

Summations of Single Terms and Successive Terms of Geometric Series

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Abstract: This paper presents the summations of separate terms and successive terms of geometric series. This will be useful for the researchers who are involving to solve the scientific problems.

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Summations of successive terms of geometric series

In this paper, the summations [1-3] of single term and successive terms of geometric series are constituted for the researchers. The geometric series are used in the areas of science, technology, and management [4].

Summation of one term of geometric series:

$$1 = \frac{x-1}{x-1}, \quad x = \frac{x^2-x}{x-1}, \quad x^2 = \frac{x^3-x^2}{x-1}, \quad x^3 = \frac{x^4-x^3}{x-1}, \dots, \quad x^n = \frac{x^{n+1}-x^n}{x-1}.$$

Summation of two successive terms of geometric series:

$$1+x = \frac{x^2-1}{x-1}, \quad x+x^2 = \frac{x^3-x}{x-1}, \quad x^2+x^3 = \frac{x^4-x^2}{x-1}, \dots, \quad x^{n-1}+x^n = \frac{x^{n+1}-x^{n-1}}{x-1}.$$

Summation of three successive terms of geometric series:

$$1+x+x^2 = \frac{x^3-1}{x-1}, \quad x+x^2+x^3 = \frac{x^4-x}{x-1}, \dots, \quad x^{n-2}+x^{n-1}+x^n = \frac{x^{n+1}-x^{n-2}}{x-1}$$

Similarly, this process continues up to multiple successive terms of geometric series. The summations of numerous successive terms [1-3] of geometric series are given below.

Summation of various successive terms of geometric series:

$$\sum_{i=k}^n x^i = x^k + x^{k+1} + x^{k+2} + \dots + x^{n-1} + x^n = \frac{x^{n+1}-x^k}{x-1}.$$

$$\sum_{i=-k}^n x^i = x^{-k} + x^{-k+1} + x^{-k+2} + \dots + x^{n-1} + x^n = \frac{x^{n+1} - x^{-k}}{x - 1}.$$

$$\sum_{i=1}^n x^i = 1 + x + x^2 + x^3 + \dots + x^{n-1} + x^n = \frac{x^{n+1} - 1}{x - 1}.$$

References

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