

Calculator Programs Can Revolutionize H.S. Algebra

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Abstract

This article develops three calculator programs for (1) exponential modeling, (2) solving systems of linear equations, and (3) displaying various types of functions encountered in high school algebra. We suggest that these calculator programs will engage students better than the pencil and paper approaches of the past, that topics covered will be retained better by students, and that teachers will find teaching the coding involved is rewarding. Calculator programs can revolutionize high school algebra; an evolution is needed.

Introduction

We hold the extreme view that students should as quickly as possible use technology to do math problems. This means that they should do a few problems with pencil and paper and then immediately investigate using spreadsheets and calculator programs to do subsequent problems.

Recently I noticed a couple of programs that seem to have a high reward for very little code writing. The two programs are in exponential modeling and solving systems of linear equations. The programs are given below.

These two programs suggested a third: a program that displays, using a menu system, the various functions studied in algebra courses: linear, quadratic, general polynomial, rational, radical, exponential, and logarithmic functions (LQPRatRadExpLog). This program is also given below. It can coordinate algebra from a central hub, we claim, in an engaging way – stirring creativity in both teacher and student. It does involve a sea change.

The normal approach of textbooks is to give tens of problems of various types without much attempt (no attempt) to automate the doing of problems. The student is expected to do the same algebraic manipulations to solve each problem, generally with pencil and paper. This is be contrasted with the usual refrain of business and, indeed, all modern life: you repetitively do something and then you demand that a computer replace you as the performer of the deeds: a TV remote control (versus walks to the TV), a car radio's tuner buttons (versus tune it manually yourself), speed dialed phone numbers (versus you remember the numbers each time) are but a few examples. Online banking, online shopping with credit cards are two other examples. Can things change so algebra becomes more like these examples?

For many algebra problems when the steps are abstracted, putting them into a calculator is rather easy. We'll show this. Given it can be done this way, how is it that student and teacher are somehow forbidden from automating the steps? Shouldn't the main focus of an algebra class be to encourage, instruct, and, indeed, insist that students get all the problems they encounter in one centralized, menu driven system as fast as possible. Isn't that the business, science, and just plain living reality of modern life. We will argue here that it is not difficult to achieve these things.

Program One: Exponential Modeling

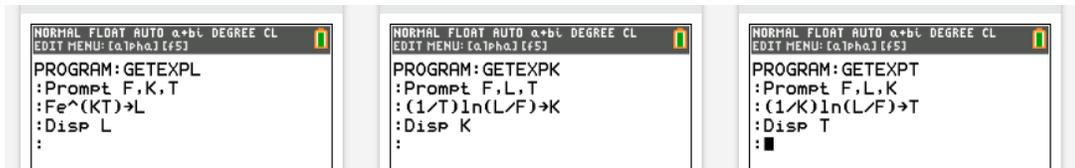


Figure 1: Three problem types in exponential modeling are easily coded into a TI-84 CE calculator using Connect.

The exponential modeling program uses a menu feature in TI-Basic. There are three variations involved with the formula:

$$L = Fe^{KT}, \tag{1}$$

where at first there are F of some quantity, such as money, population size, or radioactive material. This first amount grows or decays with time, T , per

the exponential constant K . After time T there will be a last value of the quantity, L . The more standard notation is $A_0 = Ae^{kt}$. All problems involve the student being given three of the four variables and being asked to solve for the missing fourth. Solving for K first given F , L , and T (getting the model) and then L next given the same F and K and a new T are typical. Carbon dating problems calculate T based on having calculated K and given a new L . In summary there are three functions in three variables: $L(F, K, T)$, $K(F, L, T)$, and $T(F, L, K)$.

The algebra is straight forward and simple. Given F , K , and T solve for L ; that's just (1). Given F , L , and T , solve for K : using (1)

$$e^{KT} = \frac{L}{F};$$

take \ln of both sides

$$KT = \ln \frac{L}{F} \tag{2}$$

and divide both sides by T :

$$K = \frac{1}{T} \ln \frac{L}{F}. \tag{3}$$

The last is solve for T , a *when* problem: just use (2) and arrive at

$$T = \frac{1}{K} \ln \frac{L}{F}. \tag{4}$$

In Figure 1 all three results of the above algebraic manipulations are instantiated in three programs. Next, a menu system allows for the three to be called from one central program: Figure 2 with code in Figure 3. We'll call this program EXPMENU.

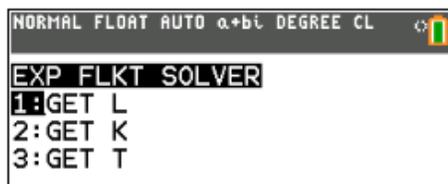


Figure 2: Menu for the exponential modeling program; it calls the three programs.

```
001 Menu("EXP FLKT SOLVER","GET L",L,"GET K",K,"GET T",T)
002 Lbl L
003 prgmGETEXPL
004 Stop
005 Lbl K
006 prgmGETEXPK
007 Stop
008 Lbl T
009 prgmGETEXPT
010 Stop
```

Figure 3: Code for making menus that call other programs.

Figure 4 shows ten typical textbook problems [1].

Here's problem 9 done with the calculator: Figure 5. The solution manual confirms the answer is correct: Figure 6. The speed of this approach can't be argued. Here's problem 11 solved with the calculator: Figure 7. The solution manual shows the suggested solution method, Figure 8; apparently, each time the student is to, in effect, derive the formulas, our (1), (3), and (4). Strange!

In Exercises 9–14, complete the table. Round projected populations to one decimal place and values of k to four decimal places.

	Country	2020 Population (millions)	Projected 2050 Population (millions)	Projected Growth Rate, k
9.	Philippines	109.6		0.0092
10.	Pakistan	220.9		0.0142
11.	Colombia	50.9	56.0	
12.	Madagascar	27.7	54.1	
13.	Germany	85.8	80.1	
14.	Bulgaria	6.9	5.4	

Source: worldpopulationreview.com

An artifact originally had 16 grams of carbon-14 present. The decay model $A = 16e^{-0.000121t}$ describes the amount of carbon-14 present after t years. Use this model to solve Exercises 15–16.

15. How many grams of carbon-14 will be present in 5715 years?
16. How many grams of carbon-14 will be present in 11,430 years?

17. The half-life of the radioactive element krypton-91 is 10 seconds. If 16 grams of krypton-91 are initially present, how many grams are present after 10 seconds? 20 seconds? 30 seconds? 40 seconds? 50 seconds?
18. The half-life of the radioactive element plutonium-239 is 25,000 years. If 16 grams of plutonium-239 are initially present, how many grams are present after 25,000 years? 50,000 years? 75,000 years? 100,000 years? 125,000 years?

Figure 4: Ten typical exponential modeling problems.

```

.....PRGM.....
PRGMEXPMENU
F=?109.6
K=?0.0092
T=?50-20
144.4361259
Done
.....

```

Figure 5: Solution to problem 9 using calculator program, option 1.

This slow and error prone way of doing problems is well demonstrated with harder problems that ask students to first solve for K and then find L

9. $P(t) = 109.6e^{0.0092t}$
 $P(30) = 109.6e^{0.0092(30)}$
 $= 144.4$
The population is projected to be 144.4 million in 2050.

Figure 6: Solution Manual's answer to problem 9.

```

.....
Pr9mEXPMENU
F=?50.9
L=?56.0
T=?50-20
0.0031829589
.....
Done

```

Figure 7: Solution to problem 11 using menu option 2.

11. $A = 50.9e^{kt}$
 $56 = 50.9e^{30k}$
 $\frac{56}{50.9} = e^{30k}$
 $\ln\left(\frac{56}{50.9}\right) = \ln e^{30k}$
 $\ln\left(\frac{56}{50.9}\right) = 30k$
 $\frac{\ln\left(\frac{56}{50.9}\right)}{30} = k$
 $k = 0.0032$
The growth rate is 0.0032.

Figure 8: Solution manuals suggested method for problem 11.

using the K . Here's a half life problem from Blitzer, Problem 28: given the half life is 7340, what is T given $L = .2$? The book's answer involves many steps: Figure 9. The student has to reason out (or be told) that the $F = 1$ and $L = .5$ for a half life problem: Figure 10. The scientific notation the calculator generates seems awkward until one realizes one can just enter K when prompted for the next, find T part of the problem: Figure 11.

Is there some benefit to repeatedly doing these exponential modeling problems by the suggested method. You must assume the student is not to see the general pattern and derive formulas for the missing L , K and T variables and then plug them in – if necessary by hand, using the calcula-

28. First find the decay equation.

$$\begin{aligned}
 0.5 &= e^{kt} \\
 0.5 &= e^{7340k} \\
 \ln 0.5 &= \ln e^{7340k} \\
 \ln 0.5 &= 7340k \\
 \frac{\ln 0.5}{7340} &= k \\
 k &= -0.000094 \\
 A &= e^{-0.000094t}
 \end{aligned}$$

Next use the decay equation answer question.

$$\begin{aligned}
 A &= e^{-0.000094t} \\
 0.2 &= e^{-0.000094t} \\
 \ln 0.2 &= \ln e^{-0.000094t} \\
 \ln 0.2 &= -0.000094t \\
 \frac{\ln 0.2}{-0.000094} &= t \\
 t &= 17121.7
 \end{aligned}$$

It will take 17121.7 years.

For greater accuracy, use $k = \frac{\ln 0.5}{7340}$.

$$A = e^{\frac{\ln 0.5}{7340}t} \text{ gives } 17043.0 \text{ years}$$

Figure 9: Solution manual suggested method for problem 28.

```

.....
Pr9mEXPMENU
F=?1
L=? .5
T=?7340
                                -9.44342208E-5
                                Done
.....

```

Figure 10: Problem 28, part 1: Solve for K using option 2.

```

.....
Pr9mEXPMENU
F=?F
L=? .2
K=?K
                                17042.95222
                                Done
.....

```

Figure 11: Problem 28, part 2: solve for T using option 3. Note you can enter variable names in for parameters.

tor for the final arithmetic (walking up to the TV to turn the channel each time). Worst: the student is not to create a calculator program that does all three types of problems with speed and efficiency. He or she is to use a

rotary phone for ever for all numbers frequently dialed. I think the suggested approach makes students dislike math and that in turn makes them not see the beauty and power of modeling ever. It is awful pedagogy!

In contrast, creativity is encouraged with the *get the calculator to do the busy work* approach. One can see various tweaks are possible and, without exaggeration, wonderful. Consider problem 17 (Figure 4). It asks for amounts of Krypton present at 20, 30, 40, and 50 seconds. This suggests the table feature. How could we, students, make tables appear from our menu and adjust it to increment by 10? Sometimes the book asks for rounded answers. Could we prompt for the number of decimal places and compute with it? Is it possible to see a graph of our model and automatically adjust the windows x and y ranges, increments? Even better: for top tier students design issues might be contemplated. Some students might naturally think about other people using their program: deeper imagination, synthesis, that brainy stuff gets called up – rather than obedience to some mindless authority like the teacher or the textbook! Pop quiz: How do we get the menu program to repeat, stay in the loop, until we want out? Hint: replace the first three STOP commands (at lines 004, 007, and 010) with prgmEXPMENU and add a "QUIT" menu option that has the one STOP command underneath its label. Try it: from this fish comes a net full later.

Systems of Linear Equations

Here's another example of how calculator programs can greatly facilitate the solving and, I suggest understanding of algebra. It may somehow be controversial but I suggest algebra should be about seeing the pattern in doing a lot of problems and distilling that understanding using algebraic manipulations to arrive at a general formula that will work in all cases. If tools, like a calculator or spreadsheet are available, it is, I argue, part of the algebraic (mathematical) spirit to seek to use them to optimize the speed and accuracy with which problems can be solved. There is no point to painstakingly using just pencil and paper and plodding through 20 problems when a calculator program might be able to do them quickly and accurately – do a few by hand, but then don't do anymore by hand ever again should be the mantra of all algebra teachers! The exponential model program exhibits this ethos. The next program, solving systems of simultaneous linear equations (SSLE), does as well.

```

001 Prompt A,B,C
002 Prompt D,E,F
003 "-A/BX+C/B"→Y1
004 "-D/EX+F/E"→Y2
005 F/E-C/B→G
006 D/E-A/B→H
007 If (H≠0)
008 Then
009 Disp "X=",G/H▶Frac
010 Disp "X=",G/H
011 Y1(G/H)→I
012 Disp "Y=",I▶Frac
013 Disp "Y=",I
014 Else
015 Disp "CHK GRAPH"
016 End

```

Figure 12: Calculator code for SSLE problems.

Given two linear equations in standard form $AX + BY = C$ and $DX + EY = F$, solve both for Y and set them equal. So

$$AX + BY = C$$

begets

$$Y_1 = -\frac{A}{B}X + \frac{C}{B}$$

and similarly algebra gives

$$Y_2 = -\frac{D}{E}X + \frac{F}{E}.$$

Set them equal and solve for X :

$$\begin{aligned}
 -\frac{A}{B}X + \frac{C}{B} &= -\frac{D}{E}X + \frac{F}{E}, \\
 \frac{D}{E}X - \frac{A}{B}X &= \left(\frac{D}{E} - \frac{A}{B}\right)X = \frac{F}{E} - \frac{C}{B}, \\
 X &= \frac{\frac{F}{E} - \frac{C}{B}}{\left(\frac{D}{E} - \frac{A}{B}\right)}. \tag{5}
 \end{aligned}$$


```

.....
Pr9mAAASLE
A=?3
B=?2
C=?4
D=?2
E=?1
F=?1■

```

Figure 14: The input phase for solving a SSLE problem.

```

X=
X= -2
Y= -2
Y= 5
..... Done

```

Figure 15: The output phase for solving a SSLE problem. Both fractions and, as might be needed, decimals are displayed.

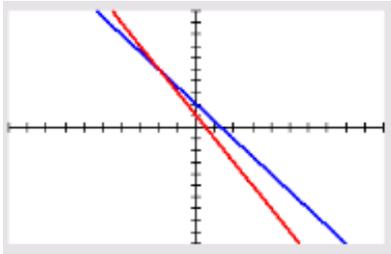


Figure 16: Division by zero prompts the user to check the graph of the system: hit F5.

y-intercepts (both intercepts) of the lines as a menu option. In real world problems, like calculating a break even point, a supply-demand equilibrium, or quantities used to create a mixture, which require rounding answers, how can we prompt for decimal accuracy. Once again tables might be a nice menu option. How can we automatically adjust these appropriately for our needs? Notice applications become easier to do because we have abstracted out of concern the large and small numbers involved in business and science. As students can now do these problems easily, I suggest they may get the big picture: humans are easily bogged down by realistic numbers, but calculators are not at all troubled by such; hence, the need and, you might say, love humans should have for technology, calculators. You can really use math to find break-even points, how many flowers to give your girlfriend, and what

quantities of rice and beans combine to make the best bioavailable protein.

It should be emphasized (as it just was) that with this automation applications are much easier; students might really enjoy the feeling that they can solve real world problems quickly without getting lost in the weeds of high precision and large numbers in the context of using the *addition* and *substitution* methods. Solving real-world applications might go from the very last thing done in a class (if ever) to the first and only thing of real concern. Figure 17: is there any point!

31.	$\begin{cases} x = 9 - 2y \\ x + 2y = 13 \end{cases}$	32.	$\begin{cases} 6x + 2y = 7 \\ y = 2 - 3x \end{cases}$
33.	$\begin{cases} y = 3x - 5 \\ 2x - 35 = 7y \end{cases}$	34.	$\begin{cases} 9x - 3y = 12 \\ y = 3x - 4 \end{cases}$
35.	$\begin{cases} 3x - 2y = -5 \\ 4x + y = 8 \end{cases}$	36.	$\begin{cases} 2x + 5y = -4 \\ 3x - y = 11 \end{cases}$
37.	$\begin{cases} x + 3y = 2 \\ 3x + 9y = 6 \end{cases}$	38.	$\begin{cases} 4x - 2y = 2 \\ 2x - y = 1 \end{cases}$
39.	$\begin{cases} \frac{x}{4} - \frac{y}{4} = -1 \\ x + 4y = -9 \end{cases}$	40.	$\begin{cases} \frac{x}{6} - \frac{y}{2} = \frac{1}{3} \\ x + 2y = -3 \end{cases}$
41.	$\begin{cases} 2x = 3y + 4 \\ 4x = 3 - 5y \end{cases}$	42.	$\begin{cases} 4x = 3y + 8 \\ 2x = -14 + 5y \end{cases}$

Figure 17: Twelve typical problems asking for solutions to SSLE.

Graphs of Functions

Spurred on by the two examples above, I thought why not take the idea up a notch: rewrite my algebra courses using a central menu that drills down to the various problems that are encountered. You can nest menus. So, as mentioned earlier, a college algebra course covers LQPRatRadExpLog functions (seek out at parties). On day one students make a menu for each of these classes of functions and during the course various programs are added. Tests and homework are just using the menus and entering answers at a web application (myLabMath). Too easy!!! Too logical!!! Too much fun!!!! Too memorable!!!!

Figure 18 gives the look of this first day of classes program (called FUNC-MENU); and Figure 19 gives the code for it.

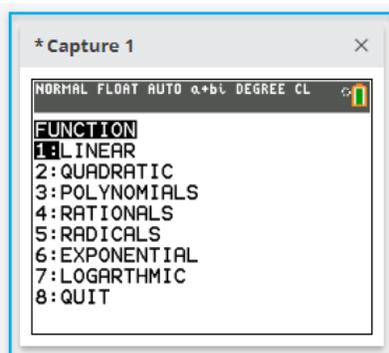


Figure 18: Look of the hub a future algebra course. Class directive: expand this, reason with regression to solve global problems right away.

```

001  prgmCLEARYS
002  Menu("FUNCTION", "LINEAR", L, "QUADRATIC", Q,
003  "POLYNOMIALS", P, "RATIONALS", R, "RADICALS", C,
004  "EXPONENTIAL", E, "LOGARITHMIC", G, "QUIT", Z)
005  Lbl L: "2X+3"→Y1
006  DispGraph:Pause
007  prgmACAFUNCS
008  Lbl Q: "-2(X-2)^2+3"→Y1
009  DispGraph:Pause
010  prgmACAFUNCS
011  Lbl P: "(X+1)(X+2)(X-1)(X-2)"→Y1
012  DispGraph:Pause
013  prgmACAFUNCS
014  Lbl R: "X/((X-1)(X+1))"→Y1
015  DispGraph:Pause
016  prgmACAFUNCS
017  Lbl C: "(X-2)^(1/3)+2"→Y1
018  DispGraph:Pause
019  prgmACAFUNCS
020  Lbl E: "10^(2-X)"→Y1
021  DispGraph:Pause
022  prgmACAFUNCS
023  Lbl G: "log(X)"→Y1
024  DispGraph:Pause
025  prgmACAFUNCS
026  Lbl Z: Stop

```

Figure 19: The code for the algebra hub. Tweak it for an *A*.

Once again, creativity and curiosity become possible with this approach. Students immediately see the graphs of the functions they will encounter

during the term and can see what happens if they vary the slope and y-intercept, the vertex coordinates, and other parameters that govern these graphs. They can ponder how one might describe the behavior of rational functions – the asymptote idea. They might really be engaged!

Conclusion

Now for the pièce de résistance: embed the first two programs into the third. Naturally, logically the exponential modeling program goes under exponentials and solving SSLE goes under linear. That’s a rhyme – but there is a reason.

Once the *embed* paradigm is established, each type of function can be given a menu option that puts into lists L_1 and L_2 , x and y values and performs a regression. Models for the math of a pandemic become possible (under exponentials), doable, pleasantly enough. Maybe we should consider doing such, maybe for all students of the world taking algebra courses, so they might be informed of the facts that will keep our species alive and thriving! Hell, global warming could be modeled, understood and acting upon before it is too late! First look at the graph and collectively weep.

References

- [1] Blitzer, R. (2018). *College Algebra: Essentials*. 5th ed. New York: Pearson.