A New Permittivity of the Rotational Electric Field

Krishna Srinivasan

***Abstract*—The electric field in Maxwell's equations can be written as a sum of the rotational and the irrotational electric fields. In this paper, it will be shown that Maxwell's equations is formulated such that the permittivity of the rotational electric field is set to 1.0, and the permittivity of the irrotational electric field is commonly denoted as ϵ_r. Faraday's law can be reformulated in a little more general equation, so that a non-unity permittivity of the rotational electric field is possible. Although only a theoretical formulation is proposed, a way by which the permittivity of the rotational electric field can be measured is discussed.**

***Index Terms*—Maxwell's equations, electric field, Faraday's law, permittivity, rotational electric field, irrotational electric field**

I. Introduction

JAMES Clerk Maxwell published 3 famous papers on electromagnetism [1] – [3]. He translated Faraday's experiment results into a mathematical equation, known as Faraday's law today, and introduced the displacement current. In his famous 1864 paper [3], he presented a comprehensive theory of the electromagnetic field, including his hypothesis of the existence of electromagnetic waves, which was confirmed by Heinrich Hertz in 1893 [4].

The motivation of this research work is to answer these questions: (1) Why is Faraday's law formulated using the electric field $\vec{E}$, instead of the electric displacement (electric-flux density) $\vec{D}$? (2) What effect does a dielectric material have on the electric field generated from Faraday's law? How is this taken into account in Maxwell's equations? (3) What is the meaning of $\vec{D} = \epsilon_o \epsilon_r \vec{E}$ in the context of Faraday's law?

An outcome of this research work has been to formulate a more general equation of Faraday's law, of which, the present formulation is a special case. A new permittivity of the rotational electric field κ is introduced, in addition to and different from the permittivity of the irrotational electric field, commonly written as ϵ_r. Additional details in this paper can be found in Ref. [5] - [6].

To enable this, the existing Maxwell's equations need to be cast in a different form in Sec. II, where the electric fields are written as a superposition of the rotational and the irrotational components. In Sec. III, the reason for introducing the permittivity ϵ_r will be presented, as well as the reason that this permittivity is associated with the irrotational electric field. A similar argument is made to introduce a new permittivity κ, associated with the rotational electric field in Sec. IV, followed by an experimental technique to measure the permittivity of the rotational and the irrotational permittivity values κ and ϵ_r.

II. Rewriting Electric Field as a Superposition of Rotational and Irrotational Components

The two sources of electric field are electric charges and a time-varying magnetic field. The electric field of electric charges is captured in Gauss's law, and the electric field generated by a time-varying magnetic field is captured in Faraday's law. In this section, the electric field will be written as a superposition of these two field components. These components are also called as rotational and irrotational fields, for reasons, which will become clear shortly.

The magnetic vector potential $\vec{A}$ is defined as

$$\vec{B} = \nabla \times \vec{A}. \tag{1}$$

Substituting the above equation in the present formulation of Faraday's law,

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{A}\right). \tag{2}$$

Rearranging the above equation,

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0. \tag{3}$$

From calculus, if the curl of a vector field is 0, the vector field can be written as the gradient of a scalar field,

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla\Phi. \tag{4}$$

Rearranging the above equation,

$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t} \tag{5}$$

$$= \vec{E}_C + \vec{E}_F. \tag{6}$$

From the above equation, $\vec{E}$ can be written as the sum of Coulomb field,

$$\vec{E}_C = -\nabla\Phi, \tag{7}$$

and the electric field generated from Faraday's law,

$$\vec{E}_F = -\frac{\partial \vec{A}}{\partial t}. \tag{8}$$

Applying the $\nabla\times$ operator on both sides of the above equation, and substituting Eq. 1,

$$\nabla \times \vec{E}_F = -\frac{\partial \vec{B}}{\partial t}, \tag{9}$$

which is the same as Faraday's law. Intuitively, it can be observed that $\vec{E}_C$ is the electric field generated by electric charges. Since $\vec{E}_F$ is the electric field generated by Faraday's law, the other source of electric field $\vec{E}_C$, must be the electric field generated by electric charges. This can be proven mathematically, using Coulomb gauge,

$$\nabla \cdot \vec{A} = 0, \tag{10}$$

to show that $\vec{E}_C$ is the instantaneous Coulomb field [7] [8]. Since $\vec{E}_C$ is the gradient of a scalar potential in Eq. 7, from calculus,

$$\nabla \times \vec{E}_C = 0. \tag{11}$$

Since $\vec{E}_C$ is "curl free", visually, the field is "radial" and not "swirling". For this reason, $\vec{E}_C$ is the irrotational electric-field component. $\nabla \times \vec{E}_F$, however, may be non-zero, as written in Faraday's law in Equation 9, and is the rotational electric-field component. Similar to Eq. 6, $\vec{D}$ at any point can be written as

$$\vec{D} = \vec{D}_C + \vec{D}_F. \tag{12}$$

Using the above equation, Ampere's law is written as

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\left(\vec{D}_C + \vec{D}_F\right). \tag{13}$$

Applying the $\nabla \cdot$ operation on both the sides of the above equation,

$$\nabla \cdot \left(\nabla \times \vec{H}\right) = \nabla \cdot \vec{J} + \frac{\partial}{\partial t}\nabla \cdot \left(\vec{D}_C + \vec{D}_F\right). \tag{14}$$

From calculus, the left-hand side reduces to 0. If

$$\nabla \cdot \left(\vec{D}_C + \vec{D}_F\right) = \rho, \tag{15}$$

Eq. 14 reduces to the current-continuity equation,

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}. \tag{16}$$

This shows that

$$\nabla \cdot \left(\vec{D}_C + \vec{D}_F\right) = \rho, \tag{17}$$

or Gauss's law, must be satisfied for time-varying fields. The existing set of Maxwell's equations can be written as

$$\nabla \cdot \left(\vec{D}_C + \vec{D}_F\right) = \rho \tag{18}$$

$$\nabla \cdot \vec{B} = 0 \tag{19}$$

$$\nabla \times \vec{E}_F = -\frac{\partial \vec{B}}{\partial t} \tag{20}$$

$$\nabla \times \vec{E}_C = \vec{0} \tag{21}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\left(\vec{D}_C + \vec{D}_F\right), \tag{22}$$

and the above equations are also valid for time-varying fields [5]. In addition to the above equations,

$$\vec{D}_F = \epsilon_o \epsilon_r \vec{E}_F \tag{23}$$

$$\vec{D}_C = \epsilon_o \epsilon_r \vec{E}_C \tag{24}$$

$$\vec{B} = \mu_o \mu_r \vec{H} \tag{25}$$

capture the effect of a material on the fields. The electric displacement and electric field at any point are

$$\vec{D} = \vec{D}_C + \vec{D}_F \tag{26}$$

$$\vec{E} = \vec{E}_C + \vec{E}_F. \tag{27}$$

III. The Permittivity ϵ_r Associated With the Irrotational Electric Field $\vec{E}_C$

The meaning of electric field and electric displacement will be explained from Faraday's experiment with spherical capacitors in this section. The reader is referred to [5] for more details.

Gauss's law for electric charges in free space, can be derived from Coulomb's law as

$$\oint_S \epsilon_o \vec{E}_C \cdot d\vec{A} = q_{enc}, \tag{28}$$

where S is a closed 3D surface enclosing charge q_{enc}. The derivation of the above equation is presented in [5], and is not repeated here.

Faraday's experimental setup is shown in Fig. 1. Faraday used two identical capacitors, A and B, with the two inner metal spheres, p and p', connected together by a wire, as well as the two outer metal spheres q and q'. A metal is an equipotential volume in electrostatics, and so are metal objects connected together. Therefore, p and p' are at the same potential, and so are q and q'. Since the inner spheres are connected, as well as the outer spheres, the potential difference between the inner sphere and the outer sphere in A, V_{pq}, is equal to that of B, $V_{p'q'}$,

$$V_{pq} = V_{p'q'}. \tag{29}$$

Since the capacitors are identical, and voltage is the path integral of the electric field, the electric fields in the cavities of A and B, $\vec{E}_A$ and $\vec{E}_B$, are equal,

$$\vec{E}_A = \vec{E}_B, \tag{30}$$

at the same point relative to the respective centers of A and B, in each of the cavities. This is true, independent of the dielectric material that fills the cavities of A and B.

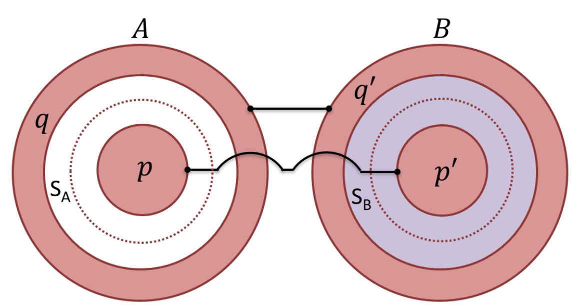

Fig. 1. The experimental setup to study the variation of the charge stored in a capacitor with different dielectric materials.

In the cavity of A, the material is always kept as air, but the material in B is varied. Capacitor A with the unfilled cavity acts as a reference for the experiment. The capacitors are charged simultaneously by connecting the inner and the outer spheres to the terminals of a Wimshurst machine. A detailed explanation of how a Wimshurst machine works is presented in [5].

Faraday studied the ratio of the charge stored in A,

Q_A, and B, Q_B. The quantity of charge stored in a capacitor can be measured using a ballistic galvanometer. The detailed methodology is presented in [5]. If the dielectric material in both A and B is air, by symmetry, the charge stored in the two identical capacitors are equal. The ratio of the charge stored $Q_B : Q_A$ is 1. However, in the case when B is filled with a dielectric material other than air, Faraday observed that the ratio is greater than 1. This ratio has a special name and is called the relative permittivity of the dielectric material, denoted by the symbol ϵ_r, where r stands for *relative*, and it means the permittivity of a material relative to air. To be precise, the cavity in Capacitor A must be vacuum, which is the absence of any material, including air. The reference dielectric material of air will be assumed for simplicity.

Faraday observed that Capacitor B, whose cavity is filled with a dielectric material, stores ϵ_r times more charge than Capacitor A, whose cavity is unfilled, or contains air. If $\vec{E}_A$ is the electric field in capacitor A, applying Gauss's law in free space, in Eq. 28,

$$\oint_{S_A} \epsilon_o \vec{E}_A \cdot d\vec{A} = Q_A, \tag{31}$$

where $\pm Q_A$ is the charge stored in the spheres of A, which reside on the outside of the inner sphere, and the inside of the outer spherical shell [5], and S_A is the spherical Gaussian surface in the cavity, as shown by the dotted line. Solving the above equation, the electric-field strength in the cavity is $\propto Q$.

If this same equation is applied to Capacitor B, the field strength in the cavity is ϵ_r times greater, since ϵ_r times more charge is stored in Capacitor B. By definition, voltage is the path integral of the electric field, and therefore, $V_{p'q'}$ is ϵ_r times greater than V_{pq}. Since $V_{pq} = V_{p'q'}$, this contradicts the experiment results. This can be resolved with the explanation presented next.

If the dielectric material in the cavity of Capacitor B reduces the electric-field strength by ϵ_r, then the electric fields in the cavities of A and B are equal. If the electric fields are equal, this means that the voltage $V_{pq} = V_{p'q'}$. This explains the reason that more charge is present in Capacitor B: the additional charge is present to overcome the reduction in the electric field caused by the dielectric material, so that the electric fields in the cavities of A and B are equal. This explanation can be captured as

$$\vec{E}_{material} = \frac{\vec{E}_{air}}{\epsilon_r}, \tag{32}$$

where $\vec{E}_{material}$ is the electric field at any point in a material with permittivity ϵ_r, and $\vec{E}_{air}$ is the electric field in air, or the electric field that would have existed at that point, if the material does not reduce the field. This observation will be used to formulate Gauss's law in the remainder of this section, taking into account the reduction of electric-field strength in a dielectric material.

The ratio of the charges stored in A and B is the relative permittivity ϵ_r,

$$Q_A = \frac{Q_B}{\epsilon_r}. \tag{33}$$

Since the potential difference between the outer and the inner conductors are the same in both the identical capacitors, as noted in Eq. 29,

$$\oint_{S_B} \epsilon_o \vec{E}_B \cdot d\vec{A} = \oint_{S_A} \epsilon_o \vec{E}_A \cdot d\vec{A}, \tag{34}$$

where S_B is a spherical Gaussian surface lying in the cavity of B, and of the same radius as S_A, shown by the dotted circles in Fig. 1. From the above equations,

$$\oint_{S_B} \epsilon_o \vec{E}_B \cdot d\vec{A} = \frac{Q_B}{\epsilon_r}. \tag{35}$$

Rearranging the above equation,

$$\oint_{S_B} \epsilon_o \epsilon_r \vec{E}_B \cdot d\vec{A} = Q_B, \tag{36}$$

is the general form of Gauss's law.

In this example, the Gaussian surface is present in a uniform dielectric material. ϵ_r is moved inside the integral, which will account for the variation in the dielectric material over S. Gauss's law is also valid in this case. However, the validity of Gauss's law in any type of material medium, uniform or non-uniform, isotropic or anisotropic, linear or non-linear, or in the case of time-varying fields, can be proven with the current-continuity equation, and is presented in [5] (See also [9] - [10]). In other words, Gauss's law is always valid!

From the above equations, the general form of Gauss's law is written as

$$\oint_S \vec{D} \cdot d\vec{A} = q_{enc}, \tag{37}$$

where S is the Gaussian surface enclosing charge q_{enc}.

The integrand in the above equation is assigned a new vector-field quantity $\vec{D}$, and is called electric displacement,

$$\vec{D} = \epsilon_o \epsilon_r \vec{E}. \tag{38}$$

$\vec{D}$ and $\vec{E}$ are related at any point by the above equation.

$\vec{D}$ is typically viewed as a vector field that satisfies Eq. 37, or as a mathematical relation in Eq. 38. However, a new meaning emerges, when Eq. 32 is substituted in Eq. 38, resulting in

$$\vec{D} = \epsilon_o \vec{E}_{air}. \tag{39}$$

The above equation states that electric displacement is the same as the electric field in air (and scaled by the constant ϵ_o), or in other words, electric displacement at any point is the same as the electric field that is not "modified", "altered", or reduced in strength by the dielectric material at that point. The electric field, however, rotational or irrotational, is defined from the equation

$$\vec{F} = q\vec{E}, \tag{40}$$

is defined from the force experienced by an electric charge q at any point, and is the net electric field, including the effect of a dielectric material at any point. Note that Equation 39 is more elegant in electrostatic units, which is written without the constant ϵ_o [5].

From this derivation, the permittivity ϵ_r is associated with the electric field due to electric charges, or the irrotational electric field. In the next section, a new permittivity κ will be introduced, which is associated with the rotational electric field of Faraday's law.

IV. Introducing a New Type of Permittivity κ, Associated with the Rotational Electric Field $\vec{E}_F$

Similar to how Gauss's law was used to introduce ϵ_r in Section III, Faraday's law will be used to introduce a new type of permittivity κ, associated with the rotational electric field. Repeating Faraday's law in Eq. 9,

$$\nabla \times \vec{E}_F = -\frac{\partial \vec{B}}{\partial t}. \tag{41}$$

Lets assume for simplicity that $\vec{B}$ is linearly time varying, so that $\vec{E}_F$ is a constant, and there is no coupling between Faraday's law and Ampere's law. Lets also assume that a dielectric material has an effect on modifying the electric field generated by a time-varying $\vec{B}$, maybe a minor, but not a NULL effect.

Two cases are analyzed in Figure 2, where Case 1 and Case 2 are two different materials. The time-varying magnetic-flux density $\vec{B}$ is equal in both the cases. Since $\vec{B}$

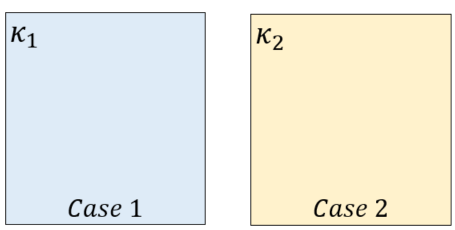

Fig. 2. Two materials with different electrical properties.

is equal in both the cases, by Faraday's law, the electric field $\vec{E}_F$ generated in each of the cases must be equal as well. In the existing formulation of Faraday's law, the effect of the material on the rotational electric field $\vec{E}_F$ is not accounted for.

This is the motivation to reformulate Faraday's law as

$$\nabla \times \left(\kappa \vec{\mathscr{E}}_F\right) = -\frac{\partial \vec{\mathscr{B}}}{\partial t}, \tag{42}$$

where κ is a new type of permittivity associated with the rotational electric field. The naming convention followed is that the fields in the reformulated equations will be written using scripted variables, as written in the above equation. The fields and equations in the existing formulation will be written using non-scripted variables.

From the above equation, a new type of electric-displacement field,

$$\vec{\mathscr{D}}_F^* = \kappa \vec{\mathscr{E}}_F, \tag{43}$$

generated in both the cases are equal, since the time-varying $\vec{\mathscr{B}}$ is the same in both the cases. The material modifies $\vec{\mathscr{D}}_F^*$ differently, resulting in

$$\vec{\mathscr{E}}_F^1 = \frac{\vec{\mathscr{D}}_F^*}{\kappa_1} \tag{44}$$

$$\vec{\mathscr{E}}_F^2 = \frac{\vec{\mathscr{D}}_F^*}{\kappa_2} \neq \vec{\mathscr{E}}_F^1, \tag{45}$$

and the results are now consistent with the assumptions made.

Note that a static $\mathscr{E}_F$, generated by a linearly time varying $\vec{\mathscr{B}}$, has been assumed only for simplicity. In general, however, Equation 42 will be assumed to be valid also for any type of a time-varying field. A way by which the formulation can be verified by experiments, and measurement of the value of κ, will be discussed in Sec. VIII.

V. The Reformulated Maxwell's Equations

The new set of equations are the same as the existing Maxwell's equations, with the exception of the modified Faraday's law,

$$\nabla \cdot \left(\vec{\mathscr{D}}_C + \vec{\mathscr{D}}_F\right) = \rho \tag{46}$$

$$\nabla \cdot \vec{\mathscr{B}} = 0 \tag{47}$$

$$\nabla \times \left(\kappa \vec{\mathscr{E}}_F\right) = -\frac{\partial \vec{\mathscr{B}}}{\partial t} \tag{48}$$

$$\nabla \times \vec{\mathscr{E}}_C = \vec{0} \tag{49}$$

$$\nabla \times \vec{\mathscr{H}} = \vec{J} + \frac{\partial}{\partial t}\left(\vec{\mathscr{D}}_C + \vec{\mathscr{D}}_F\right), \tag{50}$$

and repeating Eq. 23 – Eq. 25,

$$\vec{\mathscr{D}}_F = \epsilon_o \epsilon_r \vec{\mathscr{E}}_F \tag{51}$$

$$\vec{\mathscr{D}}_C = \epsilon_o \epsilon_r \vec{\mathscr{E}}_C \tag{52}$$

$$\vec{\mathscr{B}} = \mu_o \mu_r \vec{\mathscr{H}} \tag{53}$$

These equations are also valid for time-varying fields and time-varying sources. Note that if κ is exactly equal to 1.0, then the formulation reduces to the existing set of Maxwell's equations. If κ is exactly equal to 1.0, this implies that a dielectric material has no effect on the rotational electric field. This, however, is too ideal, and a special case of the formulation presented in this paper, which is a more general formulation.

Similar to the existing formulation, in general, permittivity and permeability are written as tensors to represent any type of material, such as an anisotropic material [5]. The above equations are valid for any material type.

VI. $\kappa \neq \epsilon_r$

Clearly, ϵ_r is different from κ. Why is this the case? A parallel-plate capacitor is used to illustrate the electric-field pattern due to electric charges. Two parallel plates, charged positive and negative, sandwiched between a dielectric material, is shown in Figure 3. The electric field between the charged plates is shown by the arrows. A simplistic view of the atoms in the dielectric is shown in the figure. The electric field in the dielectric due to the electric charges in the parallel plates, exerts a force on the positive/negative charges in the atoms, distorting them to resemble electric dipoles. This polarization of the atoms in the dielectric material, results in a reduced electric field by a factor of ϵ_r.

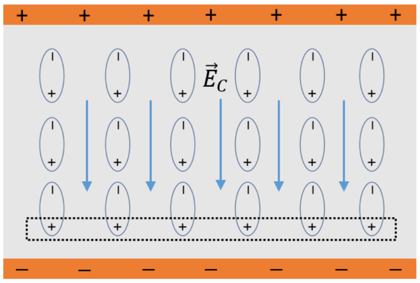

Fig. 3. The polarization of the atoms in a dielectric material due to the irrotational electric field from electric charges.

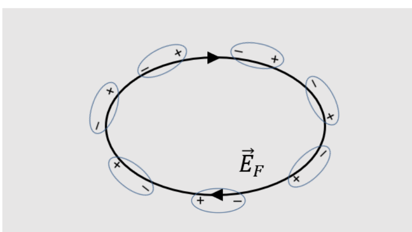

Fig. 4. The polarization of the atoms in a dielectric material due to the rotational electric field, generated by a time-varying magnetic-flux density $\vec{B}$ in Faraday's law.

Lets compare the electric-field pattern in the parallel-plate capacitor, to the electric-field pattern generated in Faraday's law. Such a "swirling" electric-field pattern can be expected to be generated by a time-varying magnetic-flux density $\vec{B}$ in Faraday's law, since $\nabla \times \vec{E}_F \neq 0$. Similar electric dipoles can be expected in the case of the electric field generated in Faraday's law, shown in the figure. These dipoles are a consequence of the force on the positive and the negative charges making up the atoms of the dielectric material. The orientation of the dipoles, however, are very different compared to Figure 3. In the simplified representation, there is a net "row" of positive charges shown by the dotted box in Figure 3, and a periodic arrangement of dipoles in the case of $\vec{E}_C$. This is not the case, however, with $\vec{E}_F$ in Figure 4.

This difference in the orientation of the electric dipoles, can be used to explain the difference in the effect of the dielectric material on an electric field in modifying the field, and the possibly large difference in the permittivity values between ϵ_r and κ.

VII. The Wave Equation in the Reformulated Maxwell's Equations

In a lossless, source-free region, and a uniform medium with permittivity ϵ_r and permeability μ_r, sufficiently far away that $\vec{\mathscr{D}}_C$ has decayed to 0, the wave equation can be derived from the reformulated equations, by following the standard procedure of the derivation from the existing equations, outlined in electromagnetics books,

$$\nabla^2 \vec{\mathscr{E}}_F = \frac{1}{\frac{\kappa\, c^2}{\mu_r \epsilon_r}} \frac{\partial^2 \vec{\mathscr{E}}_F}{\partial t^2}. \tag{54}$$

Similarly, the wave equation of the magnetic field $\vec{\mathscr{H}}$ can also be derived. This equation can be used to derive the value of κ, explained next.

VIII. Measurement of ϵ_r and κ

A measurement technique to determine the permittivity values ϵ_r and κ, will be presented in this chapter. The focus of this paper is only on the theoretical formulation. Help is needed from the research community to verify this formulation.

The different techniques to measure permittivity ϵ_r have been discussed in Ref. [11]. By sweeping the frequency and the temperature of the dielectric, the effect of these parameters on permittivity can be characterized.

In the existing formulation of Maxwell's equations, two ways to measure permittivity ϵ_r are the following: (1) Measurement of ϵ_r from the ratio of capacitances, (2) Measurement of ϵ_r from electromagnetic waves. These will be referred to as Method 1 and Method 2. In the existing formulation, both these results are assumed to be identical,

$$\epsilon_{r,Method\,1} = \epsilon_{r,Method\,2} = \epsilon_r. \tag{55}$$

In the reformulated equations, however, successive application of the above two methods, can be used to calculate ϵ_r and κ. The capacitance of a parallel-plate capacitor in Figure 3 is

$$C_d = \epsilon_r \epsilon_o \frac{A}{d}, \tag{56}$$

where ϵ_r is the permittivity of the dielectric material sandwiched between the plates, A is the surface area of the plate, and d is the distance separating the plates [5].

The capacitances of two capacitors, one filled with the dielectric material whose ϵ_r is to be characterized, and the second capacitor with air as the dielectric material, are accurately measured. In the case of air as the dielectric material, Equation 56 simplifies to

$$C_a = \epsilon_o \frac{A}{d}. \tag{57}$$

Strictly speaking $\epsilon_r = 1.0$ for vacuum, but air is used for simplicity. The ratio of the capacitances,

$$\epsilon_r = \frac{C_d}{C_a}, \tag{58}$$

is the permittivity ϵ_r of the material to be characterized.

Method 2 can be used to determine κ, after ϵ_r has been determined using Method 1. The wavelengths in a dielectric material λ_d, and in air λ_a, can be measured, similar to Ref. [12].

When the dielectric material is air,

$$\lambda_a = \frac{c}{f}, \tag{59}$$

since ϵ_r, κ, and μ_r are equal to 1.0. In the case of a non-magnetic material,

$$\lambda_d = \frac{c}{f\sqrt{\frac{\varepsilon_r}{\kappa}}}. \tag{60}$$

From the above equations,

$$\frac{\epsilon_r}{\kappa} = \left[\frac{\lambda_a}{\lambda_d}\right]^2. \tag{61}$$

Since ϵ_r is known from Method 1, this value can be used to calculate κ from the above equation,

$$\kappa = \epsilon_r \left[\frac{\lambda_d}{\lambda_a}\right]^2. \tag{62}$$

The above equations are valid for the lossless medium [13]. However, the methodology is also applicable to the lossy medium case. The experimental technique in this section, can be used to validate the new formulation, and determine the value of the permittivity associated with the rotational electric field κ, in a material.

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