

Space-Time Quantification

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The quantification of Length and Time in Kepler's laws implies an angular momentum quantum, identified with the reduced Planck's constant, showing a mass-symmetry with the Newtonian constant G . This leads to the Diophantine Coherence Theorem which generalizes the synthetic resolution of the Hydrogen spectrum by Arthur Haas, three years before Bohr. The Length quantum breaks the Planck wall by a factor 10^{61} , and the associated Holographic Cosmos is identified as the source of the Background Radiation in the Steady-State Cosmology. An Electricity-Gravitation symmetry, connected with the Combinatorial Hierarchy, defines the steady-state Universe with an invariant Hubble radius 13.812 milliard light-year, corresponding to 70.790 (km/s)/Mpc, a value deposited (1998) in a Closed Draft at the Paris Academy, confirmed by the WMAP value and the recent Carnegie-Chicago Hubble Program, and associated with the Eddington number and the Kotov-Lyuty non-local oscillation. This confirms definitely the Anthropic Principle and the Diophantine Holographic Topological Axis rehabilitating the tachyonic bosonic string theory. This specifies G , compatible with the BIPM measurements, but at 6σ from the official value, defined by merging discordant measurements.

Keywords: Holographic Principle, Holic Principle, Coherence Principle, Holophysics, Topological Axis, Kepler laws, Fundamental constants, Diophantine Coherence Theorem, Physics History, Steady-State Cosmology, Combinatorial Hierarchy, Bosonic String Theory, Tachyonic Physics, Quantum Field Theory, DNA bi-codon, Multi-dimensional Crystallography, Anthropic Principle.

1 The Diophantine Coherence Theorem (DCT)

For connecting different physical measurements, Physics uses multiplication while addition is forbidden. But multiplication is a generalization of addition [25]. This paradox may be suppressed by considering only numerical ratios of the same physical quantity, as in the third Kepler law, *introducing Space and Time quanta* L_1 and T_1 [35]. Considered as a Diophantine Equation, which uses only natural numbers n , it resolves directly :

$$(T_n/T_1)^2 = (L_n/L_1)^3 \equiv n^6 \Rightarrow T_n = n^3 T_1 ; L_n = n^2 L_1 . \quad (1)$$

This proceeds from the Holic Principle [31], a Diophantine form of the Holographic Principle, which states that the nature of a physical ratio is related to its exponent identified with its topological dimensions : 3 for Space, 2 for a 2D Time [3], 5 for Mass, and 7 for Field. This Diophantine approach, based on the even simpler Diophantine Equation $Y = X^2$ is the basis of the Topological Axis, the skeleton of the cosmic mass spectrum [34]. The n -invariant L_n^3/T_n^2 is homogeneous to Gm_G , where G is Newton's gravitational constant, and m_G is a mass (here the usual central mass is divided by the factor $4\pi^2$). The other Kepler's law states that the orbital angular momentum per unit mass is an orbital invariant. Since the corresponding term L_n^2/T_n is proportional to n , this implies an orbital momentum quantum, identified to the reduced Planck constant, or *action quantum* \hbar , privileged by the the spin concept in particle physics. While the ratio of the kinematic parts of G and \hbar are homogeneous to a speed, these two universal constants presents a symmetry by respect to the mass concept, implying the association of \hbar with a mass m_{\hbar} :

$$L_n^3/T_n^2 = Gm_G ; L_n^2/T_n = n\hbar/m_{\hbar} . \quad (2)$$

Any mass pair (m_G, m_{\hbar}) is associated to a series of Keplerian orbits (L_n, T_n) :

$$L_n = \frac{(n\hbar)^2}{Gm_G m_{\hbar}^2} ; T_n = \frac{(n\hbar)^3}{G^2 m_G^2 m_{\hbar}^3} . \quad (3)$$



33 For $n = 1$ and $m_G = m_{\hbar} = m$, the Special Non-Local Length and Time are:

$$L_{NL}(m) = \frac{\hbar^2}{Gm^3} \quad ; \quad T_{NL}(m) = \frac{\hbar^3}{G^2m^5} . \quad (4)$$

34 Introducing the *formal* velocity $V_n = L_n/T_n$, this connects the reduced Planck energy $n\hbar/T_n$ with
35 the gravitational potential energy pertaining to masses m_G and m_{\hbar} and the energy $m_{\hbar}V_n^2$:

$$V_n = L_n/T_n = Gm_Gm_{\hbar}/n\hbar \quad \Rightarrow \quad n\hbar/T_n = Gm_Gm_{\hbar}/L_n = m_{\hbar}V_n^2 . \quad (5)$$

36 With the Planck mass $m_P = \sqrt{\hbar c/G}$, where the light speed c is the third universal constant, this reads

$$\frac{n\hbar}{T_n} = \frac{Gm_Gm_{\hbar}}{L_n} = m_{\hbar}V_n^2 \equiv m_{\hbar} \left(\frac{c}{nA} \right)^2 \quad ; \quad A = \frac{m_P^2}{m_Gm_{\hbar}} . \quad (6)$$

37 This is called the *Diophantine Coherence Theorem* (DCT).

38 2 The Atom H and the Holographic Cosmos

39 Three years before Bohr, Arthur Haas [14] considered *the electron orbital period* in the Rutherford
40 model, and the corresponding Planck energy $nh\nu = nh/T_n = n\hbar v_n/L_n$ where $v_n = 2\pi V_n$ is the orbital
41 velocity. The correct Hydrogen spectrum is obtained by equalizing it with the electric potential energy
42 $\hbar c/aL_n$, where $a \approx 137.0359991$ is the electric constant, and the double (virial) kinetic electron energy
43 $m_e v_n^2$ (the useful physical constants are listed in Table 1):

$$n\hbar \frac{v_n}{L_n} = \frac{\hbar c}{aL_n} = m_e v_n^2 \equiv m_e \left(\frac{c}{na} \right)^2 . \quad (7)$$

44 Note that the so-called "properties of vacuum" ϵ_0 and μ_0 are unnecessary : they are only introduced
45 for historical reasons, leading to the cumbersome, but official, choice of electrical units, hiding the true
46 "electrical constant" a , whose inverse α , called "the fine structure constant" is of minor importance.
47 For $n = 1$, this gives the bare Hass-Bohr radius: $r_{HB} = a\lambda_e$, where $\lambda_e \equiv \hbar/(m_e c)$ is the Reduced Electron
48 wavelength (the effective electron mass effect defines the Bohr radius $r_B = r_{HB}/(1 + 1/p)$). This double
49 equation shows up the same form that the above DCT (6), where additional 2π factors are integrated in
50 the definitions of m_G and m_{\hbar} . The identification of potential energy terms implies $m_Gm_{\hbar} = m_P^2/a$, thus
51 in this case $A = a$. The simplest choice $m_{\hbar} = m_e$ implies the following m_G , where $m_N = am_e$ is *the*
52 *Nambu mass, a quasi-quantum in Particle Physics* [22]:

$$m_{\hbar} = m_e \quad ; \quad m_G = \frac{m_P^2}{m_N} \quad ; \quad A = a . \quad (8)$$

53 This last mass is $m_G \approx 3.7939 \times 10^{12}$ kg, whose corresponding Special Length (4) is :

$$d_0 = L_{NL}(m_P^2/m_N) \approx 3.051 \times 10^{-96} \text{ meter} . \quad (9)$$

54 This is the Cosmic Space Quantum breaking the "Planck Wall" by a factor 10^{61} which has been associ-
55 ated to the Cosmos holographic radius R_{hol} [34]:

$$\pi \left(\frac{R_{hol}}{l_P} \right)^2 = 2\pi \frac{R_{hol}}{d_0} . \quad (10)$$

56 This is the Bekenstein-Hawking Entropy formula of the Holographic Principle [7] where the *Planck*
57 *Length* $l_P \equiv (G\hbar/c^3)^{1/2} \equiv L_{NL}(m_P)$ is a basic holographic length. The Cosmos radius R_C has been
58 defined by the natural *mono-chromatic* holographic extension :

$$\pi \left(\frac{R_{hol}}{l_P} \right)^2 = 2\pi \frac{R_{hol}}{d_0} = 2\pi \frac{R_C}{l_P} , \quad (11)$$



59 leading to:

$$R_{hol} = 2L_{NL}(m_N) \approx 18.105 \text{ Giga light-year (Glyr)} \quad (12)$$

$$R_C = 2L_{NL}(m_N^2/m_P) \approx 9.075 \times 10^{86} \text{ meter.}$$

60 There is a direct connection with the boson Z (2 ppm) and the boson W wavelength (0.3 ppm), with
61 $p_G = \lambda_e/(2^{127/2}l_P)$, involving e^{e^2} , (0.2 and 1.4 ppm) :

$$\begin{aligned} (\beta l_P L_{NL}(m_N))^{1/2}/\lambda_e &\approx Z e^{2\pi}(6F p_G)(p/p_W)^2 \\ 3 \times (\beta l_P L_{NL}(m_N))^{1/2}/\lambda_e &\approx \lambda_e^2/\lambda_H \lambda_W \approx Z(a+1)^{3/2} \approx (a^3 6F \beta^{1/4}/p_G(pH)^{1/2})^{1/2} \\ e^{e^2}(H/p_W)^4 &\approx (a+1)^{3/2}(p/p_W)^2 \approx ((4\pi)^3/3)^3/Z. \end{aligned} \quad (13)$$

62 This means that the heart is the following holographic relations (1.9 and 10 ppm):

$$(4\pi/3) \sqrt{a}^3 \frac{a d_e}{137} \approx (4\pi/3) e^{e^2} \frac{a}{a+1} \approx 4\pi (e^\pi)^2. \quad (14)$$

63 The Cosmos radius R_C is related to the above Haas-Bohr radius r_{HB} by the 10 ppm quasi holographic
64 formula :

$$(4\pi^2/3)(p/n_i)^2 \left(\frac{R_C}{r_{HB}} \right)^3 = a^a, \quad (15)$$

65 showing a dramatic role of the Electric constant a , implying it is a *calculation basis*.

66 From $m_P/\sqrt{m_e m_N} \approx a_w n_i^3$ (Eq. (21)), these formula leads to a confirmation of the optimal G value
67 in the ppb domain, where $\beta = (H-p)^{-1}$ (Table 1) :

$$\left(\frac{P}{a_w} \right)^3 \approx \left(\frac{4\pi}{\sqrt{a}} \right)^8 \frac{(pH\beta^2)^5}{2} \approx \frac{aW}{137Z} (pH)^5 \quad (16 \text{ ppm}), \quad (16)$$

68 showing a role of the geometrical factor 4π .

69 Table 2 shows the symmetry between the Nambu mass m_N and the Planck mass m_P , whose large
70 value is the source of the "Hierarchical Problem" [30]. Now $L_{NL}(\sqrt{m_P m_N}) \approx \lambda_{CMB}/2a_s^2$, where $2a_s^2 \approx a$,
71 tying to 0.3 % the strong coupling a_s and the nominal wavelength hc/kT_{CMB} of the Cosmic Micro-onde
72 Background (CMB), whose source is lacking in the steady-state cosmology [6], [15]. The simplest
73 hypothesis is that the above Cosmos is this source. Indeed, the Wien CMB wavelength λ_{Wn} enters (0.1
74 %):

$$4\pi \left(\frac{R_{hol}}{\lambda_{Wn}} \right)^2 \approx e^a. \quad (17)$$

75 This perfect holographic formula suggests that *the CMB would be coherent, meaning it brings infor-*
76 *mation*. This could be the real signification of the CMB Anisotropy Statistics [1].

77 3 The Gravitational Dihydrogen

78 The Haas method was already applied to the *special three-body* gravitational dihydrogen [33, p.391]:

$$n\hbar \frac{v_n}{L_n} = \frac{Gm_p m_H}{L_n} = m_e v_n^2, \quad (18)$$

79 The comparison with the above Haas equation implies the substitution : $a \rightarrow a_G = m_p^2/m_p m_H$, corre-
80 sponding to the following m_G value :

$$m_{\bar{H}} = m_e \quad ; \quad m_G = m_{bc} \quad ; \quad A = a_G \quad (19)$$

81 where $m_{bc} = m_p m_H/m_e$ is close to the DNA bi-codon mass [33], (DNA = Desoxyribo Nucleic Acid).



82 For $n = 1$, this Haas-Sanchez radius R_{H_2} shows a direct Electricity-Gravitation symmetry, by respect
 83 to the Reduced Electron wavelength $\lambda_e = \hbar/m_e c$:

$$\begin{aligned} r_{HB} &= a\lambda_e = a \frac{\hbar}{m_e c} \\ R_{H_2} &= a_G \lambda_e = \frac{\hbar^2}{G m_e m_p m_H} \equiv L_{NL}(m_0) \approx 6.906 \text{ Glyr}, \end{aligned} \quad (20)$$

84 where $m_0 = (m_e m_p m_H)^{1/3}$. The above DNA bi-codon mass shows a central position in the Topological
 85 Axis [33], corresponding to the dimension 16. Indeed the corresponding topological term e^{16} is close
 86 to pHR_{hol}/R_{H_2} , and, more precisely, to $2n^4/a^3$ (0.04 %).

87 Note that a and a_G are very close to the last two terms of the Combinatorial Hierarchy 137 and
 88 $N_L + 137$, with $N_L = 2^{127} - 1$, the Lucas Number [4]. It was noted that the implied Mersenne numbers
 89 3,7,127 relates the gravitational main large number $P = m_p/m_e$ with the weak parameter $a_w = (m_F/m_e)^2$
 90 and \sqrt{a} in the following 57 ppm relation, and the rearranging of the exponents makes the neutron ratio
 91 to appear, inducing the above relation (14):

$$P^{3+7} \approx a_w^7 \sqrt{a}^{7+127} \Rightarrow P/a_w \sqrt{a} \approx (\sqrt{a}^{7+127}/P^3)^{1/7} \approx n_i^3. \quad (21)$$

92 In the gravitational dihydrogen radius formula R_{H_2} , the speed c is eliminated: for this reason, a
 93 precise approximation was immediately guessed by the c -free "dimensional analysis", the so-called
 94 *Three Minutes Formula*, from the ternary symmetry Electron-Proton-Neutron (Closed Letter to the
 95 Paris Science Academy, March 1998) [32] (Table 2). Indeed, the speed c is far too small a speed to
 96 explain the cosmic coherence manifested by the Foucault pendulum (Mach Principle).

97 The Special time $T_{NL}(m_0)$ is very close (0.9 %) to the time associated to the triplet : (\hbar, G_F, ρ_{cr}) ,
 98 with the Fermi constant G_F and the critical steady-state density $\rho_{cr} = 3c^2/8\pi GR^2$ with horizon radius
 99 $R = 2R_{H_2}$: it is $\hbar^4/G_F^{5/2} \rho_{cr}^{3/2} \approx 3m_p^2 R_{hol}/cm_e m_Z$ (0.01 %), introducing the following steady-state
 100 Universe.

101 4 The Steady-State Universe re-established

102 A salient feature of the observed Universe is its critical character, relating its horizon radius R with
 103 its mass by $R = 2GM/c^2$. However, in the initial "flat universe" model [12], the total mass M is
 104 only matter, while in the present Λ CDM standard model, it is separated between a material part with
 105 relative density Ω_m and a so-called "dark energy" part with relative density $1 - \Omega_m$ [1]. We have
 106 noted that Ω_m is compatible with 3/10, which is both the relative density of the classical gravitational
 107 energy of a critical homogeneous ball and the relative density of the steady-state *non-relativist* recession
 108 kinetic energy [34]. While the standard cosmology uses an ad-hoc inflation to justify this observed
 109 critical condition, we consider rather the Universe as a particle (Topon) in the above Cosmos, with
 110 the Topon wavelength $\lambda_M \equiv \hbar/Mc = 2\hbar/G/Rc^3 \equiv 2l_p^2/R$. Then, the critical condition results from the
 111 Bekeinstein-Hawking entropy holographic relation, as above (Eq. 10), where the Topon appears as a
 112 Length Quantum, since the wavelength λ_m associated for any particle of mass m is a whole multiple
 113 n_m of the Topon, in conformity with the Field Quantum Theory. The geometrical interpretation is
 114 clear : *it is a sphere area described by a whole number of sweeping circles, illustrating the fact that*
 115 *multiplication is a series of additions, an approximation supporting the vastness of the world* [34]:

$$4\pi \left(\frac{R_{HB}}{l_p} \right)^2 = \pi \left(\frac{R}{l_p} \right)^2 = 2\pi \frac{R}{\lambda_M} \equiv 2\pi n_m \frac{R}{\lambda_m} \Rightarrow M = \frac{Rc^2}{2G} \equiv \frac{R_{H_2} c^2}{G}, \quad (22)$$

116 identifying twice the above Haas-Sanchez's gravitational radius R_{H_2} with R , the steady-state Universe
 117 horizon radius, which is also *the limit of a theoretical star radius when its number of atoms shrinks to*
 118 *one* [10], *a central length in astrophysics*:

$$R = 2 \frac{\hbar^2}{G m_e m_p m_H} \Rightarrow M = \frac{m_p^4}{m_e m_p m_H}. \quad (23)$$



119 This is called the *Central Formula*. With the effective electron mass $m'_e = m_e m_p / (m_p + m_e) \equiv M/n_e$,
 120 this introduces n_e , the *Universe Electron Quantum Number*, canonical in *Quantum Field Theory*. The
 121 Eddington's Electron-Proton symmetry shows up in the following *resolution of the so-called Large*
 122 *Number Problem*, where λ_{pH} is the geometrical mean of the reduced wavelengths of the proton and
 123 Hydrogen:

$$\frac{m_p^2}{m_p m_e} \equiv n_e^{1/2} \equiv \frac{R}{2\lambda_{pH}}, \quad (24)$$

124 which is extended by very precise dramatic expressions involving the symmetry between the weak
 125 bosons of masses $m_W = Wm_e$ and $m_Z = Zm_e$:

$$n_e^{1/2} \approx \frac{(WZ)^4}{2} \approx \left(\frac{m_F^2}{m_p m_H} \right)^7 \left(\frac{aZ}{W} \right)^3, \quad (25)$$

126 where appears as well a Planck-Fermi symmetry, enlighting the "Hierarchical problem", specifying the
 127 known relation $a_G \approx W^8$ [9].

128 In the Topological Axis, the above Topon corresponds to the orbital number $k = 7$, while the gauge
 129 bosons corresponds to $k = 3$ (weak bosons W,Z) and $k = 5$ (strong GUT boson X), letting a single place
 130 $k = 1$ for a *non-standard massive* Gluon [34].

131 The particular values of the topological function $f(k) = e^{2k+1/2}$ for $k = 7$ and 6 show up in (0.06%):

$$n_e \approx f(7) \times 153^2, \quad (26)$$

$$R/\lambda_e \approx f(6)/6,$$

132 where $(f(6))^2 \equiv f(7)$ implies that $m_p/m_e \approx 1836 \equiv 6 \times 2 \times 153$, the Diophantine approximation of the
 133 Wyler formula $p_W = 6\pi^5$ [39]. The spectroscopic number associated to k is $2(2k + 1)$, where 2 is the
 134 spin degeneracy and $2k + 1$ the number of magnetic states [35]. For $k = 6$, this is 26, the canonical
 135 dimension in the bosonic string theory [30].

136 This *invariable* Universe radius $R \approx 13.812$ Giga light-year (Glyr) of Eq. (23) is close to c times
 137 the variable standard Universe age. So the standard theoretical approach is correct, but not its Big Bang
 138 interpretation : it seems that a confusion is made somewhere between Time and Length, which readily
 139 occurs by putting $c = 1$. Moreover, the corresponding Hubble constant c/R is 70.790 (km/s)/Mpc,
 140 which is compatible with both the WMAP and the Carnegie-Chicago Hubble Program recent direct
 141 measurements (Table 3).

142 The above Universe gravitational potential energy $(3/10)Mc^2$ shows a Neutron Quantum Number
 143 (the number of neutron masses) very close (0.05 %) to the large Eddington Number [34]. So it has
 144 nearly anticipated the correct Hubble Constant value (Table 3).

145 The theoretical prediction [9] that a is the order of $\ln a_G$ was specified in the Single Electron Cos-
 146 mical Radius R_1 , defined by [33] :

$$r_B/\lambda_e \equiv a(1 + 1/p) = \Sigma_2^{R_1/\lambda_e}(1/n)/\Sigma_2^{R_1/\lambda_e}(1/n^2) = (\ln(R_1/\lambda_e) + \gamma - 1)/(\pi^2/6 - 1), \quad (27)$$

147 leading to the 0.4 ppm connection : $R_1 \approx (RR_{hol}\beta)^{1/2} p_G/p_W$, with $p_G = m_p/\sqrt{N_L}m_e$, $p_W = 6\pi^5$.

148 The cosmos radius R_C and the holographic mass $M_{hol} = R_{hol}c^2/2G$ connect with the ratio $R/\lambda_e \equiv$
 149 T/t_e through the Cosmos-Universe couple MLT 1% Formula [35]:

$$(\ln(R_C/\lambda_e))^2 \approx (M_{hol}/m_e)^2 + (R/\lambda_e)^2 + (T/t_e)^2. \quad (28)$$

150 The Cosmos radius connects with the above radius R_{hol} and R by (0.6 ppm and 0.04 %) :

$$R_C(m_e/m_p)^2 \approx R_{hol}(WH/3)^2/\beta \approx R(2FZ^2/3). \quad (29)$$

151 With Eq (13), the elimination of Z/W implies a new 0.3 ppm formula for R (Table 3).



152 **5 The Cosmic Microwave Background (CMB)**

153 This Universe radius $R = 2R_{H_2}$ enters a 1D-2D holographic relation: $2\pi R/\lambda_e = 4\pi\lambda_p\lambda_H/l_p^2$. The
 154 extension to the 3D holographic relation using λ_{H_2} , the reduced wavelength of the dihydrogen molecule
 155 H_2 , involves the reduced wavelength of the Cosmic Microwave Background (CMB) $\lambda_{CMB} = \hbar c/kT_{CMB}$:

$$2\pi \frac{R}{\lambda_e} = 4\pi \frac{\lambda_p \lambda_H}{l_p^2} \approx \frac{4\pi}{3} \left(\frac{\lambda_{CMB}}{\lambda_{H_2}} \right)^3, \quad (30)$$

156 leading to $T_{CMB} \approx (8G\hbar^4/3\lambda_p^5)^{1/3}/k \approx 2.729$ Kelvin, which is once more, apart the holographic factor
 157 $8/3$, a c -free three-fold (Mass, Length, Time) dimensional analysis, giving the energy kT_{CMB} from the
 158 constants G, \hbar, λ_p . Moreover, by substituting $a_G = R/2\lambda_e$ with the above Lucas Number N_L , this leads
 159 to a new holographic expression (analog to the area of a 4D sphere), which gives T_{CMB} , compatible
 160 with the measured value 2.7255(6) Kelvin [34]:

$$N_L \approx 2\pi^2 \frac{\lambda_{CMB}^3}{\lambda_e \lambda_H^2} \Rightarrow T_{CMB} = \frac{hc}{k\lambda_{CMB}} \approx 2.7258205 \text{ Kelvin} \quad (31)$$

161 The standard Cosmology predicts a Neutrino background with temperature $T_{CNB} = T_{CMB} \times (4/11)^{1/3} \approx$
 162 1.946 Kelvin. The total CMB photon number is $n_{ph} = (\xi(3)/\pi)(R/\lambda_{CMB})^3$, exceeding the total Hydrogen
 163 number $n_H = M/m_H = R\lambda_H/2l_p^2$. But, in term of energy the matter dominates. So one must consider
 164 also the ratio between the critical energy density $u_{cr} = 3c^4/8\pi GR^2$ and the total background energy
 165 density $u_{cmb+cmb} = yu_{cmb}$, with $y = 1 + (21/8)(4/11)^{4/3} \approx 1.681322$ [40] and $u_{cmb} = (\pi^2/15)\hbar c/\lambda_{CMB}^4$.
 166 We observed that these ratios are tied by an Eddington's type relation:

$$\left(2 \frac{n_{ph}}{n_H} \right)^{1/2} \approx \frac{u_{cr}}{u_{cmb+cmb}} \Rightarrow T_{CMB} \approx 2.724 \text{ Kelvin} . \quad (32)$$

167 This confirms the existence of the Neutrino background. Now assuming that the total background
 168 Photon + Neutrino is the result of an on-going Hydrogen-Helium transformation, producing $e_{He} =$
 169 6.40×10^{14} Joule by kilogram of Helium, i.e. an efficiency $\epsilon_{He} = e_{He}/c^2 \approx 1/140$. The Helium
 170 mass density is $Y \times \rho_{bar}$; with the standard evaluation of baryonic density $\epsilon_{bar} = \rho_{bar}/\rho_{cr} \approx 0.045$ and
 171 $Y \approx 0.25$ [1], this leads to :

$$\left(\frac{\lambda_{CMB}^2}{l_p R} \right)^2 \approx \frac{8\pi^3 y}{45Y\epsilon_{bar}\epsilon_{He}} \approx 1.15 \times 10^5 \Rightarrow T_{CMB} \approx 2.70 \text{ Kelvin} . \quad (33)$$

172 In the standard model, the Universe age is far too small to explain a large Helium large density resulting
 173 from stellar activities [8]. Thus, it is not a real problem in the steady-state model.

174 **6 The Electron and the Kotov Non-Local Period**

175 This study confirms the central role of λ_e , the unit length in the Topological Axis and in the Single
 176 Electron Model [33]. So we look for a Diophantine series giving it for $n = 1$. This means:

$$\lambda_e \equiv \frac{\hbar}{m_e c} = \frac{\hbar^2}{Gm_G m_{\hbar}^2} \Rightarrow A \equiv \frac{m_p^2}{m_G m_{\hbar}} = \frac{m_{\hbar}}{m_e} \quad (34)$$

177 so that the fundamental ($n = 1$) energy is: $E \equiv m_{\hbar} c^2/A^2 = m_e c^2/A$. There is an *elimination of c* by
 178 considering the term A^2 as the product of the the above gravitational constant $a_G = \hbar c/Gm_p m_H$ and the
 179 electro-weak one $a_w = \hbar^3/cG_F m_e^2$ [9], where G_F is the Fermi constant:

$$A^2 = a_G a_w \Rightarrow E = \frac{m_e c^2}{\sqrt{a_G a_w}} \quad (35)$$

180 with $t_e \equiv \hbar/m_e c^2$ the electron period, this corresponds to the time:

$$t_e \sqrt{a_G a_w} \approx 9600.60 \text{ s} . \quad (36)$$

181 The identification with the Kotov P_0 period $t_K \approx 9600.606(12)$ s [18], [19] corresponds to $G \approx$
 182 6.6754527 SI, specified to 10^{-8} by the Single-Electron Radius $R_1 \approx (4\pi p/p_W)^2 a_w c t_K$ [34] and con-
 183 sistent with the BIPM measurements [28], but at 6σ from the official value, an unusual mean between
 184 *discordant* measurements. With the Fermi mass $m_F = m_e \sqrt{a_w}$, close to the mean nucleotide mass [33],
 185 the Lepton Mu mass m_μ , $u = R_{hol}/R$, the critical density $\rho_{cr} = 3c^2/8\pi GR^2$, $m_{GF} = (m_P m_F)^{1/2}$, this
 186 defines our optimal strong coupling a_s :

$$\begin{aligned}
 m_G &= \frac{m_e m_P m_H}{m_F^2} \\
 m_{\tilde{h}}/m_P &= \frac{m_F}{(m_P m_H)^{1/2}} \equiv \frac{m_\mu^2}{m_e m_N} \equiv 2\pi \frac{a_s m_P m_H}{m_e m_F} \\
 (GG_F)^{1/2} &\equiv \left(\frac{\hbar}{m_{GF}} \right)^2 = \frac{\hbar}{(m_P m_H)^{1/2}} \frac{\lambda_e^2}{t_K} \\
 \frac{G_F}{G m_P^2 l_P^2} &\approx \frac{a^4 m_P m_\mu}{m_e^2} \quad (0.2\%) \quad ; \quad \frac{\hbar}{(G_F \rho_{cr})^{1/2}} \approx \frac{\lambda_e^2}{u^{1/16} l_P} \quad (0.01\%)
 \end{aligned} \tag{37}$$

187 exhibiting a symmetry between canonical area speeds. Note that $2ct_K \approx L_{NL}(m_{bc})$, confirming once
 188 more the bi-codon mass, which enters also a relation involving the Photon and Graviton
 189 masses [34] (Table 3). Moreover, with $P = m_P/m_e$, $F = m_P/m_e$, $H = m_H/m_e$, $p = m_P/m_H$, and the
 190 precise variant (0.14 ppm) of the Golden Number: $\Phi_0 = P/(a_w H)^3 \approx ((4\pi/3)(H/p)^2)^{1/3}$, one observes :

$$\begin{aligned}
 \frac{L_{NL}(m_{GF})}{r_{HB}} &\equiv \left(\frac{P}{F^3} \right)^{1/2} \frac{1}{a} \approx \Phi_0^2 \quad (15 \text{ ppm}) \\
 cT_{NL}(m_{GF}) &\equiv l_P \left(\frac{P}{F} \right)^{5/2} \approx \left(\frac{R_{hol} \lambda_e}{2} \right)^{1/2} \frac{1}{d_e^2} \quad (74 \text{ ppm})
 \end{aligned} \tag{38}$$

191 where d_e is the canonical Excess Electron Magnetic Moment (Table 1). This specifies the holographic
 192 relations $a^2 \approx (4\pi/3)p^{3/2}$ and $F^5/Pa^3 \approx \eta$, with $\eta = 1 + 2/(3 \times 139)$ (ppb precision) [35], where
 193 139 is the complete Atiyah form [2], adding the dimensions of the four algebra (octonion, quaternion,
 194 complex, real): $139 = 137 + 2 = 2^7 + 2^3 + 2^1 + 2^0 \approx i^{-i\pi}$, and $3 \times 139 + 2 = 419$, the positive
 195 crystallographic number [37] in the superstring dimensions 10D and 11D [30], see Table 7 in [35].
 196 Moreover, $T_{NL}(m_{GF}) \approx 19.14$ ms, typical of the Human nervous system, and the third octave down the
 197 flat La tone (Lab) for $La_3(A_4) = 442.9$ Hz, an anthropic argument far more pertinent and precise than
 198 the rough standard ones, principally based on a *cosmic Big Bang scenario* [9].

199 7 Discussion

200 The Pythagoras Principle stating that all is ruled by whole numbers has been forgotten during cen-
 201 turies. This resulted in the failure of Poincaré to resolve the apparently most difficult problem of
 202 modern physics, the apparition of quanta [26]. He ought to have remember that the more difficult is
 203 a problem, the more basic feature must be revisited, in particular the Kepler laws, leading to the *ele-*
 204 *mentary* Diophantine equation, of immediate resolution, which implies directly an angular momentum
 205 quantum identifiable with the reduced Planck's constant \hbar . Strangely enough, in the same London
 206 conference (p. 102-103), Poincare explained that cosmology cannot be entirely founded on differential
 207 equations. Since the main scientific criteria is the repeatability of experiments, this implies the Perfect
 208 Cosmological Principle founding the steady-state model [6], and Poincaré could have concluded that
 209 cosmology, hence the whole physics, must be tied to the Number Theory [35].

210 This approach leads to the Diophantine Coherence Theorem (DCT) which has the same structure
 211 than the Hass formulation for the Hydrogen atom spectrum problem. This shows that the real invariant
 212 quantity is the Frequency, so that the Energy conservation would mean a Frequency Accordance, or
 213 "Coherence Principle", mandatory in Practical Holography, and conform with the Harmony Principle
 214 of Pythagoras, the father of Natural Philosophy, the very root of Science. This confirms the pertinence
 215 of the Quantum Field Theory, where any Particle Field is defined by a whole number, entering the
 216 Holographic principle in the revisited critical steady-state Universe. In particular, both the Electron



217 Quantum Number and the Neutron Quantum Number play a central role. The Universe Length Quantum (Topon) is associated to a Universe Time quantum ("Chronon" $t_M = \lambda_M/c$), which may be looked
218 as the period of the *Permanent Bang matter-antimatter oscillation* [20].
219

220 Among the two main cosmological models, the steady-state one was by far the most easily refutable,
221 so the most scientific, in the Popper sense [27]. It is why it was thought as being refuted by hasty
222 observations, however these so-called refutations were much discussed [20]. *This article firmly re-*
223 *establish the steady-state cosmology.* The galaxy recession means not at all an Universe expansion : it
224 has been shown that the repulsive force provoking expansion exceeds the attraction for larger distances
225 than a million years, typical of a galaxy group, which shows no recession, and the renewal of matter
226 inside such a group could be attributed to the giant black holes [34]. *It is now mandatory to measure*
227 *more precisely the Universe Temperature at any distance, in order to compare it with the microwave*
228 *background one.*

229 The Cosmic Length Quantum breaks the "Planck wall" by the factor 10^{-61} . The DCT shows that
230 the Haas-Bohr radius is a *secondary* length quantum, while the Universe itself appears as a ternary
231 length quantum in the Cosmos, defined by the Holographic Principle where the Planck length is an
232 intermediate holographic length, instead of the standard quantum. The unifying pseudo length quantum
233 is the reduced Electron Wavelength which shows, through the DCT and the Kotov non-local period, a
234 symmetry between gravitation and electroweak interaction. The Kotov-Lyuty *Non-Doppler oscillation*
235 plays a central role, but was overlooked : it is however a clear sign of the non-local character of
236 Quantum Cosmology which is patent in the Foucault pendulum. *It is mandatory to check the Lyuty*
237 *Non-Doppler Quasar measurements* [18]. The standard speed limit c excludes any explanation of the
238 wave packet reduction phenomena, which requires a non-local or tachyonic Physics. So, it is logical
239 that the bosonic string theory, which introduces tachyon, is confirmed by the Diophantine Topological
240 Axis. Indeed, the central bosonic dimension $d = 26$ corresponds to the non local universe radius
241 (Central Formula).

242 The Planck mass enters naturally in the DCT, while incompatible with the standard in Particle
243 Physics. However, the standard spin formulation rejoins our conclusion that the reduced Planck con-
244 stant \hbar plays a more fundamental role than h .

245 8 Conclusions

246 On the basis of the invariance of physical laws, the Mach Principle cannot enter an evolutionary Uni-
247 verse, so the steady-state model is really compatible with scientific cosmology [6]. Its invariant radius is
248 tied to 22 formula in the Table 3, in the same way that Jean Perrin [23] collected 14 formula to demon-
249 strate the atom existence. This is a *parallel between the quantification of matter and the quantification*
250 *of space-time.*

251 It is imperative for the International System to come back to a number three for the basic units :
252 Mass, Length, Time. Also, it must define electronic units by using only the electrical constant a . In
253 particular, Particle Physics must suppress the use of eV unit. Also the two inter-correlated measures,
254 the non-local Kotov-Lyuty period and G , *whose standard value is erroneous by 6* must be revisited.

255 The bosonic string theory is rehabilitated by the Topological Axis, but *its connection with the Pe-*
256 *riodic Table* must be explained [35]. The Quantum Cosmology gets definitely the status of a real
257 science. The object "Universe" is well defined : inside an external Cosmos, it is both a mono-atomic
258 star, a quasi-homogeneous black hole, a particle (ultimate gauge boson), a nuclear fusion reactor and a
259 thermal machine.

260 The Holographic Principle and the DNA bi-codon mass are both decisive. So the DNA could be an
261 helix-hologram, opening the way towards bio-computing [24].

262 The c -free Elementary Non-Local Three Minutes Formula giving the Universe half-radius is now
263 fully established: *this means a tight harmony between the Universe and Human Consciousness, a spe-*
264 *cial and decisive manifestation of the real Anthropic Principle.*
265



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270

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Table 1: Physical constants

Quantity	Value	Unit	ppb (10^{-9})
Official Gravitation Constant G_{off}	$6.674\ 30(15) \times 10^{-11}$	$\text{kg}^{-1}\text{m}^3\text{s}^{-2}$	1.7×10^6
BIPM Gravitation Constant [28]	$6.67545(18) \times 10^{-11}$	$\text{kg}^{-1}\text{m}^3\text{s}^{-2}$	2.7×10^3
Optimal Gravitation Constant G [34]	$6.67545272 \times 10^{-11}$	$\text{kg}^{-1}\text{m}^3\text{s}^{-2}$	
Electrical Constant a	137.035999084(21)	-	0.15
Electron Excess Magnetic moment d_e	1.00115965218096	-	0.26
Reduced Planck constant $\hbar = h/2\pi$	$1.054\ 571\ 81\ 10^{-34}$	J s	exact
Speed of light in vacuum c	299 792 458	m s^{-1}	exact
Optimal Fermi Constant $G_F = \hbar^3/cm_F^2$	$61.435\ 85110^{-62}$	J m^3	
Optimal Weak Coupling Constant $a_w = \hbar^3/cG_F m_e^2$ [35]	$3.283374406 \times 10^{11}$	-	
Optimal Fermi mass ratio $m_F/m_e = F = a_w^{1/2}$	573007.3652	-	
Official Strong Coupling constant	8.446(50)(8)	-	7.6×10^6
Optimal Strong Coupling Constant a_s [35]	8.434502914	-	
Proton/Electron mass ratio p	1836.152 673 43	-	0.06
Proton/Electron Wyler mass ratio p_W [39]	$6\pi^5$	-	exact
Neutron/Electron mass ratio nt	1838.683 661 7	-	0.5
Hydrogen/Electron mass ratio H	1837.152 660 14	-	0.06
Hydrogen Relativist correction factor $\beta = 1/(H - p)$	1.0000266	-	
Optimal Muon/Electron mass ratio μ [34]	206.768 286 9	-	
Optimal Higgs Boson mass m_{H_g} [35]	$495^2 m_e$	-	
W boson mass ratio $W = m_W/m_e$	157298 ± 23	-	1.5×10^5
Z boson mass ratio $Z = m_Z/m_e$	178450 ± 4	-	2.3×10^4
Electron mass m_e	$9.109\ 383\ 701\ 510^{-31}$	kg	0.3
Boltzmann Constant k	1.38064910^{-23}	J K^{-1}	exact
Reduced Electron Wavelength λ_e	$3.861\ 592\ 675\ 10^{-13}$	m	0.3
Single Electron Universe radius R_1	$1.492\ 365\ 473\ 10^{26}$	m	
Measured CMB temperature T_{CMB}	2.725 5(6)	Kelvin	
Optimal CMB Temperature T_{CMB}	2.725 820 138	K	
Optimal CMB Wien wavelength λ_{Wn}	$1.063\ 082\ 472\ 10^{-3}$	m	
Optimal CMB reduced wavel. $\hbar\lambda_{CMB} = \hbar c/kT_{CMB}$	$8\ 400\ 716\ 617^{-4}$	m	
Optimal CNB Temperature $T_{CNB} \equiv T_{CMB}(11/4)^{-1/3}$	1 945 597	K	
Water Triple Point Temperature T_{H_2O}	273.16	K	
Optimal CNB reduced wavelength $\lambda_{CNB} = \hbar c/kT_{CNB}$	$1\ 176\ 956\ 918^{-3}$ [34]	m	
Optimal critical density $\rho_{cr} = 3c^2/8\pi GR^2$	$9.411\ 979\ 89\ 10^{-27}$	$\text{kg m}^{-1/3}$	
Kotov P_0 period t_K	9600.606(12) [19]	s	1200

Table 2: Values of the DCT Fundamental ($n = 1$) Radius $\hbar^2/Gm_G m_{\tilde{h}}^2$ for specific values of m_G and $m_{\tilde{h}}$. Planck mass: m_P . Nambu mass : $m_N = am_e$. Holographic ratio $u = R_{hol}/R$. Proton mass: m_P . Hydrogen mass : m_H . Mean Atomic mass : $m_0 = (m_e m_p m_H)^{1/3}$. Bicodon mass $m_{bc} = m_p m_H/m_e$. Photon mass $m_{ph} = \hbar/c^2 t_K \approx 1.2222 \times 10^{-55}$ kg. Graviton mass : $m_{gr} = m_{ph}/a_w \approx 3.7223 \times 10^{-67}$ kg [34]. Optimal Higgs boson mass: $m_{H_g} = 495^2 m_e$.

m_G	$m_{\tilde{h}}$	Length	Symbol	Precision/offset
m_P^2/m_N	m_P^2/m_N	Space Quantum	d_0	exact
m_P^2/m_0	m_P^2/m_0	Topon	λ_M	exact
m_{bc}/a_w	$m_e \sqrt{a_w a_G}$	Reduced Electron Wavelength	λ_e	exact
m_P^2/m_N	m_e	Hass-Bohr radius $r_{HB} = a\lambda_e = r_B/(1 + 1/p)$	r_{HB}	exact
$a^3 m_P$	$\sqrt{m_p m_H}$	Background Wien Wavelength	λ_W	3.2×10^{-4}
m_{bc}	m_{bc}	Twice Kotov Length	$2l_K$	6.3×10^{-3}
m_{H_g}	m_{H_g}	$R\lambda_e/4\lambda_{CMB}$ $Ra_w^{1/2}/WZ^2$		- 0.23 % + 0.25 %
m_{bc}	m_e	Half Universe Radius	$R_{H_2} \equiv R/2$	exact
m_N	m_N	Half Holographic Cosmos radius	$R_{hol}/2$	exact
m_N^2/m_P	m_N^2/m_P	Half Cosmos Radius	$R_C/2$	exact
$u \times m_{bc}$	$\sqrt{m_{ph} m_{gr}}$	Cosmos radius	R_C	1.7×10^{-3}

Table 3: Implication of Eddington Number ($N_E = 136 \times 2^{256}$) and Holo-physics formula for the *invariant* Hubble radius $R \approx 13.812$ Giga light-year (Gly = 1 billion light-year) and the corresponding Hubble constant $H_0 = c/R$, which uses the length unit Megaparsec, compared to the main measurements. Lucas Number $N_L = 2^{127} - 1$. Topological Function $f(k) \equiv e^{2^{k+1/2}}$. Holographic ratio $u = R_{hol}/R$. For comparison, the so-called standard "Universe Age" is also presented, with unit in the c ratio (Gy = 1 billion year). The optimal WZ value (Table 1) is defined from the identification to the Central Formula $R = 2 \times$ gravitational H_2 radius, which is also $2 \times$ the "Three Minutes Formula" (closed draft 1998) where the neutron mass is replaced by the hydrogen mass. The last Euler idoneal number is $s_{65} = 1848$.

Date	Source $R = 2GM/c^2$	Hubble radius Gly	Hubble Cst. $\text{km s}^{-1}/\text{Mpc}$	Univ. "Age" Gyr
1945	Eddington Number N_E [11] ; $N_E \approx (3/10)M/m_n$	13.805	70.826	
1927	Lemaître [21]	1.6	620	
1929	Hubble [16]	1.8	540	
1956	Humason, Maydal and Sandage [17]	5.4	180	
1958	Sandage [36]	13	75	
1998	$2\hbar^2/Gm_em_p m_n$ Twice Closed Draft	13.800	70.852	
2006	$2\hbar^2/Gm_em_p m_n$ [32]	13.800	70.852	
2006	$2 N_L \lambda_e$ [32]	13.889	70.397	
2017	$(WZ)^4 (\lambda_p \lambda_H)^{1/2}$ [9] [33]	13.796 ± 0.002	70.87 ± 0.01	
2017	$\lambda_e f(6)/6$ [33]	13.821	70.744	
2017	$(2 \lambda_e/3) (\lambda_{CMB}/\lambda_{H_2})^3$ [33] Holography Eq. (30)	13.897	70.357	
2017	$\lambda_e (3^3)^3 / u$ [33] From $R_{hol}/\lambda_e \approx (3^3)^3$	13.812	70.790	
2017	$2\hbar^2/Gm_em_p m_H$ [33] CENTRAL FORMULA	13.812	70.790	
2017	$2(ct_K)^2/a_w \lambda_e$ [33] Non-Local Oscillation	13.812	70.790	
2017	$\lambda_e (H/p_w)(2\pi^2 a^3)^5$ [33] Holic Principle	13.812	70.790	
2017	$(hc/kT_{H_2O})^2/ul_p$ [33] From $\sqrt{R_{hol} l_p} \approx \lambda_{H_2O}$	13.840	70.647	
2019	$\lambda_e (2/u)^{2 \times 3 \times 5 \times 7}$ [34] Complete Holic Principle	13.856	70.565	
2021	$\lambda_e (6/\pi)^{r_B/\lambda_e}$ [35]	13.776	70.975	
2021	$\lambda_e (n_i/p)^{1/2} \pi^{5 \times 31/2}$ [35]	13.812	70.790	
2021	$\lambda_p (d_e/2)(pH)^{3a_s/4}$ [35]	13.812	70.790	
2021	$2\lambda_e((1837 + s_{65})/2 + 1)^{\sqrt{a}}$ [35] $s_{65} = 1848$	13.812	70.790	
2021	Cosmos-Universe Couple MLT Formula (28) [35]	13.726	71.273	
2022	$(3R_{hol} \lambda_{CMB}^4 / \lambda_e^3)^{1/2}$	13.832	70.769	
2022	$\lambda_e (a - 136)^{1/2} (e^e)^{\sqrt{a}/2}$	13.814	70.780	
2022	$(2\pi/3) \lambda_e p^{\sqrt{a}}$	13.804	70.831	
2022	$(1 + 1/a)^6 \lambda_e^5 / 18a_w N_L l_p^4$ From Eqs. (13, 29)	13.812	70.790	
2022	$R_1^2 N_L l_p^2 p_w^2 / R_{hol} \lambda_e^2 \beta$ From $R_1 \approx (RR_{hol})^{1/2}$	13.812	70.790	
2022	$2R_1^2 / a_w^3 (4\pi p/p_w)^4$ From $R_1 \approx ct_K a_w (4\pi p/p_w)^4$	13.812	70.790	
1998	PDG (Particle Data Group)	14 ± 2	70 ± 10	11.5 ± 1.5
2002	PDG	13.7 ± 0.3	71 ± 3	15 ± 3
2005	Hubble Space Telescope	13.6 ± 1.5	72 ± 8	13.7 ± 0.2
2012	WMAP [5]	14.1 ± 0.2	69.3 ± 0.8	13.77 ± 0.06
2019	Riess group [29]	13.2 ± 0.3	74.2 ± 1.4	
2020	Planck mission [1]	14.5 ± 0.1	67.4 ± 0.5	13.82 ± 0.04
2020	HOLICOW [38]	13.4 ± 0.3	73.3 ± 1.8	
2021	Carnegie-Chicago Hubble Program [13]	14.0 ± 0.3	69.8 ± 1.6	