Mass Spectrum of the Charmed-Strange Mesons

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Abstract

This paper is about the mesons PDG calls charmed-strange (**cs** mesons). However, according to the results of n-sphere surface volume factoring, it appears that only two or three of those mesons are actually **cs** mesons. Most of the rest of them are either **cc**, **cb,** or **bt** mesons. One of them, the Ds(2713), is a tetraquark. Obviously, the rules currently used to determine quark content do not give correct results in all cases, but you can decide for yourself if the conclusions about the quark content of these mesons are correct by analyzing the reasoning behind their quark content determinations, which is explained in the paper.

Contents

1. Introduction

Using the technique of n-sphere surface volume factoring, the *quark content* of subatomic particles can be narrowed down to just a few possibilities. In some cases, it can be narrowed down to a single possibility. Even though the technique cannot always narrow down quark content possibilities of a given hadron to a single possibility, it can corroborate (say it's possibly correct) or invalidate (say it's definitely not correct) quark content determinations that have been made using other methods. This feature of the n-sphere factoring technique is how it was determined that most of the mesons in the cs meson category listed by PDG are not cs mesons. Details of what exactly n-sphere surface volume factoring is and how to use it to determine the exact masses and quark content of hadrons follows.

3. A Tutorial on n-Sphere Surface Volume Factoring

It has been discovered by trial and error, that the 3d mass of a hadron is a simple multiple of the n-sphere surface volume formulae associated with its quarks multiplied together along with Planck's constant's coefficient 'h', which is 6.62607015. The n-sphere surface volume formulae associated with each quark are given in the table below. (Sn is an abbreviation for the surface volume formula of an n-sphere.)

u	S2 =	$2\ \pi^1 r^1$
d	S3 =	$4\ \pi^1 r^2$
s	S4 =	$2\ \pi^2 r^3$
c	S5 =	$8/3\ \pi^2 r^4$
b	S6 =	$\pi^3 r^5$
t	S7 =	$16/15\ \pi^3 r^6$

Notice that this quark model can be extended to allow for an unlimited number of different types of quarks - one for each dimension of n-sphere. As an example of how to factor a hadron, we'll factor the Ds+ meson. The experimentalists say the Ds+ meson has quark content 'cs' and has a mass of 1967.0 +/-1.0 MeV. We'll assume that quark content is correct. So, to find the correct factoring unit for Ds+, multiply the associated n-sphere surface volume formulae for 'c' and 's' together (while setting r=1), then multiply by 'h'. The resulting factoring unit is **csh = 3442.3438 MeV**. That is the factoring unit needed to factor the Ds+ meson. Now divide that into the mass of the Ds+, which is 1967.0 MeV. The result should be an integer or small denominator fraction.

csh = $S_5 S_4 h = (8/3\ \pi^2 r^4)(2\ \pi^2 r^3)h = 3442.3438$ MeV

1967.0 / 3442.3438 = 0.571412994 = **4/7**

And it is (a small denominator fraction). 1967.0 = **4/7** csh. Maybe this is a fluke. Let's try factoring Ds(2460). Its experimental mass (one of them) is 2458.9 +/- 1.5. Dividing 2458.9 by csh should again result in an integer or small denominator fraction.

csh = $S_5 S_4 h = (8/3\ \pi^2 r^4)(2\ \pi^2 r^3)h = 3442.3438$ MeV

2458.9 / 3442.3438 = 0.714309817 = **5/7**

And again the result is a small denominator fraction, 2458.9 = **5/7** csh. These two examples are not flukes or coincidences. All hadrons can be factored this way. More examples of n-sphere surface volume factoring of hadrons are given in *Appendix B: Examples of Hypersphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each*. (The examples in Appendix B are actually factored using a slightly different method than the method just described – using a single n-sphere surface volume formula - which will be covered next.)

This method of factoring, using the quark content of a hadron to calculate a unit of factorization, works well if the quark content of a hadron is known, but in many cases the quark content is not known. From factoring experience, the quark content of only about 25% of hadrons have been determined correctly. It might be even less. It's definitely not more. When the quark content of a hadron is not known or is in doubt, there is an alternative method of factoring that can be employed. It is based on the fact that when two or more n-sphere surface volume formulae are multiplied together, the result is another n-sphere surface volume formula (usually), except for a difference in the constant of multiplication. This single n-sphere surface formula can then be used, after multiplying by 'h', as a unit of factorization. If a hadron factors with that formula, say it is S6 (S6h is the factoring unit), then you know the quark content of that hadron has to be one of the combinations of quarks that when their associated surface volume formulae are multiplied together, results in an S6 similar formula.

This reduces the search for the correct quark content, because only three combinations of quarks when multiplied together form an **S6** similar formula. They are **ddu**, **sd**, and **cu**. There are about 150 different quark combinations that are compatible with n-sphere surface formulae between S4 and S21, so first determining which n-sphere surface formula will factor the hadron in question narrows down the search of its quark content to just the possibilities associated with the factoring unit employed, which could be from 1 to about 12 possibilities, instead of 150. A table showing the quark combinations associated with each single n-sphere factoring unit from dimension 4 to 21 is found in *Appendix A, Quark Content Possibilities by Factoring Unit Used*. Only quark combinations that result in an n-sphere surface volume formula are listed. Not all quark combinations, when multiplied together, result in an n-sphere surface volume formula. Any quark combination containing two or more even dimension n-sphere quarks (**uu**, **ss**, **bb**, **su**, **bu**, **bs**, **ssu**, **bbs**, etc.), does not result in an n-sphere surface volume formula when the quarks' n-sphere surface volume formulae are multiplied together, so quark combinations of that description are not included in the table. It is assumed that the only quark combinations that exist are the ones that yield an n-sphere surface volume formula when the n-sphere surface volume formulae of the quarks in the combination are multiplied together.

You may agree that **ss** mesons do not exist, because there is no PDG category of **ss** mesons, but you may not agree that **bb** and **bs** mesons do not exist because PDG has categories of those types of mesons filled with particles. However, n-sphere surface volume factoring of the masses of the particles listed in those PDG categories shows that the quark contents assigned to those particles are incorrect. For instance, the first bb meson listed by PDG, the ηb(1S), factors very convincingly with S14h, as shown below, and S14 has (π, r) powers of (7,13), whereas 'bb' has (π, r) powers of (6,10). So, the ηb(1S) cannot be a 'bb' meson (contingent on whether the S14h factoring is the correct factoring, of course, which it seems to be). Several other so called 'bb' mesons factor convincingly as 'cccc' tetraquarks, which have (π, r) powers of (8,16). That's not even close to the (π, r) powers of 'bb', which again are (6, 10).

Particle	ExpMass	Error	Factoring	ThrMass	dm	dm/Error
ηb(1S)	9394.8	2.7/3.1	**169.000 S14h** =	9394.8390	.0390	1.4%

Likewise for the first particle in PDG's 'bs' category, the Bs^0. It factors convincingly as a 'cccc' tetraquark, as shown below.

Particle	ExpMass	Error	Factoring	ThrMass	dm	dm/Error
Bs^0	5366.90	.28/.23	**67 c^4h / $2^1 3^4 5^1 7^2$** =	5366.9017	.0017	0.6%

Although *all* the bb and bs mesons have *not* been factored, the ones that have been are *not* bb or bs mesons. So there is some evidence supporting the claim that the only quark combinations that exist are the ones that yield an n-sphere surface volume formula when the n-sphere surface volume formulae of the quarks in the combination are multiplied together.

Source of ExpMass and Error data: R.L. Workmanet al.(Particle Data Group), Prog.Theor.Exp.Phys.2022, 083C01 (2022)

2. Key Factorings
of the
Charmed-Strange Mesons

PDG#	Meson	ExpMass	Error	Factoring		ThrMass	Compatible Quark Content
1.	Ds	1967.0	1.0/1.0	**64/7 S8h**	=	1967.053	cs
2.	Ds*	2112.2	0.4	**12.5000 S10h**	=	2112.195	cb
		2106.6	2.1/2.7	**12.4666 S10h**	=	2106.563	cb
3.	Ds0(2317)	2317.3	0.4/0.8	**$1/2^2 7^3$ c^4h**	=	2317.265	cccc
4.	Ds1(2460)	2458.9	1.5	**80/7 S8h**	=	2458.817	cs
5.	Ds1(2536)	2434.6	0.3/0.7	**15.0000 S10h**	=	2534.634	cb
6.	Ds2(2573)	2568.39	.29/.26	**15.2000 S10h**	=	2568.430	cb
		2572.2	0.3/1.0	**15.2222 S10h**	=	2572.185	cb
7.	Ds0(2590)	2591	6/7	**15.3333 S10h**	=	2590.960	cb
8.	Ds1(2700)	2688	4/3	**13.6666 S9h**	=	2688.306	cc
		2710	2	**13.7777 S9h**	=	2710.162	cc
9.	Ds1(2860)	2859	12/24				
10.	Ds2(2860)	2866.6	(avg)	**27.0000 S12h**	=	2866.604	bt
11.	X0(2900)	2866	7/2				
12	X1(2900)	2904	5/1				
13.	Dsj(3044)	3044	8	**28.6666 S12h**	=	3043.555	bt

Commentary

As can be seen by examining the column on the extreme right in the table above, not all mesons listed in PDG's **cs** meson category are **cs** mesons. According to how they factor, there are at least four and perhaps five different types of particles in this group. Their units of factorization are: S8h, S9h, S10h, S12h, and c^4h. (c^4h is a 'cccc' tetraquark unit of factorization. S17 can also be used to factor cccc tetraquarks, but it is 200,200 times smaller than c^4h, so, to get smaller integer multipliers, c^4h is the better choice for factoring 'cccc' particles.)

The *cs mesons* that factor with S10h and are assigned quark content **bc** in the table above, could possibly be pentaquarks with quark content **ddddu**, because that quark content is also compatible with S10h factoring, and, because the masses of some particles recently discovered by the LHCb Collaboration factor similarly, and are thought to be pentaquarks. Three of them are shown factored below with S10h.

Particle	ExpMass	Error	Factoring		ThrMass	Compatible Quark Content
Pc(4337)	**4337**	7/4	**25.6666 S10h**	=	4337.0419	ddddu
Pc(4440)	**4440.3**	1.3	**26.2777 S10h**	**=**	4440.3048	ddddu
Pc(4450)	**4449.8**	1.7/2.5	**26.3333 S10h**	=	4449.6923	ddddu

The **Pc** designation the experimenters gave these particles seems to indicate they believe those particles are pentaquarks that contain at least one 'c' quark. However, that's impossible if the particles factor with S10h or S11h (which they do), because **ddddc** factors with S12h or S13h not with S10h or S11h. Since their masses factor with S10h or S11h, the **Pc** particles recently discovered must have quark content **ddddu** (if they factor with S10h) or **ddddd** (if they factor with S11h) , contingent on if they are pentaquarks, that is. (For a table showing which n-sphere surface volumes factor which quark combinations, see *Appendix A, Quark Content Possibilities by Factoring Unit Used*.)

Source of ExpMass and Error data: R.L. Workman et al.(Particle Data Group), Prog.Theor.Exp.Phys.2022, 083C01 (2022)

Mass Spectrum of D_s^+ Data

S8h Factoring / **cs** Mesons (**S_5S_4** Mesons)

Res = (4/4900) **S8h** = **0.175629788** MeV/c[2]

Range = 7.90 MeV/c[2]

	n	(n / 4900) S8h	ExpMass	Error	dm	dm/Error	
	44788	1966.5267					
	44792	1966.7024					
	44796	1966.8780					
700(64) S8h/4900	**44800**	**1967.0536**	**1967.0**	1.0/1.0	0.053	5.3%	**64/7 S8h**
	44804	1967.2293					
	44808	1967.4049					
	44812	1967.5805					
	44816	1967.7561					
	44820	1967.9318					
	44824	1968.1074					
	44828	1968.2830	**1968.3**	0.7/0.7	0.017	2.4%	
	44832	1968.4587					
	44836	1968.6343					
	44840	1968.8099					
	44844	1968.9856					
	44848	1969.1612					
	44852	1969.3368	**1969.3**	1.4/1.4	.036	2.6%	
	44856	1969.5124					
	44860	1969.6881					
701(64) S8h/4900	**44864**	**1969.8637**	**1970**	5/5	0.136	2.7%	
	44868	1970.0393					
	44872	1970.2150					
	44876	1970.3906					
	44880	1970.5662					
	44884	1970.7418					
	44888	1970.9175					
	44892	1971.0931					
	44896	1971.2687					
	44900	1971.4444					
	44904	1971.6200					
	44908	1971.7956					
	44912	1971.9713					
	44916	1972.1469					
	44920	1972.3225	**1972.4**	3.7/3.7	0.078	2.1%	
	44924	1972.4981					
702(64) S8h/4900	**44928**	**1972.6738**	**1972.7**	1.5/1.0	0.026	1.7%	
	44932	1972.8494					
	44936	1973.0250					
	44940	1973.2007					
	44944	1973.3763					
	44948	1973.5519	**1973.6**	2.6/3.0	0.048	1.9%	
	44952	1973.7276					
	44956	1973.9032					
	44960	1974.0788					
	44964	1974.2544					
	44968	1974.4301					
	44972	1974.6057					
	44976	1974.7813					
	44980	1974.9570	**1975**	9/10	0.043	0.5%	
	44984	1975.1326					
	44988	1975.3082					
703(64) S8h/4900	44992	1975.4839					
	44996	1975.6594					
	45000	1975.8351					
	45004	1976.0107					

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of Ds^{*+} Data

S10h Factoring / cb Mesons ($\mathbf{S_5S_6}$ Mesons)

Res = (1/90) **S10h** = **1.877507314** MeV/c^2

Range = 5.63 MeV/c^2

		n	(n/90) S10h	ExpMass	Error	dm	dm/Error
		1098	2061.503				
		1099	2063.381				
		1100	2065.258				
		1101	2067.136				
		1102	2069.013				
		1103	2070.891				
		1104	2072.768				
		1105	2074.646				
		1106	2076.523				
		1107	2078.401				
		1108	2080.278				
		1109	2082.156				
		1110	2084.033				
		1111	2085.911				
		1112	2087.788				
		1113	2089.666				
		1114	2091.543				
		1115	2093.421				
		1116	2095.298				
		1117	2097.176				
		1118	2099.053				
		1119	2100.931				
		1120	2102.808				
		1121	2104.686				
	12.4666 S10h	**1122**	**2106.563**	**2106.6**	2.1/2.7	0.037	1.8%
		1123	2108.441				
		1124	2110.318				
Key Mass	**12.5000 S10h**	**1125**	**2112.196**	**2112.2**	0.4	0.004	0.1%
		1126	2114.073				
		1127	2115.951				
		1128	2117.828				
		1129	2119.706				
		1130	2121.583				
		1131	2123.461				
		1132	2125.338				
		1133	2127.216				
		1134	2129.093				
		1135	2130.971				
		1136	2132.848				
		1137	2134.726				
		1138	2136.603				
		1139	2138.481				
		1140	2140.358				
		1141	2142.236				
		1142	2144.113				
		1143	2145.991				
		1144	2147.868				
		1145	2149.746				
		1146	2151.623				
		1147	2153.501				
		1148	2155.378				
		1149	2157.256				
		1150	2159.133				
		1151	2161.011				
		1152	2162.888				

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of $D_{s0}^{*}(2317)^{+}$ Data

$(S_5)^4$ h/ $(7^3 1000)$ Factoring / **cccc** Tetraquark

TQU = $(1/7^3 1000)$ $(S_5)^4$**h** = **9.269062702** MeV/c^2

Range = 3.00 Mev/c^2

	n	(250+n/128) TQU	ExpMass	Error	dm	dm/Error
	-16	2316.1070				
	-15	2316.1795				
	-14	2316.2519				
	-13	2316.3243				
	-12	2316.3967				
	-11	2316.4691				
	-10	2316.5415				
	-9	2316.6139				
	-8	2316.6864				
	-7	2316.7588				
	-6	2316.8312	**2316.8**	0.4/3.0	.0312	7.8%
	-5	2316.9036				
	-4	2316.9760				
	-3	2317.0484				
	-2	2317.1208				
	-1	2317.1933	**2317.2**	0.5/0.9	.0067	1.3%
250.000 TQU	**0**	**2317.2657**	**Key Mass**			
	0.5	2317.3018	**2317.3**	0.4/0.8	.0018	0.5%
	1	2317.3381				
	2	2317.4105				
	3	2317.4829				
	4	2317.5553				
	5	2317.6277	**2317.6**	1.3	.0277	2.1%
	6	2317.7002				
	7	2317.7726				
	8	2317.8450				
	9	2317.9174				
	10	2317.9898				
	11	2318.0622				
	12	2318.1347				
	13	2318.2071				
	14	2318.2795				
250.111 TQU	14.222	2318.2955	**2318.3**	1.2/1.2	.005	0.4%
	15	2318.3519				
	16	2318.4243				
	17	2318.4967				
	18	2318.5691				
	19	2318.6416				
	20	2318.7140				
	21	2318.7864				
	22	2318.8588				
	23	2318.9312				
	24	2319.0036				
	25	2319.0760				
	26	2319.1485				
	27	2319.2209				
	28	2319.2933				
	29	2319.3657				
	30	2319.4381				
	31	2319.5105				
250.250 TQU	**32**	2319.5829	**2319.6**	0.2/1.4	.0171	8.5%
	33	2319.6554				
	34	2319.7278				
	35	2319.8002	**2319.8**	2.1/2.0	.0002	0.0%
	36	2319.8726				
	37	2319.9450				
	38	2320.0174				
	39	2320.0898				
	40	2320.1623				

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of $D_{s1}(2460)^+$ Data

S8h Factoring / **cs** Mesons (**S_5S_4** Mesons)

Res = (2/4900) **S8h** = **0.087814894** MeV/c^2

Range = 3.07 MeV/c^2

	n	(n / 4900) S8h	ExpMass	Error	dm	dm/Error	
	55976	**2457.7633**					
	55978	2457.8511					
	55980	2457.9389					
	55982	2458.0267	**2458.0**	1.0/1.0	0.026	2.6%	
	55984	**2458.1145**					
	55986	2458.2023					
	55988	2458.2901					
	55990	2458.3780					
	55992	**2458.4658**					
	55994	2458.5536					
	55996	2458.6414	2458.6	1.0/2.5	0.041	4.1%	
	55998	2458.7292					
7000(8) =	**56000**	**2458.8170**	**2458.9**	1.5	0.083	5.5%	**80/7 S8h**
	56002	2458.9048					
	56004	2458.9927					
	56006	2459.0805					
7001(8) =	**56008**	**2459.1683**	**2459.2**	1.6/2.0	0.032	2.0%	
	56010	2459.2561					
	56012	2459.3439					
	56014	2459.4317					
7002(8) =	**56016**	**2459.5195**	**2459.5**	1.2/3.7	0.019	1.6%	PDG FIT
	56018	2459.6074					
	56020	2459.6952					
	56022	2459.7830					
7003(8) =	**56024**	**2459.8708**	**2459.9**	0.9/1.6	0.029	3.2%	
	56026	2459.9586					
	56028	2460.0464					
	56030	2460.1343	2460.1	0.2/0.8	0.034	3.4%	
7004(8) =	**56032**	**2460.2221**	**2460.2**	0.2/0.8	0.022	11.0%	
	56034	2460.3099					
	56036	2460.3977					
	56038	2460.4855					
	56040	**2460.5733**					
	56042	2460.6611					
	56044	2460.7490					
	56046	2460.8368					
	56048	**2460.9246**					
	56050	2461.0124					
	56052	2461.1002	**2461.1**	1.6	0.000	0.0%	
	56054	2461.1880					
	56056	**2461.2758**					

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of Ds1(2536)$^+$ Data

S10h Factoring / **cb** Mesons (**S_5S_6** Mesons)

Res = (1/3072) **S10h** = **0.055005097** MeV/c^2

Range = 3.63 Mev/c^2

	n	(15 + n/3072) S10h	ExpMass	Error	dm	dm/Error
	-14.000	2533.865				
	-13.000	2533.920				
-4/1024 S10h	-12.000	2533.975				
	-11.000	2534.030				
	-10.000	2534.085	**2534.1**	0.6	0.015	2.5%
	-9.000	2534.140				
	-8.000	2534.195	**2534.2**	1.2	0.005	0.4%
	-7.000	2534.250				
-2/1024 S10h	-6.000	2534.305				
	-5.000	2534.360				
	-4.000	2534.415				
	-3.000	2534.470				
	-2.000	2534.525				
	-1.000	2534.580				
	-0.500	2534.607	**2534.6**	0.3/0.7	0.007	2.3%
15.000 S10h	**0.000**	**2534.635**	**Key Mass**			
	1.000	2534.690				
	2.000	2534.745				
	2.500	2534.772	**2534.78**	.31/.40	0.008	2.6%
	3.000	2534.800	**2534.8**	0.6/0.6	0.000	0.0%
	4.000	2534.855				
	5.000	2534.910				
+2/1024 S10h	6.000	2534.965				
	7.000	2535.020	**2535**	0.6/1	0.020	3.3%
	8.000	2535.075	**2535.08**	.01/.15	0.005	50.0%
	9.000	2535.130				
	10.000	2535.185	**2535.2**	0.5/1.5	0.015	3.0%
	11.000	2535.240				
+4/1024 S10h	12.000	2535.295	**2535.3**	0.7	0.005	0.7%
	13.000	2535.350				
	14.000	2535.405				
	15.000	2535.460				
	16.000	2535.515				
	17.000	2535.570	**2535.57**	.44/.41	0.000	0.0%
	18.000	2535.625				
	19.000	2535.680				
	20.000	2535.735	**2535.7**	0.6/0.5	0.035	5.0%
	21.000	2535.790				
	22.000	2535.845				
	23.000	2535.900	**2535.9**	0.6/2.0	0.000	0.0%
	24.000	2535.955				
	25.000	2536.010				
	26.000	2536.065				
	27.000	2536.120				
	28.000	2536.175				
	29.000	2536.230				
	30.000	2536.285				
	31.000	2536.340				
	32.000	2536.395				
	33.000	2536.450				
	34.000	2536.505				
	35.000	2536.560				
	36.000	2536.615	**2536.6**	0.7/0.4	0.015	2.1%
	37.000	2536.670				
	38.000	2536.725				
	56.000	2537.715	**2537.7**	0.6/0.6	0.015	2.5%

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of $D_{s2}(2573)$, $D_{s0}(2590)^+$ Data

S10h Factoring / **cb** Mesons (**S_5S_6** Mesons)

Res = (1/900) **S10h** = **0.187750731** MeV/c^2

Range = 22.53 MeV/c^2

	n	(15 + n/900) S10h	ExpMass	Error	dm	dm/Error	Meson
	174	2567.304					
	175	2567.491					
	176	2567.679					
	177	2567.867					
	178	2568.055					
	179	2568.242					
15.200 S10h	180	2568.430	**2568.39**	.29/.26	0.040	13.8%	
	181	2568.618	**2568.6**	3.2	0.018	0.6%	
	182	2568.806					
	183	2568.993					
	184	2569.181					
	185	2569.369	**2569.4**	1.6/0.5	0.031	1.9%	
	186	2569.557					
	187	2569.744					
	188	2569.932	**2570.0**	4.3	0.068	1.6%	
	189	2570.120					
	190	2570.308					
	191	2570.495					
	192	2570.683	**2570.7**	2.0/1.7	0.017	0.9%	
	193	2570.871					
	194	2571.059					
	195	2571.246					
	196	2571.434					
	197	2571.622					
	198	2571.810					
	199	2571.997					
15.222 S10h	200	2572.185	**2572.2**	0.3/1.0	0.015	5.0%	
	201	2572.373					
	202	2572.561					
	203	2572.748					
	204	2572.936					
	205	2573.124	**2573.2**	1.7/1.6	0.076	4.5%	
	206	2573.312					
	207	2573.499					
	208	2573.687					
	209	2573.875					
	210	2574.063					
	211	2574.250					
	212	2574.438	**2574.5**	3.3/1.6	0.062	1.9%	
	213	2574.626					
	214	2574.814					
	215	2575.001					
	216	2575.189					
	217	2575.377					
	218	2575.565					
	219	2575.752					
	220	2575.940					
15.333 S10h	300	2590.960	**2591**	13	0.040	0.3%	**$D_{s0}(2590)^+$**
	301	2591.147					
	302	2591.335					
	303	2591.523					
	304	2591.711					
	305	2591.898					
	306	2592.086					

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of Ds1*(2700)+ Data

S9h Factoring / **cc** Mesons (**S_5S_5** Mesons)

Res = (.0625/9)**S9h** = **1.366009461** MeV/c^2

Range = 21.85 MeV/c^2

	n	(n/9) S9h	ExpMass	Error	dm	dm/Error
	.50000	2677.379				
	.56250	2678.745				
	.62500	2680.111				
	.68750	2681.477				
	.75000	2682.843				
	.81250	2684.209				
	.87500	2685.575				
	.93750	2686.941				
13.6666 S9h	**123.00000**	2688.307	**2688**	4	0.307	7.7%
	.06250	2689.673				
	.12500	2691.039				
	.18750	2692.405				
	.25000	2693.771	**2694**	8	0.226	2.8%
	.31250	2695.137				
	.37500	2696.503				
	.43750	2697.869				
13.7222 S9h	**123.50000**	2699.235	**2699**	14/7	0.235	1.7%
	.56250	2700.601				
	.62500	2701.967				
	.68750	2703.333				
	.75000	2704.699				
	.81250	2706.065				
	.87500	2707.431	**2707**	8/8	0.431	5.4%
	.90625	2708.114	**2708**	9	0.114	1.3%
	.93750	2708.797				
	.95555	2709.191	**2709.2**	1.9/4.5	0.009	0.6%
13.7777 S9h	**124.00000**	2710.163	**2710**	2	0.163	8.2%
	.06250	2711.529				
	.12500	2712.895				
	.18750	2714.261				
	.25000	2715.627				
	.31250	2716.993				
	.37500	2718.359				
	.43750	2719.725				
	.50000	2721.091				

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of Ds1*(2860)$^+$, Ds3*(2860)$^+$ Data

S12h Factoring / **bt** Mesons (**S_6S_7** Mesons)

Res = (1/210) **S12h** = **0.505573987** MeV/c^2

Range = 9.95 MeV/c^2

	n	(27 + n/210) S12h	ExpMass	Error	dm	dm/Error
	-24	2854.471				
	-23	2854.976				
	-22	2855.482				
	-21	2855.987				
	-20	2856.493				
26.90625 S12h		2856.651	**2856.6**	1.5/5.0	0.051	3.4%
	-19	2856.999				
	-18	2857.504				
	-17	2858.010				
	-16	2858.515				
	-15	2859.021	**2859**	12/24	0.021	0.2%
	-14	2859.526				
	-13	2860.032				
	-12	2860.538	**2860.5**	2.6/6.5	0.038	1.5%
	-11	2861.043				
	-10	2861.549				
	-9	2862.054	**2862**	2	0.054	2.7%
	-8	2862.560				
	-7	2863.065				
	-6	2863.571				
	-5	2864.077				
	-4	2864.582				
	-3	2865.088				
	-2	2865.593				
	-1	2866.099	**2866.1**	1.0/6.3	0.001	0.1%
27.00000 S12h	**0**	**2866.605**	**Key Mass**			
	1	2867.110	**2867.1**	6.2	0.010	0.2%
	2	2867.616				
	3	2868.121				
	4	2868.627				
	5	2869.132				
	6	2869.638				
	7	2870.144				
	8	2870.649				
	9	2871.155				
	10	2871.660				
	11	2872.166				
	12	2872.671				
	13	2873.177				
	14	2873.683				
	15	2874.188				
	16	2874.694				
	17	2875.199				
	18	2875.705				
	19	2876.210				
	20	2876.716				
	21	2877.222				
	22	2877.727				
	23	2878.233				
	24	2878.738				

Source of ExpMass and Error data: R.L. Workman et al.(Particle Data Group), Prog.Theor.Exp.Phys.2022, 083C01 (2022)

Mass Spectrum of D(3000)^0, Dsj(3040)$^+$ Data

S12h Factoring / **bt** Mesons (**S_6S_7**Mesons)

	n	(28 + n/90) S12h	ExpMass	Error	dm	dm/Error	Meson
	-0.8	2971.831	**2971.8**	8.7	0.031	0.4%	**D(3000)^0**
28.0000 S12h	**0**	**2972.775**	**Key Mass**				
	1	2973.955					
	2	2975.134					
	3	2976.314					
	4	2977.494					
	5	2978.673					
	6	2979.853					
	7	2981.033					
	8	2982.212					
	9	2983.392					
	10	2984.572					
	11	2985.751					
	12	2986.931					
	13	2988.111					
	14	2989.290					
	15	2990.470					
	16	2991.650					
	17	2992.829					
	18	2994.009					
	19	2995.189					
	20	2996.368					
	21	2997.548					
	22	2998.728					
	23	2999.908					
	24	3001.087					
	25	3002.267					
	26	3003.447					
	27	3004.626					
	28	3005.806					
	29	3006.986					
28.3333 S12h	30	3008.165	**3008.1**	4.0	0.065	1.6%	**D(3000)^0**
	31	3009.345					
	32	3010.525					
	33	3011.704					
	34	3012.884					
	35	3014.064					
	36	3015.243					
	37	3016.423					
	38	3017.603					
	39	3018.782					
	40	3019.962					
	41	3021.142					
	42	3022.321					
	43	3023.501					
	44	3024.681					
	45	3025.860					
	46	3027.040					
	47	3028.220					
	48	3029.399					
	49	3030.579					
	50	3031.759					
	51	3032.938					
	52	3034.118					
	53	3035.298					
	54	3036.477					
	55	3037.657					
	56	3038.837					
	57	3040.016					
	58	3041.196					
	59	3042.376					
28.6666 S12h	60	3043.555	**3044**	8	0.445	5.6%	**DsJ(3040)$^+$**
	61	3044.735					
	62	3045.915					

Source of ExpMass data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

9.0 Conclusions

As shown by n-sphere surface volume factoring, seven of the thirteen particles listed by PDG in its Charmed-Strange Meson category are definitely *not cs mesons*. Those are the four that factor with **S10h**, the two that factor with **S12h**, and the tetraquark - Ds(2317) - that factors with **c^4h**. Three of the *cs mesons* were not factored because of having only one data point to factor, and/or, having mass measurements too inaccurate to be confident of the correctness of any factoring.

The three mesons remaining factor with S8h/S9h, meaning they could have **cs** quark content. But they could also have **cc** quark content, because that quark content is also compatible with S8h/S9h factoring. But even if these three are correct, seven of ten cs meson quark content determinations are definitely wrong.

The main conclusion to draw from all this is that the method, or, some of the methods of quark content determination currently being used are flawed. Unfortunately, n-sphere surface volume factoring cannot narrow down the quark content of a given hadron to a single possibility, except for hadrons that factor with S4h or S5h. However, it can provide a short list of quark content possibilities for any hadron for which a convincing factoring can be found, which can then be used to corroborate or invalidate quark content determinations found using other methods.

10.0 References

[1] P.A.Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

[2] Mass spectra and strong decays of charmed and charmed-strange mesons, arXiv:2110.05024v2 [hep-ph] 1 March 2022

APPENDIX A

Quark Content Possibilities by Factoring Unit Used

	Factoring Unit If……. Mass factors with		Quark Content Possibilities Then……. Hadron has one of these Quark Contents								
u	S2h	= (1, 1)									
d	S3h	= (1 ,2)									
s	S4h	= (2, 3)	du								
c	S5h	= (2, 4)	dd								
b	S6h	= (3, 5)	ddu	sd, cu							
t	S7h	= (3, 6)	ddd	cd							
v	S8h	= (4, 7)	dddu	dds	cs , bd, tu						
w	S9h	= (4, 8)	dddd	ddc	cc , td						
x	S10h	= (5, 9)	ddddu	ddds	dcs	bc, ts					
y	S11h	= (5, 10)	ddddd	dddc	dcc	t c					
z	S12h	= (6, 11)	dddddu	sdddd	csdd	ccs	tb, vc				
	S13h	= (6, 12)	dddddd	cdddd	ccdd	ccc	t t,				
	S14h	= (7, 13)	ddddddu	ddddds	dddcs	dccs	bcc	tv			
	S15h	= (7, 14)	ddddddd	dddddc	dddcc	dccc	t cc	tw			
	S16h	= (8, 15)	dddddddu	dddddds	ddddcs	ddccs	cccs	btc	wv		
	S17h	= (8, 16)	dddddddd	ddddddc	ddddcc	ddccc	cccc	t t s	ww		
	S18h	= (9, 17)	ddddddddu	ddddddds	dddddcs	dddccs	dcccs	cccb	t t b	wx	
	S19h	= (9, 18)	ddddddddd	dddddddc	dddddcc	dddccc	dcccc	ccct	t t t	wy	
	S20h	= (10, 19)	dddddddddu	dddddddds	ddddddcs	ddddccs	ddcccs	ccccs	cccv	t t w	xy
	S21h	= (10, 20)	dddddddddd	ddddddddc	ddddddcc	ddddccc	ddcccc	ccccc	cccw	t t x	yy

Note: s=du c=dd b=sd=cu t=cd

All quark combinations for the factoring units from S4h to S9h are shown. For the factoring units from S10h to S21h not all possible quark combinations are shown, especially for the triquarks (qqq, baryons) and the diquarks (qq, mesons). This was done so the table wouldn't look too complex and potentially confusing.

The parentheses enclosing two integers separated by a comma that is just to the right of the factoring units, such as the (1,2) in the line S3h = (1,2), means the surface volume formula of that factoring unit has the powers 1 and 2 for 'π' and 'r'. In the case of S3h, $S3 = 4\pi^1 r^2$. 'π' is raised to the power 1, and 'r' is raised to the power 2, that's why it's written S3h = (1,2). Using this parentheses notation for surface volume formula representation makes it easy to determine which factoring unit will factor which quark combinations, or vice versa, which quark combinations can be factored by which factorung unit.

For instance, if you want to know which factoring unit will factor 'ddd', since 'd' = S3 = (1,2), just add the corresponding integers together of the product (1,2)(1,2)(1,2). You are multiplying numbers together ('π′ and 'r') that are raised to integer powers, and, powers add, so you get (3,6). Now find the line with (3,6) in it. It is S7h = (3,6). So the factoring unit needed to factor 'ddd' is S7h.

APPENDIX B

Examples of n-Sphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each

Subatomic Particle	ExpMass	Error	HSSV Factoring	ThrMass	Compatible QuarkContent
ρ(770)	775.02	0.35	**4.44444 S5h** =	775.071	dd
η	547.865	0.031	**2.66666 S6h** =	547.8660	ds
Δ(1232)	1232.9	1.2	**6.00000 S6h** =	1232.698	ddu
K(1430)	1438	8/4	**7.00000 S6h** =	1438.148	ds
Δ(1700)	1643	6/3	**8.00000 S6h** =	1643.598	ddu
Xi^0	1314.86	0.20	**6.00000 S7h** =	1314.878	ddd
Xi^-	1321.71	0.07	**6.03125 S7h** =	1321.727	ddd
a2(1700)	1721	11/44	**8.00000 S8h** =	1721.172	cs
Ds	1967.0	1.0/1.0	**64/7 S8h** =	1967.053	cs
Ds(2460)	2458.9	1.5	**80/7 S8h** =	2458.817	cs
B2(5747)	5737.2	0.7	**26.66666 S8h =**	5737.239	bd
Ds	1967.0	1.0/1.0	**10.00000 S9h** =	1967.053	cc
Ds(2460)	2458.9	1.5	**12.50000 S9h** =	2458.817	cc
Ds(2700)	2688	4	**13.66666 S9h** =	2688.307	cc
Ds(2700)	2710	2	**13.77777 S9h** =	2710.163	cc
Bj(5732)	5704	4/10	**29.00000 S9h** =	5704.455	cc
Ds(2212)	2112.2	0.4	**12.5000 S10h** =	2112.195	bc
Ω(2250)	2253	13	**13.3333 S10h** =	2253.008	dcs
Ds1(2536)	2534.6	0.3/0.7	**15.0000 S10h** =	2534.634	bc
Ds2(2572)	2572.2	0.3/1.0	**15.2222 S10h** =	2572.185	bc
Ds0(2590)	2591	13	**15.3333 S10h** =	2590.960	bc
Pc(4337)	4337	7/4	**25.6666 S10h** =	4337.041	ddddu
Pc(4457)	4449.8	1.7/2.5	**26.3333 S10h** =	4449.692	ddddu
Y(4500)	4506	11	**26.6666 S10h** =	4506.017	ddddu
b1(1235)	1236	16	**9.0000 S11h** =	1235.936	ddddd
X(2175)	2197.4	4.4	**16.0000 S11h** =	2197.219	ddddd
Z(3985)	3982.5	1.8	**29.0000 S11h** =	3982.461	ddddd
X(4660)	4669	21/3	**34.0000 S11h** =	4669.092	ddddd
Ds(2860)	2866.6(avg)		**27.0000 S12h** =	2866.605	bt
$D(3000)^0$	2971.8	8.7	**28.0000 S12h** =	2972.775	bt
$D(3000)^0$	3008.1	4.0	**28.3333 S12h** =	3008.165	bt
Dsj(3040)	3044	8	**28.6666 S12h** =	3043.555	bt
Λ	1115.59	0.08	**14.2222 S13h** =	1115.599	ccc
Ω	1673.4	1.7	**21.3333 S13h** =	1673.398	ccc
Xi(1950)	1952	11	**24.8888 S13h** =	1952.298	ccc
Xi(2500)	2505	10	**31.9375 S13h** =	2505.195	ccc
fj(2220)	2223.9	2.5	**40.0000 S14h** =	2223.630	vt
Xc0(1P)	3415.5	0.4/0.4	**61.4400 S14h** =	3415.496	ccsd
Xc2(1P)	3557.8	0.2/4	**64.0000 S14h** =	3557.808	ccsd
ηb(1S)	9394.8	2.7/3.1	**169.0000 S14h** =	9394.839	vt
f0(980)	977.3	0.9/3.7	**99.7500 S18h** =	977.298	cccb
f0(980)	982.2	1.0/8.1	**100.2500 S18h** =	982.197	cccb
f0(980)	984.7	0.4/2.4	**100.5000 S18h** =	984.646	cccb

Source of ExpMass and Error data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

APPENDIX C

Hypersphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

Sphere Dimension	Sn	Surface Volume Formula	(π, r) Powers
2	S2 =	2 $\pi^1 r^1$	(1, 1)
3	S3 =	4 $\pi^1 r^2$	(1, 2)
4	S4 =	2 $\pi^2 r^3$	(2, 3)
5	S5 =	8/3 $\pi^2 r^4$	(2, 4)
6	S6 =	$\pi^3 r^5$	(3, 5)
7	S7 =	16/15 $\pi^3 r^6$	(3, 6)
8	S8 =	1/3 $\pi^4 r^7$	(4, 7)
9	S9 =	32/105 $\pi^4 r^8$	(4, 8)
10	S10 =	1/12 $\pi^5 r^9$	(5, 9)
11	S11 =	64 / 945 $\pi^5 r^{10}$	(5, 10)
12	S12 =	1 / 60 $\pi^6 r^{11}$	(6, 11)
13	S13 =	128 / 10395 $\pi^6 r^{12}$	(6, 12)
14	S14 =	1 / 360 $\pi^7 r^{13}$	(7, 13)
15	S15 =	256 / 135135 $\pi^7 r^{14}$	(7, 14)
16	S16 =	1 / 2520 $\pi^8 r^{15}$	(8, 15)
17	S17 =	512 / 2027025 $\pi^8 r^{16}$	(8, 16)
18	S18 =	1 / 20160 $\pi^9 r^{17}$	(9, 17)
19	S19 =	1024 / 34459425 $\pi^9 r^{18}$	(9, 18)
20	S20 =	1 / 181440 $\pi^{10} r^{19}$	(10, 19)
21	S21 =	2048 / 654729075 $\pi^{10} r^{20}$	(10, 20)

APPENDIX D

Values of Hypersphere Surface Volume Units of Factorization

(Dimension 2 - Dimension 21)

Sphere Dimension	Unit of Factorization		Formula		Value (MeV/c^2)
2	S2h	=	2 $\pi^1 r^1 h$	=	41.63282661
3	S3h	=	4 $\pi^1 r^2 h$	=	83.26565322
4	S4h	=	2 $\pi^2 r^3 h$	=	130.7933822
5	S5h	=	8/3 $\pi^2 r^4 h$	=	174.3911763
6	S6h	=	$\pi^3 r^5 h$	=	205.4497644
7	S7h	=	16/15 $\pi^3 r^6 h$	=	219.1464153
8	S8h	=	1/3 $\pi^4 r^7 h$	=	215.1464901
9	S9h	=	32/105 $\pi^4 r^8 h$	=	196.7053624
10	S10h	=	1/12 $\pi^5 r^9 h$	=	168.9756582
11	S11h	=	64 / 945 $\pi^5 r^{10} h$	=	137.3262492
12	S12h	=	1 / 60 $\pi^6 r^{11} h$	=	106.1705373
13	S13h	=	128 / 10395 $\pi^6 r^{12} h$	=	78.44057013
14	S14h	=	1 / 360 $\pi^7 r^{13} h$	=	55.59076334
15	S15h	=	256 / 135135 $\pi^7 r^{14} h$	=	37.91204905
16	S16h	=	1 / 2520 $\pi^8 r^{15} h$	=	24.94907624
17	S17h	=	512 / 2027025 $\pi^8 r^{16} h$	=	15.88056197
18	S18h	=	1 / 20160 $\pi^9 r^{17} h$	=	9.797479330
19	S19h	=	1024 / 34459425 $\pi^9 r^{18} h$	=	5.869441980
20	S20h	=	1 / 181440 $\pi^{10} r^{19} h$	=	3.419965454
21	S21h	=	2048 / 654729075 $\pi^{10} r^{20} h$	=	1.940989032