputs for the numerical integration program:
  M = 1.4693e+27 kg
  rMin = 1e+08 m
  rDiff = 1.0000e+04 m
The ball apexes at 1.0000e+04 m after 4.5160e+01 s
```

Input those values into the numerical integration program, along with the values above for `dLabel`, `tLabel`, `G`, and `c`. Set `stepCount = 1e+7`. Run the program. The output is

```
The ball apexes at 1.0000e+04 m after 4.5154e+01 s
Its velocity v = 0.0000% of c at the apex
```

When the following values for Earth are input:

```
M = 5.9724e+24 kg
rMin = 6.3781e+06 m
```

the output is

```
The ball apexes at 1.0000e+04 m after 4.5227e+01 s
Its velocity v = 0.0000% of c at the apex
```

Here are finer points of the numerical integration program:

The `rDiff`, the  $r$ -coordinate (radial coordinate or reduced circumference) distance, doesn't mean the same as the maximum height `H` that's input into the helper program. The height `H` is the distance  $d$  covered by the ground in the ball's frame. The helper program finds `rDiff` for a uniform gravitational field by using the Relativistic Rocket equation

$$t = \sqrt{(d/c)^2 + 2d/a} \quad (11)$$

The inputted `H` is plugged in for  $d$ . This gives the elapsed time in the ball's frame. Then that value for  $t$  is plugged into

$$d = \frac{1}{2}at^2 \quad (12)$$

from [Equations for a falling body](#). This gives the  $r$ -coordinate distance, or  $r_{\text{Diff}}$ . This distance is length contracted in the ball's frame until the ball apexes (as shown by the [muon experiments](#)), so  $r_{\text{Diff}} > H$ .

The code is similar to that in [How the twins each age less than the other](#) for a twin paradox experiment. The thrower is a traveling twin, as if decelerating away from the ball in a rocket. The ball is a stationary twin, at rest relative to the starting and ending points between which the rocket/ground moves. The ground's speed  $v$  in the ball's frame is given by

$$v_i = \sqrt{2GM \left( \frac{1}{r} - \frac{1}{r+d} \right)} \quad (13)$$

from [Equations for a falling body](#), described therein as the “Instantaneous velocity  $v_i$  of a falling object that has travelled distance  $d$  on a planet with mass  $M$  and radius  $r$ .” We can use this speed as well for the ball that rises, since an object's fall reverses its rise. Except that this speed isn't relativistic/valid because it can be  $\geq c$ . So it's converted to a relativistic speed using

$$v = \frac{v_{\text{old}}}{\sqrt{1 + (v_{\text{old}}/c)^2}} \quad (14)$$

This equation derives by substituting the two terms  $at$  in (1) with the  $v_{\text{old}}$  from (2). This is the same as dividing  $v_{\text{old}}$  by its gamma factor  $\gamma$ , as shown by

$$\gamma = \sqrt{1 + (at/c)^2} \quad (15)$$

from The Relativistic Rocket. That is, a Newtonian speed  $v_{\text{old}}$  converts to a relativistic speed  $v$  by dividing it by its gamma factor, using (15), which accepts a Newtonian speed. This conversion effectively changes general relativity's (GR's) equation for [escape velocity](#) (which is (13) when  $d$  is infinite) such that it returns a value  $< c$  for an  $r$ -coordinate  $r > 0$ . So black holes aren't predicted. More on this below. See also the corresponding new metric for Schwarzschild geometry at [A solution to the black hole information paradox](#).

The message “Reduce the stepCount” means that division by zero would otherwise occur, due to limited precision.

## 4 Space itself expanding is superfluous

The [gamma factor](#) equation shows that all objects have a gamma factor  $\gamma$  as a function of their velocity. As we on Earth measure, a radially moving free-falling object is length contracted by its current factor of  $\gamma$  along that axis of motion, and so is its proper distance from us. For example, both the Andromeda galaxy (which approaches us) and its proper distance from us are somewhat length contracted as we measure, by the same gamma factor. Imagine that a string of unconnected beads, that are all momentarily at rest relative to Andromeda, extends from it all the way to you. See that the string is length contracted. (The tidal force is ignorable, hence the gravitational field needn't be uniform, since we're using only the instantaneous values  $\gamma$  and the

speed  $v$ —it takes time for the tidal force to distort a measurement. By using unconnected beads, the string isn't stretched by the tidal force either.) The space (or string of momentarily co-moving beads) between you and the object either further contracts as the object approaches and its speed  $v$  increases, due to gravity, or expands as the object recedes and its speed  $v$  decreases.

Let a rocket launch from Earth and accelerate and decelerate at  $1 g$  to arrive at the center of our galaxy, 30,000 light years away from us, at relative rest. The Relativistic Rocket shows that its crew ages just  $\sim 20$  years (see “Here are some of the times you will age when journeying to a few well known space marks”). At the midpoint of the trip, 15,000 light years away from the center as we measure, the rocket is  $< 10$  light years away from the center as the crew measures, due to length contraction; that's how they'll have aged just  $\sim 10$  more years by when they reach the center, despite moving at  $< c$  relative to stars passing by. The whole Milky Way is contracted by the same gamma factor, lest it be physically deformed. Let a ball be free-floating at the midpoint. A plot of the ball's height over time, in the rocket's frame during the deceleration half of the trip, is the left half of the plot given by (6)-(9).

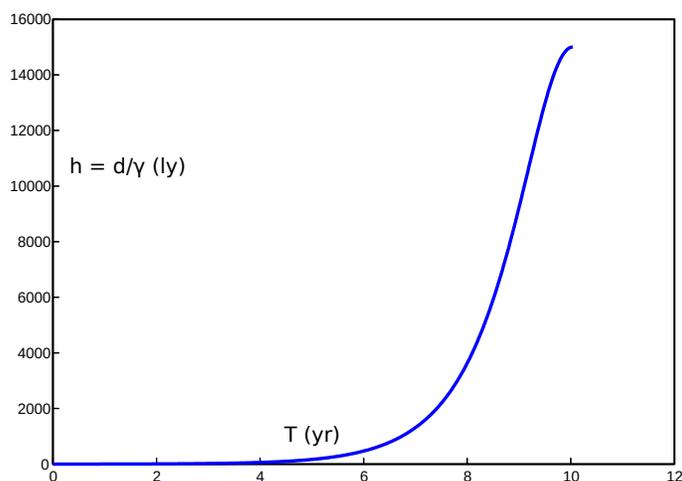


Figure 6: Ball passed by a rocket at the midpoint of a trip from Earth to the center of our galaxy, arriving at relative rest. The rocket decelerates at  $1 g$ .

Fig. 6 shows that the ball accelerates up initially. This is due to length expansion of the space between the rocket and the ball. The whole Milky Way length expands by the same change in the gamma factor. When the rocket arrives at the center of the galaxy at relative rest, the galaxy is no longer length contracted. If the rocket kept its engines running and accelerated back to the ball, then the plot would be the full bell-shaped plot given by (6)-(9). This is as if the crew had thrown the ball (or Milky Way) upward to a maximum height  $H = 15,000$  light years, and then it fell back to the rocket/“ground”, all in a time  $T \approx 20$  years.

Since a whole galaxy length contracts when a rocket accelerates across it, why isn't a whole black hole contracted to zero length when an observer reaches its event horizon at the speed of light? That's an illogical discrepancy. When the whole galaxy contracts, so must any black holes

in it, lest the galaxy physically deform. If the numerical integration program didn't do the conversion described above using (14), which effectively changes GR's equation for escape velocity such that black holes aren't predicted, then it couldn't predict that an object thrown upward can accelerate up, and otherwise wouldn't always approximate (6)-(9) in a uniform gravitational field (i.e. locally, where the ground accelerates in a LIF). This shows that the Schwarzschild metric's prediction of black holes violates the EP. More on this below.

Our cosmology is illogical without incorporating the idea that space expands and contracts relatively at cosmic scales. For the rocket that travels to the center of our galaxy, our [Local Group](#), and so on outward, must length contract in the crew's frame during the acceleration half of the trip, or else there'd be a physical deformity somewhere. Reversely, the Local Group, and so on outward, must length expand in their frame during the deceleration half of the trip. Below, the idea that space itself expands is rejected as superfluous. Then only our decreasing speed  $v$  relative to receding galaxies, due to gravity, explains the (relative) expansion of space and concomitant stretching of light that we observe. This eliminates the [flatness problem](#), which depends on the assumption that space itself is expanding. The Relativistic Rocket says "For distances greater than about a thousand million light years, the formulae given here are inadequate because the universe is expanding." When space expands only relatively then spacetime can be asymptotically flat for any distance in principle, thus SR can apply at any scale. Parallel light rays can remain parallel for any distance. The new metric for Schwarzschild geometry can also apply at any scale.

Space itself needn't expand in order for most or all galaxies to recede from one another. Sprinkle an infinite universe, in which space itself neither expands nor contracts, with countless galaxies. Nothing in principle prevents all of the galaxies from coalescing under gravity. "Run the film backward" to see that nothing in principle prevents all of the galaxies from receding from one another as well.\*

## 5 An object thrown upward in a highly non-uniform gravitational field

Let's use the numerical integration program to plot a ball thrown upward to a great height in a highly non-uniform gravitational field. Change the inputs at the top to  $M = 1e+14$ ,  $rMin = 1e+10$ ,  $rDiff = 1e+12$ ,  $dLabel = "ly"$ ,  $tLabel = "yr"$ ,  $G = 1$ , and  $c = 1$ . Set  $stepCount = 1e+7$ . Run the program. The output is

```
The ball apexes at 1.0328e+11 ly after 2.9044e+10 yr
Its velocity v = 0.4428% of c at the apex
```

Increasing the  $stepCount$  to  $2e+10$  (on one's own machine) improves the results to

```
The ball apexes at 1.0328e+11 ly after 2.9077e+10 yr
Its velocity v = 0.0099% of c at the apex
```

---

\* For an informative description of an expanding flat universe, see the chapter "The Big Bang" in the book *Relativity Visualized*, by [Lewis Carroll Epstein](#).

which gives the plot:

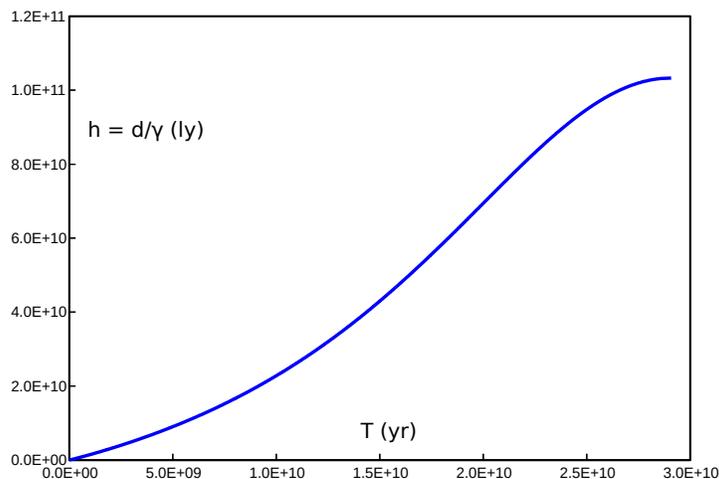


Figure 7: Ball thrown upward to a great height in a highly non-uniform gravitational field.

The ball apexes at  $\sim 103$  billion light years after  $\sim 29$  billion years. The acceleration of gravity ranges from  $\sim 1$  one-millionth of  $g$  at  $r_{\text{min}}$  (the ground, if only the “surface” of a sphere that contains the mass  $M$ ) to  $\sim 1$  ten-billionth of  $g$  at the ball’s apex. So see that an object can accelerate up (due to a large *relative* expansion of space) even when the acceleration of gravity is small, as predicted locally by (10).

## 6 More on the Schwarzschild metric’s violation of the equivalence principle

The Relativistic Rocket says about the EP:

Einstein postulated that any experiment done in a real gravitational field—provided that experiment has a “small” extent in space and time—will give a result indistinguishable from the same experiment done in the above “uniformly accelerating” rocket.

For example, an experiment done in either scenario in Fig. 1.

For experiment #2 for a uniformly accelerating rocket (the result for which is depicted by Fig. 3), we got an indistinguishable result (depicted by Fig. 5) in a real gravitational field with a negligible tidal force only by effectively changing GR’s equation for escape velocity using (14), to convert the speed it returns to a relativistic speed. Without that change, an object thrown upward (or light shone upward) can’t accelerate up. Eq. (10) shows that the Schwarzschild metric, or equations derived from it, approximate (6)-(9) in a LIF (to obey the EP) only when both the acceleration of gravity  $a$  and the maximum height  $H$  are small enough, so that the initial or final speed  $v$  is less enough than  $c$ . This means that the Schwarzschild metric violates the EP in every LIF.

## 7 Why the JWST won't see the first galaxies

Eqs. (6)-(9) and the numerical integration program show that

- An object can in principle be thrown upward to any height and fall back to the ground, all in an arbitrarily short time  $T$ . Then an object or light can in principle rise from the ground to any height, or fall from any height to the ground, in an arbitrarily short time  $T$ . This eliminates the [horizon problem](#). A region of space that's any distance from us can have been in causal contact with a region of space that's any distance from us in the opposite direction, since any age  $> 0$  of our length-expanding universe as we measure. The [observable universe](#) can now be arbitrarily large.
- Galaxies that recede from us at a speed  $v$  close enough to  $c$  accelerate away as we measure. This obviates the need for [dark energy](#). The evidence for dark energy includes high-redshift supernovae that are accelerating away from us. They can simply be receding from us fast enough, rather than dark energy needed. (Space expands only relatively in the new model of the universe, so recession is in no way due to space itself expanding.)

It follows that the [first galaxies](#) can in principle be much farther away or much more redshifted than the current model predicts. For this reason, and because we've already seen galaxies that look much older than expected, such as [ALESS 073.1](#), likely the JWST won't be able to image the first galaxies.

By changing the Schwarzschild metric to obey the EP, and changing cosmology so that space expands only relatively, thereby letting SR and the new metric apply at any scale, we can predict the anomalous observations of the JWST and solve several problems in cosmology.

### Appendix A - Code for the programs

Below is the [Go language](#) code for the numerical integration program that's referenced in section 3, in case the link to the code is broken. You can run the code at the [Go Playground](#) after fixing the formatting.

```
package main

import (
    "fmt"
    "math"
)

const (
    // The mass of the massive body
    M = 1.0300e+12

    // The ground's radial coordinate (r-coordinate)
    rMin = 1e+06

    // The ball is thrown upward from rMin and apexes at rMin + rDiff
```

```

rDiff = 6.1500e+01

// The number of steps in the numerical integration below. More
steps gives greater accuracy
stepCount = 1e+7

// The distance label used for output
dLabel = "ly"

// The time label used for output
tLabel = "yr"

// The gravitational constant
G = 1

// The speed of light
c = 1

// The number of data points to output
outputCount = 0

// End of user input

// The maximum r-coordinate
rMax = rMin + rDiff

// The r-coordinate distance the ball covers in each step
rStep = rDiff / stepCount
)

func main() {
// Test whether "+Inf" would be output, due to limited precision
if rMin + (float64(stepCount - 1) * rStep) == rMax {
    fmt.Println("Reduce the stepCount")
    return
}

outputEvery := 0
if outputCount > 0 {
    // Output data every outputEvery-th iteration
    // Adjust for: The origin and last data points are always
output
    outputCount2 := math.Max(1, float64(outputCount) - 2)
    outputEvery = int(math.Ceil(stepCount / outputCount2))
    fmt.Printf("T, h = d/\gamma r\n") // Print data header
    fmt.Printf("%0.4e, %0.4e\r\n", 0.0, 0.0) // Print origin
data point
}

// The twins in this twin paradox experiment are the ball and the
thrower
// The thrower is like the crew of a rocket, which in this case is
the ground
d := 0.0 // The distance the ground covers in the ball's frame
T := 0.0 // The thrower's aging

```

```

var v float64 // Their velocity relative to each other
var gamma float64 // The gamma factor

for i := 0; i < stepCount; i++ {
    r := rMin + (float64(i) * rStep)

    // From
https://en.wikipedia.org/wiki/Equations\_for\_a\_falling\_body
    vOld := math.Sqrt((2 * G * M) * ((1 / r) - (1 / rMax)))

    // From the Relativistic Rocket site
    gamma = math.Sqrt(1 + math.Pow(vOld / c, 2))

    // Convert vOld to a relativistic velocity, as explained in
the paper
    v = vOld / gamma

    // Get the distance the ground covers in the ball's frame
during this step
    // rStep is divided by gamma because r-coordinate distances
are length contracted in the ball's frame
    // As the ball rises, r-coordinate distances length expand
in its frame
    // For example, see the muon experiment at
https://en.wikipedia.org/wiki/Experimental\_testing\_of\_time\_dilation
    // As the muon falls, r-coordinate distances further length
contract in its frame
    dStep := rStep / gamma

    d += dStep

    // During this step:
    // dStep / v is the elapsed time in the ball's frame
    // dStep / gamma is the distance the ball covers in the
ground's frame
    // dStep / (v * gamma) is the thrower's aging
    T += dStep / (v * gamma)

    if outputCount > 0 {
        if i % outputEvery == 0 || i == (stepCount - 1) {
            fmt.Printf("%0.4e, %0.4e\r\n", T, d / gamma)
        }
    }
}

fmt.Printf("The ball apexes at %0.4e %s after %0.4e %s\r\n", d /
gamma, dLabel, T, tLabel)
fmt.Printf("Its velocity v = %0.4f%% of c at the apex\r\n", (v /
c) * 100)
}

```

Below is the Go language code for the helper program that's referenced in section 3.

```
package main
```

```

import (
    "fmt"
    "math"
)

const (
    // The acceleration of gravity at rMin
    a = 1.03

    // The ball's maximum height
    H = 10

    // The mass label used for output
    MLabel = "ly"

    // The distance label used for output
    dLabel = "ly"

    // The time label used for output
    tLabel = "yr"

    // The gravitational constant
    G = 1

    // The speed of light
    c = 1

    // Increase this number to make the gravitational field more
    uniform, at the expense of larger numbers output for "Use these inputs"
    // Decrease this number (even to a negative value) to make the
    gravitational field less uniform
    uniformityLevel = 5

    // End of user input

    aLabel = dLabel + "/" + tLabel + "^2"
)

func main() {
    // These equations are from the Relativistic Rocket site
    // The ball's aging
    tExpected := math.Sqrt(math.Pow(H / c, 2) + 2 * H / a)
    // The thrower's aging
    TExpected := (c / a) * math.Acosh(a * H / math.Pow(c, 2) + 1)

    // Calculate the difference d between rMin and rMax, given t
    // This equation is from
https://en.wikipedia.org/wiki/Equations\_for\_a\_falling\_body
    rDiff := a * math.Pow(tExpected, 2) / 2

    // Let the acceleration of gravity at rMin be the acceleration a
    // Calculate an rMin where the acceleration of gravity at rMax
    won't differ much
    rMin := math.Pow(10, math.Floor(math.Log10(rDiff)) +
    uniformityLevel)
}

```

```
M := math.Pow(rMin, 2) * a / G

rMax := rMin + rDiff

fmt.Printf("Use these inputs for the numerical integration
program:\n")
fmt.Printf("\tM = %0.4e %s\n", M, MLabel)
fmt.Printf("\trMin = %0.0e %s\n", rMin, dLabel)
fmt.Printf("\trDiff = %0.4e %s\n", rDiff, dLabel)
fmt.Printf("Acceleration of gravity at\n")
fmt.Printf("\trMin = %0.4e %s\n", a, aLabel)
fmt.Printf("\trMax = %0.4e %s\n", G * M / math.Pow(rMax, 2),
aLabel)
fmt.Printf("The ball apexes at %0.4e %s after %0.4e %s\n",
float64(H), dLabel, TExpected, tLabel)
}
```

---

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