Evidence of Planck-constant-like constant in five planetary systems and its significances

Huaiyang Cui
Department of Physics, Beihang University, Beijing, 102206, China
Email: hycui@buaa.edu.cn

Abstract: In a many-body system with the total mass M, a constituent particle moves at the speed v, its wavelength of matter wave is modified as $2\pi hM/v$, where h is a Planck-constant-like constant, it is found that this modified de Broglie's matter wave quantizes orbits correctly. The solar system, Jupiter's satellites, Saturn's satellites, Uranus' satellites, Neptune's satellites are five different many-body systems under investigation with Bohr orbit model using the modified matter wave, by fitting observational data, it is found that these five systems have almost the same Planck-constant-like constant. Considering that the Bohr orbit model completely ignores interactions between those constituent particles, the same-magnitude Planck-constant-like constant is a remarkable discovery, has great significance for introducing wave-particle duality to gravity.

This year is 99th anniversary of the initiative of de Broglie's matter wave. In 1922, the Louis de Broglie considered blackbody radiation as a gas of light quanta [1], he tried to reconcile the concept of light quanta with the phenomena of interference and diffraction. In 1923 and 1924, the concept that matter behaves like a wave was proposed by Louis de Broglie [2,3,4]. It is also referred to as the de Broglie hypothesis, matter waves are referred to as de Broglie waves [5,6,7]. Direct generalization of matter wave to planetary structure never achieves success. This letter reports that there is an approach to quantize the planetary structure by modified de Broglie's matter wave. In a many-body system with the total mass M, a constituent particle has the mass m and moves at the speed v, its wavelength of matter wave is modified as

$$\lambda = \frac{2\pi\hbar}{mv} \quad \Rightarrow \text{modify} \Rightarrow \quad \lambda = \frac{2\pi hM}{v} \; . \tag{1}$$

where h is a Planck-constant-like constant. It is found that this modified de Broglie's matter wave works for quantizing orbits correctly. In the Bohr orbit model, the circular quantization condition is given by

$$\left.\begin{aligned} \frac{2\pi r}{\lambda} = \frac{2\pi r}{2\pi hM/v} = n \\ v = \sqrt{\frac{GM}{r}} \end{aligned}\right\} \quad \Rightarrow \quad \sqrt{r} = h\sqrt{\frac{M}{G}}n \tag{2}$$

It indicates that there is a linear relation between the square root of radius and the quantum number

n. After fitting observational data, the Planck-constant-like constant h is obtained in Table 1, the predicted quantization in Fig.1, Fig.2, Fig.3, Fig.4 and Fig.5 agrees well with experimental observations for those inner constituent particles. The key point is that the various systems have almost same Planck-constant-like constant h in Table 1 with the mean value of 3.51e-16 $m^2s^{-1}kg^{-1}$, at least have the same magnitude!

Table 1 Planck-constant-like constant h in various systems, N is constituent particle number with smaller inclination.

system	N	h ($m^2s^{-1}kg^{-1}$)	M/M_{earth}	Prediction
Solar planets	9	4.574635e-16	333000	Fig.1
Jupiter' satellites	7	3.531903e-16	318	Fig.2
Saturn's satellites	7	6.610920e-16	95	Fig.3
Uranus's satellites	18	1.567124e-16	14.5	Fig.4
Neptune 's satellites	7	1.277170e-16	17	Fig.5

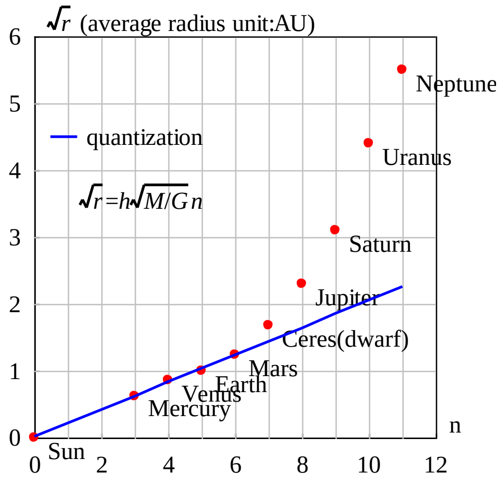

Fig.1 The orbital radii are quantized for inner 4 planets in the solar system with h=4.574635e-16 ($m^2s^{-1}kg^{-1}$). The relative error is less than 3.9%.

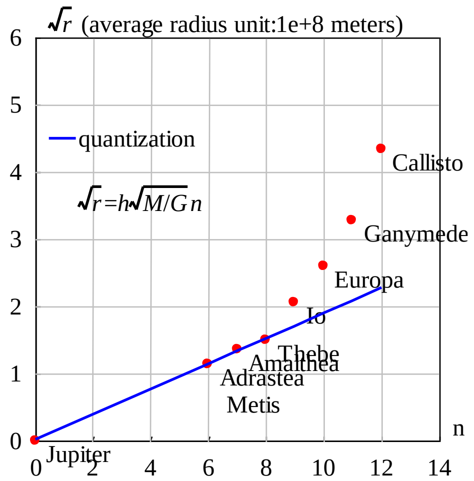

Fig.2 The orbital radii are quantized for inner 4 satellites in the Jupiter system with h=3.531903e-16 ($m^2s^{-1}kg^{-1}$). Metis and Adrastea are assigned the same quantum number for their almost same radius. The relative error is less than 1.9%.

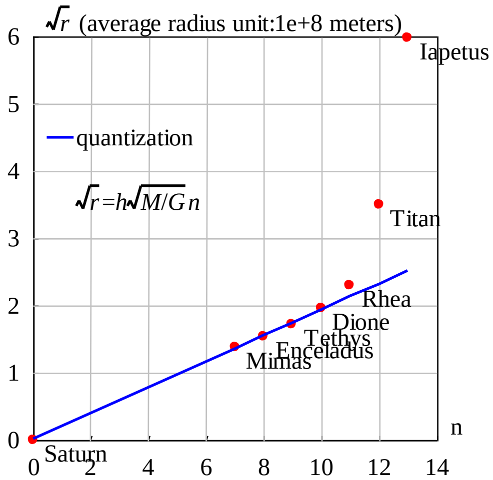

Fig.3 The orbital radii are quantized for inner 4 satellites in the Saturn system with h=6.610920e-16 ($m^2s^{-1}kg^{-1}$). The relative error is less than 1.1%.

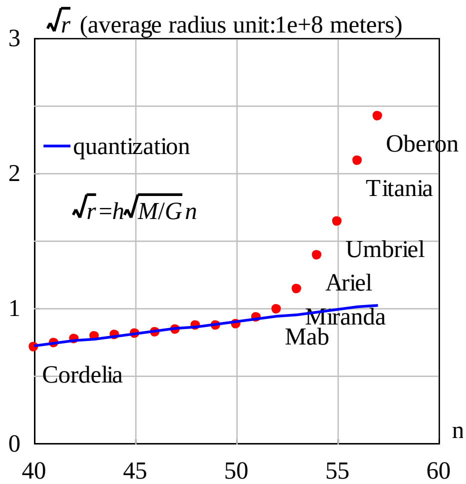

Fig.4 The orbital radii are quantized for inner 12 satellites in the Uranus system with h=1.567124e-16 ($m^2s^{-1}kg^{-1}$). n=0 is assigned to the Uranus. The relative error is less than 2.5%.

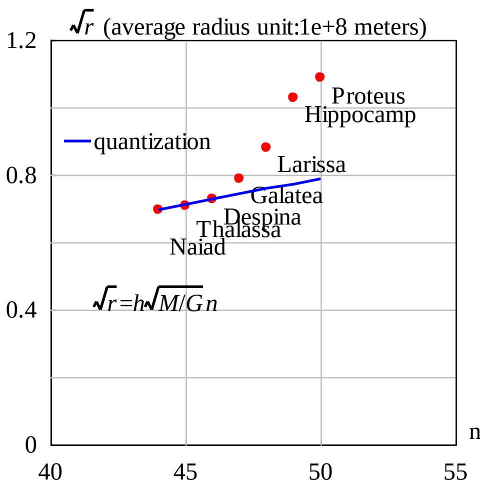

Fig.5 The orbital radii are quantized for inner 3 satellites in the Neptune system with h=1.277170e-16 ($m^2s^{-1}kg^{-1}$). n=0 is assigned to the Neptune. The relative error is less than 0.17%.

The offset of quantum number n has no influence on the h value, because h corresponds to the slope of linear regression line in the figures, but we insist that the line goes through n=0. Appendix A presents the related data and detailed calculations. The first clue of this approach came from the author's early superconductivity research [9], where de Broglie's matter waves must subject to SU(n) group symmetry in a strongly correlated system.

Considering that the Bohr orbit model completely ignores the interactions between those constituent particles, the same-magnitude Planck-constant-like constant h in Table 1 is a remarkable discovery. There are three important significances for physics, at the first, the Planck-constant-like constant is an "official license" for introducing quantum theory to gravity; the next, its fluctuation about the mean value can be used to estimate dark matter mass and constituent interactions; the third, as the total mass decreases, the wavelength drops down dramatically, tiny exoplanet systems may have easily-observable wave-particle duality.

In conclusions, in a many-body system with the total mass M, a constituent particle moves at the speed v, its wavelength of matter wave is modified as $2\pi hM/v$, where h is a Planck-constant-like constant, it is found that this modified de Broglie's matter wave quantizes orbits correctly. The solar system, Jupiter's satellites, Saturn's satellites, Uranus' satellites, Neptune's satellites are five different many-body systems under investigation with Bohr orbit model using the modified matter wave, by fitting observational data, it is found that these five systems have almost the same Planck-constant-like constant. Considering that the Bohr orbit model completely ignores interactions between those constituent particles, the same-magnitude Planck-constant-like constant is a remarkable discovery, has great significance for introducing wave-particle duality to gravity.

References

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Appendix A: related data and detailed calculations

Many-body systems can be divided into inner constituent particles (such as Mercury, Venus, Earth, Mars in the solar system) and outer constituent particles (such as Jupiter, Saturn, Uranus and Neptune in solar system), we focus on inner constituent particles in the fallowing systems.

(1) The solar system

The data of the solar system is listed in Table 2. Every planet is assigned a quantum number n, the sun is assigned to n=0.

Table 2 Planets in the solar system [8].

n	Name	Mean Semimajor axis (AU)	Eccentricity
0	Sun	0	0
3	Mercury	0.3870	0.2056
4	Venus	0.7233	0.0067
5	Earth	1	0.0167
6	Mars	1.5236	0.0934
7	Ceres	2.78	0
8	Jupiter	5.2033	0.0483
9	Saturn	9.5370	0.0541
10	Uranus	19.1912	0.0471
11	Neptune	30.0689	0.0085

n	Name	r_sqrt	r_sqrt_predict	h_predict ($m^2s^{-1}kg^{-1}$)
3	Mercury	0.622093	0.612508	4.646225e-16
4	Venus	0.850470	0.816677	4.763929e-16
5	Earth	1.000000	1.020846	4.481218e-16
6	Mars	1.234342	1.225016	4.609462e-16

C Source Codes:

```
int i,N; double GravityC,error,Earth_Mass,M,r_unit,h,h_predict,r_mean,r_sqrt,r_sqrt_predict;
double r_orbit[20]={0,0.3870,0.7233,1,1.5236,2.78,5.2033,9.5370,19.1912,30.0689,};
double Eccentricity[20]={0,0.2056,0.0067,0.0167,0.0934,0.0483,0.0541,0.0471,0.0085,};
int quantum_number[20]={0,3,4,5,6,7,8,9,10,11,12,13,14,15};
int main(){GravityC=6.67408E-11;Earth_Mass=5.97237e24;M=333000*Earth_Mass;
r_unit=1.496E11; h=4.574635e-16; N=10;
for(i=1;i<N;i+=1) { r_mean= r_orbit[i];
r_sqrt=sqrt(r_mean); r_sqrt_predict=h*sqrt(M/GravityC)*quantum_number[i]/sqrt(r_unit);
h_predict=sqrt(r_mean*r_unit)/(sqrt(M/GravityC)*quantum_number[i]);
error=(r_sqrt_predict-r_sqrt)/r_sqrt;
printf("n=%d, r_mean=%f, r_sqrt=%f, r_sqrt_predict=%f, relative_error=%f, h_predict=%e \r\n",
quantum_number[i],r_mean,r_sqrt,r_sqrt_predict,error,h_predict);}}
```

The orbital radii are quantized for inner 4 planets in the solar system with h=4.574635e-16($m^2s^{-1}kg^{-1}$). The relative error is less than 3.9%.

(2) The Jupiter's satellites

Jupiter is the fifth planet from the Sun and the largest in the solar system with the mass 317.9 Earths. Jupiter has 79 known natural satellites. Table3 collects 8 Jupiter's satellites with inclination less than 3 degrees.

Table 3 Jupiter's satellites with smaller inclination[8].

n	Name	Semimajor axis (1e+8m)	Eccentricity	Inclination (°)
0	Jupiter	0	0	
6	Metis	1.28852	0.0077	2.226
6	Adrastea	1.29000	0.0063	2.217
7	Amalthea	1.81366	0.0075	2.565
8	Thebe	2.22452	0.0180	2.909
9	Io	4.21700	0.0041	0.050
10	Europa	6.71034	0.0094	0.471
11	Ganymede	10.70412	0.0011	0.204
12	Callisto	18.82709	0.0074	0.205

n	Name	r_sqrt	r_sqrt_predict	h_predict ($m^2s^{-1}kg^{-1}$)
6	Metis	1.133578	1.130450	3.541678e-16
6	Adrastea	1.135782	1.130450	3.548562e-16
7	Amalthea	1.345362	1.318858	3.602882e-16
8	Thebe	1.489966	1.507266	3.491365e-16

C Source Codes

```
int i,N; double GravityC,error,Earth_Mass,M,r_unit,h,h_predict,r_mean,r_sqrt,r_sqrt_predict;
double r_orbit[20]={0,1.285,1.29,1.81,2.22,4.21,6.71,10.7,18.82,};
double Eccentricity[20]={0,0.0077,0.0063,0.0075,0.0180,0.0041,0.0094,0.0011,0.0074,0,0,0,};
int quantum_number[20]={0,6,6.1,7,8,9,10,11,12,13,14,15,16,17,18,19,20};
int main(){ GravityC=6.67408E-11;Earth_Mass=5.97237e24;M=318*Earth_Mass;
r_unit=1E8; h=3.531903e-16; N=9;
for(i=1;i<N;i+=1) { r_mean= r_orbit[i]*(1+sqrt(1-Eccentricity[i]*Eccentricity[i]))/2;
r_sqrt=sqrt(r_mean); r_sqrt_predict=h*sqrt(M/GravityC)*quantum_number[i]/sqrt(r_unit);
h_predict=sqrt(r_mean*r_unit)/(sqrt(M/GravityC)*quantum_number[i]);
error=(r_sqrt_predict-r_sqrt)/r_sqrt;
printf("n=%d, r_mean=%f, r_sqrt=%f, r_sqrt_predict=%f, relative_error=%f, h_predict=%e \r\n",
quantum_number[i],r_mean,r_sqrt,r_sqrt_predict,error,h_predict);}}
```

The orbital radii are quantized for inner 4 satellites in the Jupiter system with h=3.531903e-16($m^2s^{-1}kg^{-1}$). Metis and Adrastea are assigned the same quantum number for their almost same radii. The relative error is less than 1.9%.

(3) The Saturn's satellites

Saturn is the sixth planet from the Sun and the second-largest in the Solar System, after Jupiter. Saturn has 82 known moons, 53 of which have formal names. In addition, there is evidence of dozens to hundreds of moonlets with diameters of 40–500 meters in Saturn's rings, which are not considered to be true moons. Table 4 collects 7 Saturn's satellites with smaller inclination.

Table 4 Saturn's satellites with smaller inclination[8].

n	Name	Orbital radius(1e+8m)
0	Saturn	0
7	Mimas	1.85539
8	Enceladus	2.37948
9	Tethys	2.94,619
10	Dione	3.77396
11	Rhea	5.27108
12	Titan	12.21870
13	Iapetus	35.60820

n	Name	r_sqrt	r_sqrt_predict	h_predict ($m^2s^{-1}kg^{-1}$)
7	Mimas	1.360147	1.349272	6.664205e-16
8	Enceladus	1.539480	1.542025	6.600012e-16
9	Tethys	1.714643	1.734778	6.534189e-16
10	Dione	1.941649	1.927531	6.659341e-16

C Source Codes

```
int i,N; double GravityC,error,Earth_Mass,M,r_unit,h,h_predict,r_mean,r_sqrt,r_sqrt_predict;
double r_orbit[20]={0,1.85,2.37,2.94,3.77,5.27,12.21,35.6,};
double Eccentricity[20]={0,0,0,0,0,0,0,0,0,0,0,0,0,};
int quantum_number[20]={0,7,8,9,10,11,12,13,14,15,16,17,18,19,20};
int main(){ GravityC=6.67408E-11;Earth_Mass=5.97237e24;M=95*Earth_Mass;
r_unit=1E8; h=6.610920e-16; N=8;
for(i=1;i<N;i+=1) { r_mean= r_orbit[i]*(1+sqrt(1-Eccentricity[i]*Eccentricity[i]))/2;
r_sqrt=sqrt(r_mean); r_sqrt_predict=h*sqrt(M/GravityC)*quantum_number[i]/sqrt(r_unit);
h_predict=sqrt(r_mean*r_unit)/(sqrt(M/GravityC)*quantum_number[i]);
error=(r_sqrt_predict-r_sqrt)/r_sqrt;
printf("n=%d, r_mean=%f, r_sqrt=%f, r_sqrt_predict=%f, relative_error=%f, h_predict=%e \r\n",
quantum_number[i],r_mean,r_sqrt,r_sqrt_predict,error,h_predict);}}
```

The orbital radii are quantized for inner 4 satellites in the Saturn system with h=6.610920e-16($m^2s^{-1}kg^{-1}$). The relative error is less than 1.1%.

(4) The Uranus' satellites

Uranus is the seventh planet of the Solar System, has 27 known moons. Table 5 collects 18 Uranus' satellites with inclination less than 5 degrees.

Table 5 Uranus' satellites with smaller inclination[8]

n	Name	Semimajor axis (1e+8m)	Eccentricity	Inclination (°)
0	Uranus	0	0	
40	Cordelia	0.49770	0.00026	0.08479
41	Ophelia	0.53790	0.00992	0.1036
42	Bianca	0.59170	0.00092	0.193
43	Cressida	0.61780	0.00036	0.006
44	Desdemona	0.62680	0.00013	0.11125
45	Juliet	0.64350	0.00066	0.065
46	Portia	0.66090	0.00005	0.059
47	Rosalind	0.69940	0.00011	0.279
48	Cupid	0.74800	0.0013	0.100
49	Belinda	0.75260	0.00007	0.031

50	Perdita	0.76400	0.0012	0.0
51	Puck	0.86010	0.00012	0.3192
52	Mab	0.97700	0.0025	0.1335
53	Miranda	1.29390	0.0013	4.232
54	Ariel	1.91020	0.0012	0.260
55	Umbriel	2.66300	0.0039	0.205
56	Titania	4.35910	0.0011	0.340
57	Oberon	5.83520	0.0014	0.058

n	Name	r_sqrt	r_sqrt_predict	h_predict ($m^2s^{-1}kg^{-1}$)
40	Cordelia	0.705479	0.714044	1.548325e-16
41	Ophelia	0.733417	0.731895	1.570382e-16
42	Bianca	0.769220	0.749746	1.607829e-16
43	Cressida	0.786003	0.767597	1.604700e-16
44	Desdemona	0.791707	0.785448	1.579611e-16
45	Juliet	0.802185	0.803300	1.564949e-16
46	Portia	0.812958	0.821151	1.551488e-16
47	Rosalind	0.836301	0.839002	1.562080e-16
48	Cupid	0.864870	0.856853	1.581787e-16
49	Belinda	0.867525	0.874704	1.554263e-16
50	Perdita	0.874071	0.892555	1.534670e-16
51	Puck	0.927416	0.910406	1.596403e-16

C Source Codes

```
int i,N; double GravityC,error,Earth_Mass,M,r_unit,h,h_predict,r_mean,r_sqrt,r_sqrt_predict;
double r_orbit[20]={0,0.49770,0.53790,0.59170,0.61780,0.62680,0.64350,0.66090,0.69940,0.74800,
0.75260,0.76400,0.86010,0.97700,1.29390,1.91020,2.66300,4.35910,5.83520,};
double Eccentricity[20]={0,0.00026,0.00992,0.00092,0.00036,0.00013,0.00066,0.00005,
0.00011,0.0013,0.00007,0.0012,0.00012,0.0025,0.0013,0.0012,0.0039,0.0011,0.0014,};
int quantum_number[22]={0,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60};
int main(){ GravityC=6.67408E-11;Earth_Mass=5.97237e24;M=14.5*Earth_Mass;
r_unit=1E8; h=1.567124e-16; N=19;
for(i=1;i<N;i+=1) { r_mean= r_orbit[i]*(1+sqrt(1-Eccentricity[i]*Eccentricity[i]))/2;
r_sqrt=sqrt(r_mean); r_sqrt_predict=h*sqrt(M/GravityC)*quantum_number[i]/sqrt(r_unit);
h_predict=sqrt(r_mean*r_unit)/(sqrt(M/GravityC)*quantum_number[i]);
error=(r_sqrt_predict-r_sqrt)/r_sqrt;
printf("n=%d, r_mean=%f, r_sqrt=%f, r_sqrt_predict=%f, relative_error=%f, h_predict=%e \r\n",
quantum_number[i],r_mean,r_sqrt,r_sqrt_predict,error,h_predict);}}
```

The orbital radii are quantized for inner 12 satellites in the Uranus system with h=1.567124e-16($m^2s^{-1}kg^{-1}$). To note that n=0 is in the linear relation. The relative error is less than 2.5%.

(5) The Neptune's satellites

The planet Neptune has 14 known moons, which are named for minor water deities in Greek mythology. Table 6 collects 7 Neptune's satellites with inclination less than 5 degrees.

Table 6 Neptune's satellites with smaller inclination[8].

n	Name	Semimajor axis (1e+8m)	Eccentricity	Inclination (°)
0	Neptune	0	0	
44	Naiad	48224	0.0047	4.691
45	Thalassa	50074	0.0018	0.135

46	Despina	52526	0.0004	0.068
47	Galatea	61953	0.0001	0.034
48	Larissa	73548	0.0012	0.205
49	Hippocamp	105283	0.0005	0.064
50	Proteus	117646	0.0005	0.075

n	Name	r_sqrt	r_sqrt_predict	h_predict ($m^2s^{-1}kg^{-1}$)
44	Naiad	0.694435	0.693112	1.279608e-16
45	Thalassa	0.707630	0.708865	1.274945e-16
46	Despina	0.724748	0.724617	1.277401e-16

C Source Codes

```
int i,N; double GravityC,error,Earth_Mass,M,r_unit,h,h_predict,r_mean,r_sqrt,r_sqrt_predict;
double r_orbit[10]={0,0.48224,0.50074,0.52526,0.61953,0.773548,1.05283,1.17646,};
double Eccentricity[10]={0,0.0047,0.0018,0.0004,0.0001,0.0012,0.0005,0.0005,0,0,};
int quantum_number[20]={0,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,};
int main(){ GravityC=6.67408E-11;Earth_Mass=5.97237e24;M=17*Earth_Mass;
r_unit=1E8; h=1.277170e-16; N=8;
for(i=1;i<N;i+=1) { r_mean= r_orbit[i]*(1+sqrt(1-Eccentricity[i]*Eccentricity[i]))/2;
r_sqrt=sqrt(r_mean); r_sqrt_predict=h*sqrt(M/GravityC)*quantum_number[i]/sqrt(r_unit);
h_predict=sqrt(r_mean*r_unit)/(sqrt(M/GravityC)*quantum_number[i]);
error=(r_sqrt_predict-r_sqrt)/r_sqrt;
printf("n=%d, r_mean=%f, r_sqrt=%f, r_sqrt_predict=%f, relative_error=%f, h_predict=%e \r\n",
quantum_number[i],r_mean,r_sqrt,r_sqrt_predict,error,h_predict);}}
```

The orbital radii are quantized for inner 3 satellites in the Neptune system with h=1.277170e-16($m^2s^{-1}kg^{-1}$). To note that n=0 is in the linear relation. The relative error is less than 0.17%.