

Mass Spectrum of the Kaons

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Abstract

All subatomic particle masses can be expressed as either integer multiples of, or, small denominator fraction multiples of hypersphere surface volumes times ‘h’ – Planck’s constant’s coefficient. For example, the Eta meson’s mass can be expressed as $(8/3)S6h$, where S6 represents the formula for the surface volume of a 6-sphere: $(8/3)S6h = 547.866 \text{ MeV}/c^2$. One experimental mass of the Eta meson reported by Particle Data Group is: $547.865 +/- 0.031 \text{ MeV}/c^2$, which matches $(8/3)S6h$ closely. The purpose of this paper is to show that the masses of all kaons can also be expressed as simply defined multiples of $S6h$, by matching experimental kaon masses with their theoretical values in the mass spectrums generated by $S6h$ that are presented in this paper.

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1. Introduction

Kaons are mesons, which are subatomic particles composed of two quarks. The two quarks in kaons are the ‘down’ and ‘strange’ quarks. According to Quark Theory, the two quarks inside mesons orbit one another in 3d space under the influence of a central force called the Strong Force. The mathematics for predicting meson masses using this model of meson structure is very complicated and uncertain since two key elements necessary for making accurate predictions are missing, namely the exact masses of the quarks, and an exact mathematical expression for the Strong Force. In this paper, much simpler mathematics is used for specifying meson masses, based on the assumption that matter at the subatomic particle level occupies hypersphere surface volumes, and that therefore, subatomic particle masses can be expressed as multiples of hypersphere surface volumes times a constant. This is what the kaon mass spectrums presented here illustrate.

This Hypersphere Surface Volume Theory of subatomic particle structure defines quarks as masses occupying simply defined multiples of hypersphere surface volumes. Thus the mass of a quark does not have one specific value, but rather a series of possible values based on the hypersphere surface volume equation defining that quark. In HSV Theory, the base mass of the ‘down’ quark corresponds to the surface volume of a 3-sphere, and the base mass of the ‘strange’ quark corresponds to the surface volume of a 4-sphere. (Mass throughout this paper is in units of MeV/c²)

Surface volume of a 3-sphere: $S_3 = 4 \pi r^2$

Surface volume of a 4-sphere: $S_4 = 2\pi^2 r^3$

To get the 3d mass of a meson, the two hypersphere surface volume formulae representing its quark content must be multiplied together, along with ‘h’ - Planck’s constant’s coefficient ($h = 6.62607015$). Also, the r ’s in the hypersphere surface volume formulae must be set equal to one. So, to get the 3d base mass of the kaon, set the r ’s equal to one, then multiply S_3 , S_4 , and ‘h’ together.

$$S_3 S_4 = (4\pi r^2)(2\pi^2 r^3) = 8 \pi^3 r^5$$

$$S_3 S_4 h = (4\pi)(2\pi^2)h = 8 \pi^3 h$$

$$S_3 S_4 h = 1643.598 \text{ MeV}/c^2$$

Notice that the surface volume of a 6-sphere is $\pi^3 r^5$, which means that the volume of $S_3 S_4$ is exactly eight times bigger.

$$S_6 = \pi^3 r^5$$

$$S_3 S_4 = 8 \pi^3 r^5$$

Because of this equivalence ($S_3 S_4 = 8 S_6$), S_6 will be used as the factoring unit throughout this paper, rather than $S_3 S_4$, because the notation for S_6 is more concise, and, because S_6 is eight times smaller than $S_3 S_4$. To get the value of the factoring unit that will be used, set $r=1$, then multiply S_6 by the coefficient of Planck’s constant ($h = 6.62607015$).

$$S_6 h = \pi^3 h$$

$$S_6 h = 205.4497644 \text{ MeV}/c^2 = (1/8)(S_3 S_4 h)$$

This is the factoring unit which will be used to construct the mass spectrums throughout this paper.

A Note About Factoring Kaon Masses

All kaon masses are multiples of **S6h** (multiples of S3S4h actually, but S6h = (1/8)S3S4h).

Mass of any Kaon = **nS6h**

But **n** is not just any number. It is a fraction of the form **a/b** where ‘**a**’ is a multiple of a power of two and ‘**b**’ is a power of 3 times 100. (The 100 can be folded into the value of ‘**h**’ used, and may not actually be a part of ‘**b**’.)

$$\mathbf{n} = \mathbf{a/b}$$

a can be: m(1), m(2), m(4), m(8), m(16), m(32), m(64), m(128), etc – where m is an integer

b can be: $3^2(100)$, $3^3(100)$, $3^4(100)$, $3^5(100)$, $3^6(100)$, $3^7(100)$, etc.

There is no theoretical reason currently known why kaons factor this way. This finding came from trial and error searches for the correct factoring coefficients.

Here are the divisors needed to factor some kaon masses.

(The list actually shows the smallest divisors necessary to factor those kaon masses. Larger power divisors will factor all kaon masses that are factored by lower power divisors, but at an unnecessarily higher resolution.)

Divisor	Kaon
$3^2(100)$	K(1630)
$3^3(100)$	K(1270)
$3^4(100)$	the majority of kaon masses factor using this divisor
$3^5(100)$	K(700), K(1580), K(1820), K(1830)
$3^6(100)$	
$3^7(100)$	
$3^8(100)$	K(493), K(497)
$3^9(100)$	K(493), K(497)

2 Mass Spectrums of the Experimental Mass Data Associated with Individual Kaons

Mass Spectrum of K(493) Data

	n	<u>n</u> $3^8(100)$	S6h	ExpMass	Error	dm	dm / Error
	98524	(16)	493.625547				
	98525	(16)	493. 630 557	493.631	0.007	.000442	6.3%
	98526	(16)	493. 635 567	493.636	0.011	.000433	3.9%
	98526.500	(16)	493. 638 072	493.638	0.035	.000072	0.2%
	98527	(16)	493. 640 578	493.640	0.054	.000577	1.0%
12316 (128) =	98528	(16)	493.645588				
	98529	(16)	493.650598				
	98530	(16)	493.655608				
	98530.333	(16)	493. 657 278	493.657	0.020	.000278	1.4%
	98530.500	(16)	493. 658 113	493.658	0.019	.000113	0.6%
	98531	(16)	493.660618				
	98532	(16)	493.665629				
	98533	(16)	493. 670 639	493.670	0.029	.000638	2.2%
	98534	(16)	493. 675 649	493.675	0.026	.000649	2.5%
	98535	(16)	493.680659				
12317 (128) =	98536	(16)	493.685669				
	98537	(16)	493. 690 680	493.691	0.040	.000320	0.8%
	98538	(16)	493. 695 690	493.696	0.007	.000310	4.4%
	98539	(16)	493.700700				
	98540	(16)	493.705710				
	98540.666	(16)	493. 709 050	493.709	0.073	.000050	0.06%
	98541	(16)	493.710721				
	98542	(16)	493.715731				
	98543	(16)	493.720741				
12318 (128) =	98544	(16)	493.725751				
	98545	(16)	493.730761				
	98546	(16)	493.735772				
	98547	(16)	493.740782				
	98547.333	(16)	493. 742 451	493.742	0.081	.000451	0.5%
	98548	(16)	493.745792				
	98549	(16)	493.750802				
	98549.500	(16)	493. 753 072	493.753	0.042	.000307	0.7%
	98550	(16)	493.755812				
	98551	(16)	493.760823				
12319 (128) =	98552	(16)	493.765833				
	98553	(16)	493.770843				
	98554	(16)	493.775853				
	98555	(16)	493.780863				
	98556	(16)	493.785874				
	98557	(16)	493.790884				
	98558	(16)	493.795894				
	98559	(16)	493.800904				
385 (4096) =	98560	(16)	493. 805 914	493.806	0.095	.000085	0.09%
	98561	(16)	493.810925				
	98562	(16)	493.815935				

Mass Spectrum of K(497) Data

	n	<u>n</u> $3^8(100)$	S6h	ExpMass	Error	dm	dm / Error
	99310	(16)	497.563569				
	99311	(16)	497.568579				
12414 (128) =	99312	(16)	497.573589				
	99313	(16)	497.578600				
	99314	(16)	497. 583 610	497.583	0.005	.000609	12.2%
	99315	(16)	497.588620				
	99316	(16)	497.593630				
	99317	(16)	497.598640				
	99318	(16)	497.603651				
	99318.666	(16)	497. 606 990	497.607	0.007	.000009	0.1%
	99319	(16)	497.608661				
12415 (128) =	99320	(16)	497.613671				
	99321	(16)	497.618681				
	99322	(16)	497.623692				
	99322.250	(16)	497. 624 944	497.625	0.001	.000056	5.6%
	99323	(16)	497.628702				
	99324	(16)	497.633712				
	99325	(16)	497.638722				
	99326	(16)	497.643732				
	99327	(16)	497.648743				
388 (4096) =	99328	(16)	497.653753				
	99329	(16)	497.658763				
	99329.500	(16)	497. 661 268	497.661	0.033	.000268	0.8%
	99330	(16)	497.663773				
	99331	(16)	497.668783				
	99332	(16)	497.673794				
	99333	(16)	497.678804				
	99334	(16)	497.683814				
	99335	(16)	497.688824				
12417 (128) =	99336	(16)	497.693834				
	99337	(16)	497.698845				
	99338	(16)	497.703855				
	99339	(16)	497.708865				
	99340	(16)	497.713875				
	99341	(16)	497.718885				
	99342	(16)	497.723896				
	99343	(16)	497.728906				
12418 (128) =	99344	(16)	497.733916				
	99345	(16)	497.738926				
	99345.666	(16)	497. 742 266	497.742	0.085	.000266	3.1%
	99346	(16)	497.743936				
	99347	(16)	497.748947				
	99348	(16)	497.753957				

Mass Spectrum of K(700) Data

	<u>n</u>	S6h				
	n	$3^5(100)$	ExpMass	Error	dm	dm / Error
19 (4096)	= 152 (512)	657.980	658	13	0.020	0.2%
	153 (512)	662.309				
	154 (512)	666.638				
	155 (512)	670.967				
	156 (512)	675.296				
	157 (512)	679.624				
	158 (512)	683.953				
	159 (512)	688.282				
20 (4096)	= 160 (512)	692.611	694	53	1.389	2.6%
	161 (512)	696.940				
	162 (512)	701.269				
	163 (512)	705.597				
	164 (512)	709.926				
	165 (512)	714.255				
	166 (512)	718.584				
	167 (512)	722.913				
21 (4096)	= 168 (512)	727.241	727		0.241	
	169 (512)	731.570				
	170 (512)	735.899				
	171 (512)	740.228				
	172 (512)	744.557				
	173 (512)	748.886				
	174 (512)	753.214				
	175 (512)	757.543				
22 (4096)	= 176 (512)	761.871				
	706 (128)	764.036	764	63	0.036	0.1%
	177 (512)	766.201				
	178 (512)	770.530				
	179 (512)	774.858				
	180 (512)	779.187				
	181 (512)	783.516				
	182 (512)	787.845				
	183 (512)	792.174				
23 (4096)	= 184 (512)	796.503	797	19	0.497	2.6%
	185 (512)	800.831				
	186 (512)	805.160				
	187 (512)	809.489				
	188 (512)	813.818				
	189 (512)	818.147				
	190 (512)	822.475				
	191 (512)	826.804	826	49	0.804	1.6%
24 (4096)	= 192 (512)	831.133				
	193 (512)	835.462				
	194 (512)	839.791				
	195 (512)	844.120				
	196 (512)	848.448				
	197 (512)	852.777				
	198 (512)	857.106				
	199 (512)	861.435				
26 (4096)	= 208 (512)	900.394				
	209 (512)	904.723	905	65	0.277	0.4%
	210 (512)	909.052				
	211 (512)	913.381				
	212 (512)	917.709				

Mass Spectrum of K(892) Data

n	$\frac{n}{3^4(100)}$	S6h				
		ExpMass	Error	dm	dm / Error	
	4386 (8)	889.977	890.0	2.3	.023	1.0%
	4387 (8)	890.180				
	4388 (8)	890.383	890.4	0.2	.017	8.5%
	4389 (8)	890.586				
	8779 (4)	890.688	890.7	0.9	.012	1.3%
	4390 (8)	890.789				
	4391 (8)	890.992	891	1	.008	0.8%
549 (64)	= 4392 (8)	891.195				
	4393 (8)	891.398				
	4394 (8)	891.601				
	8789 (4)	891.702	891.7	0.6	.002	0.6%
	4395 (8)	891.804				
	8791 (4)	891.905	891.9	0.7	.005	0.7%
	4396 (8)	892.007	892.0	2.6	.007	0.3%
	4397 (8)	892.209	892.2	1.5	.009	0.6%
	4398 (8)	892.412				
	4399 (8)	892.615	892.6	0.5	.015	3.0%
550 (64)	= 4400 (8)	892.818	892.8	1.6	.018	1.1%
	4401 (8)	893.021	893	1	.021	2.1%
	35215 (1)	893.199	893.2	0.1	.001	1.0%
	4402 (8)	893.224				
	4403 (8)	893.427				
	8807 (4)	893.528	893.5	1.1	.028	2.5%
	35231 (1)	893.605	893.6	0.1	.005	5.0%
	4404 (8)	893.630				
	4405 (8)	893.833				
	4406 (8)	894.036	894.0	1.3	.036	2.8%
	4407 (8)	894.239	894.2	2.0	.039	1.9%
551 (64)	= 4408 (8)	894.442				
	8817 (4)	894.543	894.53	0.17	.013	7.6%
	4409 (8)	894.644	894.63	0.76	.014	1.8%
	4410 (8)	894.847	894.9	0.5	.053	10.6%
	4411 (8)	895.050	895	1	.050	5.0%
	4412 (8)	895.253				
	17649 (2)	895.304	895.3	0.2	.004	2.0%
	17651 (2)	895.405	895.41	0.32	.004	1.3%
	4413 (8)	895.456	895.47	0.20	.014	7.0%
	4414 (8)	895.659	895.6	0.8	.059	7.3%
	4415 (8)	895.862	895.9	0.5	.038	7.6%
552 (64)	= 4416 (8)	896.065	896.0	0.6	.065	10.8%
	17667 (2)	896.217	896.2	0.3	.017	5.7%
	4417 (8)	896.268				
	4418 (8)	896.471	896.4	0.9	.071	7.9%
	4419 (8)	896.674				
	4420 (8)	896.876				
	8841 (4)	896.978	897	1	.021	2.1%
	4421 (8)	897.079	897.1	0.7	.021	3.0%
	4422 (8)	897.282				
	4423 (8)	897.485				
	8847 (4)	897.587	897.6	0.9	.013	1.4%
553 (64)	= 4424 (8)	897.688				
	4425 (8)	897.891	897.9	1.1	.009	0.8%
	8851 (4)	897.993	898.0	0.7	.007	1.0%
	4426 (8)	898.094	898.1	1.0	.006	0.6%
	4427 (8)	898.297				
	8855 (4)	898.398	898.4	1.3	.002	0.2%

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(1270) Data

n	$\frac{n}{3^3(100)}$	S6h	ExpMass	Error	dm	dm / Error
	1018 (16)	1239.395				
	1019 (16)	1240.612				
	1020 (16)	1241.830	1242	9/10	0.170	1.9%
	1021 (16)	1243.047				
	1022 (16)	1244.265				
	1023 (16)	1245.482				
16,384 =	1024 (16)	1246.700				
	1025 (16)	1247.917	1248.1	3.3/1.4	0.183	5.5%
	1026 (16)	1249.135				
	1027 (16)	1250.352				
	1028 (16)	1251.570				
	1029 (16)	1252.787				
	1030 (16)	1254.004	1254	33/34	0.004	0.0%
	1031 (16)	1255.222				
	1032 (16)	1256.439				
	1033 (16)	1257.657				
	1034 (16)	1258.874				
	1035 (16)	1260.092	1260	~	0.092	
	1036 (16)	1261.309				
	1037 (16)	1262.527				
	1038 (16)	1263.744				
	1039 (16)	1264.962				
	1040 (16)	1266.179				
	1041 (16)	1267.397				
	1042 (16)	1268.614				
	1043 (16)	1269.832	1270	10	0.168	1.7%
	1044 (16)	1271.049				
	1045 (16)	1272.267				
	1046 (16)	1273.484				
	1047 (16)	1274.702	1275	10	0.298	3.0%
	1048 (16)	1275.919	1276	~		
	1050.5 (16)	1278.962	1279	10	0.038	0.4%
	1059 (16)	1289.311	1289	25	0.311	1.2%
	1063 (16)	1294.181	1294	10	0.181	1.8%
	1068 (16)	1300.269	1300	~		

Mass Spectrum of K(1400), K(1410), and K(1430) Data

	<u>n</u>	8100	S6h	ExpMass	Error	dm	dm / Error
6.50 (8192)	= 416 (128)	1350.591	1350	~			
	417 (128)	1353.837					
	418 (128)	1357.084					
	419 (128)	1360.331					
	420 (128)	1363.577					
	421 (128)	1366.824	1367	54	0.160	0.3%	
	3371 (16)	1368.042	1368	18	0.042	0.2%	
	423 (128)	1373.318	1373	14/18	0.318	2.3%	
	424 (128)	1376.564					
	425 (128)	1379.811	1380	21/19	0.189	0.9%	
	426 (128)	1383.057					
	427 (128)	1386.303					
	428 (128)	1389.550					
	3430 (16)	1391.986	1392	18	0.014	0.1%	
	429 (128)	1392.797					
	430 (128)	1396.044					
	431 (128)	1399.290					
6.75 (8192)	= 3456 (16)	1402.537					
	3457 (16)	1402.943	1403	7	0.057	0.8%	
	3458 (16)	1403.349					
	3459 (16)	1403.755					
	3460 (16)	1404.160	1404	10	0.160	1.6%	
	3461 (16)	1404.566					
	3462 (16)	1404.972					
	3463 (16)	1405.378					
433 (128)	= 3464 (16)	1405.784					
	3465 (16)	1406.189	1406	29	0.189	0.7%	
	3466 (16)	1406.595					
	3467 (16)	1407.001					
	3468 (16)	1407.407					
	3469 (16)	1407.813					
	3470 (16)	1408.219					
	3471 (16)	1408.624					
434 (128)	= 3472 (16)	1409.030					
	3473 (16)	1409.436					
	3474 (16)	1409.842	1410	25	0.159	0.6%	
	3475 (16)	1410.248					
	3476 (16)	1410.654					
	3477 (16)	1411.059					
	3478 (16)	1411.465					
	3479 (16)	1411.871	1412	6	0.129	2.2%	
435 (128)	= 3480 (16)	1412.277					
	3481 (16)	1412.683					
	3482 (16)	1413.089					
	3483 (16)	1413.494					
	3484 (16)	1413.900	1414	130.	0.100	0.8%	
	3485 (16)	1414.306					
	3486 (16)	1414.712					
	3487 (16)	1415.118	1415	15	0.118	0.8%	

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

436(128) =	3488(16)	1415.524			
	3489(16)	1415.929	1416	10	0.071 0.7%
	3490(16)	1416.335			
	3491(16)	1416.741			
	3492(16)	1417.147			
	3493(16)	1417.553			
	3494(16)	1417.958	1418	8	0.042 0.5%
	3495(16)	1418.364			
437(128) =	3496(16)	1418.770			
	3497(16)	1419.176	1419.1	3.7	0.076 2.1%
	3498(16)	1419.582			
	3499(16)	1419.988	1420.0	3.1	0.013 0.4%
	3500(16)	1420.393			
	3501(16)	1420.799			
	3502(16)	1421.205	1421.1	2.6	0.105 4.0%
	3503(16)	1421.611	1421.6	4.2	0.010 0.2%
438(128) =	3504(16)	1422.017			
	3505(16)	1422.423			
	3506(16)	1422.828	1423	5	0.172 3.4%
	3507(16)	1423.234			
	3508(16)	1423.640	1423.8	4.6	0.160 3.5%
	3509(16)	1424.046			
	3510(16)	1424.452			
	3511(16)	1424.858	1425	8	0.143 1.8%
439(128) =	3512(16)	1425.263			
	3513(16)	1425.669			
	3514(16)	1426.075	1426	8/24	0.075 0.9%
	3515(16)	1426.481			
	3516(16)	1426.887	1427	12	0.113 0.9%
	3517(16)	1427.292	1427.3	1.5	0.008 0.5%
	3518(16)	1427.698			
	3519(16)	1428.104	1428	3	0.104 3.3%
440(128) =	3520(16)	1428.510	1428.5	3.8	0.010 0.3%
	3521(16)	1428.916			
	3522(16)	1429.322			
	3523(16)	1429.727			
	3524(16)	1430.133	1930	3.2	0.133 4.2%
	3525(16)	1430.539			
	3526(16)	1430.945			
	3527(16)	1431.351	1431.2	1.8	0.150 8.3%
441(128) =	3528(16)	1431.757			
	3529(16)	1432.162			
	3530(16)	1432.568	1432.7	0.7	0.132 18.9%
	3531(16)	1432.974	1433	6/10	0.026 4.3%
	3532(16)	1433.380			
	3533(16)	1433.786			
	3534(16)	1434.192	1434	4/6	0.191 4.8%
	3535(16)	1434.597			
442(128) =	3536(16)	1435.003	1435	6	0.003 0.1%
	3537(16)	1435.409			
	3538(16)	1435.815	1436	8	0.185 2.3%
	3539(16)	1436.221			
	3540(16)	1436.627			
	3541(16)	1437.032	1437	8/16	0.032 0.4%
	3542(16)	1437.438			
	3543(16)	1437.844	1438	8/4	0.156 2.0%

443(128) =	3544(16)	1438.250				
	3545(16)	1438.656				
	3546(16)	1439.061				
	3547(16)	1439.467				
	3548(16)	1439.873	1440	10	0.127	1.3%
	3549(16)	1440.279				
	3550(16)	1440.685				
	3551(16)	1441.091				
	444(128)	1441.496				
	445(128)	1444.743				
	446(128)	1447.990				
	447(128)	1451.236				
7.00 (8192) =	3584(16)	1454.483	1455	20/15	0.517	2.6%
	3585(16)	1454.888				
	3586(16)	1455.294				
	3587(16)	1455.700				
	3588(16)	1456.106				
	3589(16)	1456.512				
	3590(16)	1456.917				
	3591(16)	1457.323				
449(128) =	3592(16)	1457.729				
	3593(16)	1458.135				
	3594(16)	1458.541				
	3595(16)	1458.946	1459	9	0.054	0.6%
	3596(16)	1459.352				
	3597(16)	1459.758				
	3598(16)	1460.164				
	3599(16)	1460.570				
450(128) =	3600(16)	1460.976	1461.0	4.0/2.1	0.024	0.6%
	3601(16)	1461.382				
	3602(16)	1461.788				
	3603(16)	1462.194				
	3604(16)	1462.599				
	3605(16)	1463.005	1463	64/68	0.005	0.01%
	3606(16)	1463.411				
	3607(16)	1463.817				
451(128) =	3608(16)	1464.223				
	3609(16)	1464.629				
	3610(16)	1465.034				
	3611(16)	1465.440				
	3612(16)	1465.846				
	3613(16)	1466.252				
	3614(16)	1466.658	1466.6	0.7/3.4	0.002	0.3%
	3615(16)	1467.064				
452(128) =	3616(16)	1467.469				
	453(128)	1470.716	1471	12	0.284	2.4%
	454(128)	1473.963	1474	25	0.037	0.1%
	455(128)	1477.209				
	456(128)	1480.456				
	457(128)	1483.702				
	458(128)	1486.949				
	459(128)	1490.196				
	460(128)	1493.442				
	461(128)	1496.689				
	462(128)	1499.935	1500	30	0.065	0.2%
	463(128)	1503.182				
7.25 (8192)	464(128)	1506.429				

Mass Spectrum of K(1580) Data

n	<u>n</u> $3^5(100)$	S6h	ExpMass	Error	dm	dm / Error
364 (512)	1575.689					
365 (512)	1580.018	1580	~	.018		
366 (512)	1584.347					

Mass Spectrum of K(1630) Data

n	<u>n</u> $3^2(100)$	S6h	ExpMass	Error	dm	dm / Error
221 (32) =	7072	1614.378				
222 (32) =	7104	1621.683				
223 (32) =	7136	1628.988	1629	7	0.011	0.2%
224 (32) =	7168	1636.293				
225 (32) =	7200	1643.598				

Mass Spectrum of K(1650), K(1680) Data

n	<u>n</u> S6h 8100	ExpMass	Error	dm	dm / Error
	508 (128)	1649.279			
	509 (128)	1652.526			
	510 (128)	1655.772			
	511 (128)	1659.019			
64 (1024)	= 512 (128)	1662.266			
	513 (128)	1665.513			
	514 (128)	1668.759			
	515 (128)	1672.006			
	516 (128)	1675.253			
	516.5 (128)	1676.875	1650	50	0.721
	517 (128)	1678.499	1677	10/32	0.125
	518 (128)	1681.746	1678	64	0.499
	519 (128)	1684.992			
65 (1024)	= 520 (128)	1688.239			
	521 (128)	1691.486			
	522 (128)	1694.732			
	523 (128)	1697.979			
	524 (128)	1701.226			
	525 (128)	1704.472			
	526 (128)	1707.719			
	527 (128)	1710.965			
66 (1024)	= 528 (128)	1714.212			
	529 (128)	1717.459			
	530 (128)	1720.705			
	530.5 (128)	1722.328	1722	20	0.328
	531 (128)	1723.952			
	532 (128)	1727.198			
	533 (128)	1730.445			
	534 (128)	1733.692			
	534.5 (128)	1735.314	1735	10/20	0.314
	535 (128)	1736.938			
67 (1024)	= 536 (128)	1740.185			
	537 (128)	1743.431			
	538 (128)	1746.678			
	539 (128)	1749.925			
	540 (128)	1753.171			
	541 (128)	1756.418			
	542 (128)	1759.665			
	543 (128)	1762.911			
68 (1024)	= 544 (128)	1766.158			
	545 (128)	1769.404			
	546 (128)	1772.651			
	547 (128)	1775.898			
	548 (128)	1779.144			
	549 (128)	1782.391			
	550 (128)	1785.637			
	551 (128)	1788.884			
69 (1024)	= 552 (128)	1792.131	1793	59	0.869
	553 (128)	1795.377			
	554 (128)	1798.624	1800	70	1.376
	555 (128)	1801.871			
	556 (128)	1805.117			
	557 (128)	1808.364			
	558 (128)	1811.610			
	559 (128)	1814.857			
70 (1024)	= 560 (128)	1818.104			
	561 (128)	1821.350			
	562 (128)	1824.597			
	563 (128)	1827.843			
	564 (128)	1831.090			
	565 (128)	1834.337			
	566 (128)	1837.583			
	567 (128)	1840.830	1840	~	
71 (1024)	= 568 (128)	1844.077			

Mass Spectrum of K(1770) and K(1780) Data

	<u>n</u>	S6h				
	8100		ExpMass	Error	dm	dm/Error
	524 (128)	1701.226				
	525 (128)	1704.472				
	526 (128)	1707.719				
	527 (128)	1710.965				
8.25 (8192) =	528 (128)	1714.212				
	529 (128)	1717.459				
	530 (128)	1720.705				
	531 (128)	1723.952				
	532 (128)	1727.198				
	533 (128)	1730.445	1730	~		
	534 (128)	1733.692				
	535 (128)	1736.938				
	536 (128)	1740.185	1740	14/15	0.185	1.3%
	537 (128)	1743.431	1743	15	0.431	2.9%
	1075 (64)	1745.054	1745	20	0.054	0.3%
	538 (128)	1746.678				
	539 (128)	1749.925				
	540 (128)	1753.171				
	541 (128)	1756.418				
	542 (128)	1759.665	1760	15	0.335	2.2%
	543 (128)	1762.911				
8.50 (8192) =	544 (128)	1766.158				
	544.25 (128)	1766.969	1767	6	0.031	0.5%
	545 (128)	1769.404				
	546 (128)	1772.651	1773	8	0.349	4.4%
	547 (128)	1775.898	1776	26	0.102	0.4%
	548 (128)	1779.144	1779	11	0.144	1.3%
	1097 (64)	1780.767	1781	8/4	0.233	2.9%
	549 (128)	1782.391				
	1099 (64)	1784.014	1784	9	0.014	0.2%
	550 (128)	1785.637	1786	8	0.363	4.5%
	551 (128)	1788.884				
	1103 (64)	1790.507	1790	15	0.507	3.4%
	552 (128)	1792.131				
	553 (128)	1795.377				
	554 (128)	1798.624				
	555 (128)	1801.871				
	556 (128)	1805.117				
	557 (128)	1808.364				
	558 (128)	1811.610	1812	28	0.390	1.4%
	559 (128)	1814.857				
8.750 (8192) =	560 (128)	1818.104				
	561 (128)	1821.350				
	562 (128)	1824.597				
	563 (128)	1827.843				
	564 (128)	1831.090				

Mass Spectrum of K(1820), K(1830) Data

	n	<u>n</u> $3^5(100)$	S6h	ExpMass	Error	dm	dm / Error
106 (2048) =	834 (256)	1805.117					
	835 (256)	1807.281					
	836 (256)	1809.445					
	837 (256)	1811.610					
	838 (256)	1813.775					
	839 (256)	1815.939	1816	13	0.061	0.5%	
	840 (256)	1818.104					
	841 (256)	1820.268					
	842 (256)	1822.432					
	843 (256)	1824.597					
423 (512) =	844 (256)	1826.761					
	845 (256)	1828.926					
	846 (256)	1831.090	1830	~			
	847 (256)	1833.254					
	848 (256)	1835.419					
	849 (256)	1837.583					
	850 (256)	1839.748	1840	~			
	851 (256)	1841.912					
	852 (256)	1844.077					
	853 (256)	1846.241					
107 (2048) =	854 (256)	1848.405					
	855 (256)	1850.570					
	856 (256)	1852.734	1853	27	0.266	1.0%	
	857 (256)	1854.899					
	858 (256)	1857.063					
	859 (256)	1859.227					
	860 (256)	1861.392					
	861 (256)	1863.556					
	862 (256)	1865.721					
	863 (256)	1867.885					
433 (512) =	864 (256)	1870.049					
	865 (256)	1872.214					
	866 (256)	1874.378	1874	43	0.378	0.9%	
	867 (256)	1876.543					
	868 (256)	1878.707					

Mass Spectrum of K(1950) Data

	<u>n</u>	S6h 8100	ExpMass	Error	dm	dm / Error
	577 (128)	1873.296				
	578 (128)	1876.543				
	579 (128)	1879.789				
	580 (128)	1883.036				
	581 (128)	1886.282				
	582 (128)	1889.529				
	583 (128)	1892.776				
9.125 (8192) =	584 (128)	1896.022				
	585 (128)	1899.269				
	586 (128)	1902.516				
	587 (128)	1905.762				
	588 (128)	1909.009				
	589 (128)	1912.255				
	590 (128)	1915.502				
	590.5 (128)	1917.125	1917	12	0.125	1.0%
	591 (128)	1918.749				
9.250 (8192) =	592 (128)	1921.995				
	593 (128)	1925.242				
	594 (128)	1928.488				
	595 (128)	1931.735				
	596 (128)	1934.982				
	597 (128)	1938.228				
	598 (128)	1941.475				
	599 (128)	1944.721	1945	10/20	0.279	2.8%
9.375 (8192) =	600 (128)	1947.968				
	601 (128)	1951.215				
	602 (128)	1954.461				
	603 (128)	1957.708				
	604 (128)	1960.955				
	605 (128)	1964.201				
	606 (128)	1967.448				
	607 (128)	1970.694				
9.500 (8192) =	608 (128)	1973.941				
	609 (128)	1977.188				
	610 (128)	1980.434				
	611 (128)	1983.681				
	612 (128)	1986.927				
	613 (128)	1990.174				
	614 (128)	1993.421				
	615 (128)	1996.667				

Mass Spectrum of K(1980) Data

	<u>n</u>	8100	S6h	ExpMass	Error	dm	dm / Error
	136 (512)	1766.158					
	137 (512)	1779.144					
	138 (512)	1792.131					
	139 (512)	1805.117					
	140 (512)	1818.104					
	141 (512)	1831.090					
	142 (512)	1844.077					
	143 (512)	1857.063					
	143.8437 (512)	1868.020		1868	8/40	0.020	0.3%
9.0 (8192)	= 144 (512)	1870.049					
	145 (512)	1883.036					
	146 (512)	1896.022					
	147 (512)	1909.009					
	148 (512)	1921.995					
	149 (512)	1934.982					
	150 (512)	1947.968					
	151 (512)	1960.955					
	151.9375 (512)	1973.129		1973	8/25	0.129	1.6%
9.5 (8192)	= 152 (512)	1973.941					
	153 (512)	1986.927					
	154 (512)	1999.914					
	155 (512)	2012.900					
	156 (512)	2025.887					
	157 (512)	2038.873					
	158 (512)	2051.860					
	159 (512)	2064.846					
10.0 (8192)	= 160 (512)	2077.833					
	161 (512)	2090.819					
	162 (512)	2103.806					
	163 (512)	2116.792					
	164 (512)	2129.778					
	165 (512)	2142.765					
	166 (512)	2155.751					
	167 (512)	2168.738					
	168 (512)	2181.724					

Mass Spectrum of K(2045) Data

	<u>n</u>	S6h				
	8100		ExpMass	Error	dm	dm / Error
9.750 (8192)	= 624 (128)	2025.887				
	625 (128)	2029.133				
	626 (128)	2032.380				
	627 (128)	2035.627				
	628 (128)	2038.873	2039	10	0.127	1.3%
	629 (128)	2042.120				
	630 (128)	2045.367				
	631 (128)	2048.613				
	632 (128)	2051.860				
	633 (128)	2055.106				
	634 (128)	2058.353				
	635 (128)	2061.600	2062	14/13	0.400	2.9%
	636 (128)	2064.846				
	637 (128)	2068.093				
	638 (128)	2071.339				
	639 (128)	2074.586				
10.000 (8192)	= 640 (128)	2077.833				
	640.5 (128)	2079.455	2079	7	0.455	6.5%
	641 (128)	2081.079				
	642 (128)	2084.326				
	643 (128)	2087.573	2088	20	0.427	2.1%
	644 (128)	2090.819	2090	9/11	0.819	9.1%
	645 (128)	2094.066				
	646 (128)	2097.312				
	647 (128)	2100.559				
	648 (128)	2103.806				
	649 (128)	2107.052				
	650 (128)	2110.299				
	651 (128)	2113.545	2115	46	1.455	3.2%
	652 (128)	2116.792				
	653 (128)	2120.039				
	654 (128)	2123.285				
	655 (128)	2126.532				
10.250 (8192)	= 656 (128)	2129.778				

Mass Spectrum of K(2250), K(2320), and K(2500) Data

	<u>n</u>	S6h	ExpMass	Error	dm	dm / Error	Kaon
	n	8100					
10 (8192)	=	160 (512)	2077.833				
		161 (512)	2090.819				
		162 (512)	2103.806				
		163 (512)	2116.792				
		164 (512)	2129.778				
		165 (512)	2142.765				
		166 (512)	2155.751				
		167 (512)	2168.738				
		168 (512)	2181.724				
		169 (512)	2194.711				
		170 (512)	2207.697				
		171 (512)	2220.684				
		172 (512)	2233.670	2235	50	1.330	2.7%
		173 (512)	2246.657	2247	17	0.343	2.0%
		174 (512)	2259.643	2260	20	0.357	1.8%
		175 (512)	2272.629				
11 (8192)	=	176 (512)	2285.616				
		177 (512)	2298.602				
		178 (512)	2311.589				
		179 (512)	2324.575	2324	24	0.575	2.4%
		180 (512)	2337.562				
		181 (512)	2350.548				
		182 (512)	2363.535				
		183 (512)	2376.521				
		184 (512)	2389.508				
		185 (512)	2402.494				
		186 (512)	2415.480				
		187 (512)	2428.467				
		188 (512)	2441.453				
		189 (512)	2454.440				
		190 (512)	2467.426				
		191 (512)	2480.413				
		191.75 (512)	2490.152	2490	20	0.152	0.8%
12 (8192)	=	192 (512)	2493.399				
		193 (512)	2506.386				
		194 (512)	2519.372				
		195 (512)	2532.359				
		196 (512)	2545.345				
		197 (512)	2558.331				
		198 (512)	2571.318				
		199 (512)	2584.304				
		200 (512)	2597.291				
		201 (512)	2610.277				
		202 (512)	2623.264				
		203 (512)	2636.250				
		204 (512)	2649.237				
		205 (512)	2662.223				
		206 (512)	2675.210				
		207 (512)	2688.196				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(2380) Data

<u>n</u>	n 8100	S6h				
			ExpMass	Error	dm	dm / Error
	168 (512)	2181.724				
	169 (512)	2194.711				
	170 (512)	2207.697				
	171 (512)	2220.684				
	172 (512)	2233.670				
	173 (512)	2246.657				
	174 (512)	2259.643				
	175 (512)	2272.629				
11.0 (8192)	= 176 (512)	2285.616				
	177 (512)	2298.602				
	178 (512)	2311.589				
	179 (512)	2324.575				
	180 (512)	2337.562				
	181 (512)	2350.548				
	182 (512)	2363.535				
	183 (512)	2376.521				
	183.5 (512)	2383.014	2382	14/19	1.014	7.2%
11.5 (8192)	= 184 (512)	2389.508				
	185 (512)	2402.494				
	186 (512)	2415.480				
	187 (512)	2428.467				
	188 (512)	2441.453				
	189 (512)	2454.440				
	190 (512)	2467.426				
	191 (512)	2480.413				
	767 (128)	2490.152				
12.0 (8192)	= 192 (512)	2493.399				
	193 (512)	2506.386				
	194 (512)	2519.372				
	195 (512)	2532.359				
	196 (512)	2545.345				
	197 (512)	2558.331				
	198 (512)	2571.318				
	199 (512)	2584.304				
	200 (512)	2597.291				

Mass Spectrum of K(3100) Data

n	(128)	8100	<u>n</u> S6h				
			ExpMass	Error	dm	dm / Error	
937.5	(128)	3043.700					
938.0	(128)	3045.323	3045	8/20	0.323	4.0%	
938.5	(128)	3046.946					
939.0	(128)	3048.570					
939.5	(128)	3050.193					
940.0	(128)	3051.816	3052	8/20	0.184	2.3%	
940.5	(128)	3053.440					
941.0	(128)	3055.063	3055	7/20	0.063	0.9%	
941.333	(128)	3056.145	3056	7/20	0.145	2.1%	
941.5	(128)	3056.686					
942.0	(128)	3058.309					
942.5	(128)	3059.933	3060	8/20	0.067	0.8%	
943.0	(128)	3061.556					
943.5	(128)	3063.179					
14.75 (8192)	=	944.0 (128)	3064.803				
		944.5 (128)	3066.426				
		944.666 (128)	3066.967	3067	6/20	0.033	0.6%
		945.0 (128)	3068.049				
		945.5 (128)	3069.673				
		946.0 (128)	3071.296				
		946.5 (128)	3072.919				
		947.0 (128)	3074.543				
		947.5 (128)	3076.166				
		948.0 (128)	3077.789				
		948.5 (128)	3079.412				
		949.0 (128)	3081.036				
		949.5 (128)	3082.659				
		950.0 (128)	3084.282				
		950.5 (128)	3085.906				
		951.0 (128)	3087.529				
		951.5 (128)	3089.152				
		952.0 (128)	3090.776				
		952.5 (128)	3092.399				
		953.0 (128)	3094.022				
		953.333 (128)	3095.104	3095	30	0.104	0.3%
		953.5 (128)	3095.646				
		954.0 (128)	3097.269				
		954.5 (128)	3098.892				
		955.0 (128)	3100.515				
		955.5 (128)	3102.139				
		956.0 (128)	3103.762				
		956.5 (128)	3105.385	3105	30	0.385	1.3%
		957.0 (128)	3107.009				
		957.5 (128)	3108.632				
		958.0 (128)	3110.255				
		958.5 (128)	3111.879				
		959.0 (128)	3113.502				
		959.5 (128)	3115.125	3115	30	0.125	0.4%
15.00 (8192)	=	960.0 (128)	3116.749				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

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n	<u>n</u> $3^4(100)$	S6h	ExpMass	Error	dm	Reference
	150.000 (512)	1947.968				
	151.000 (512)	1960.954				
	152.000 (512)	1973.941				
	153.000 (512)	1986.927				
	153.250 (512)	1990.174	1990	80	0.174	arXiv:2012.07360
	154.000 (512)	1999.913				
	155.000 (512)	2012.900				
	156.000 (512)	2025.886				
	156.625 (512)	2034.003	2034	13/9	0.003	arXiv:2009.08099
	157.000 (512)	2038.873				
	158.000 (512)	2051.859				
	159.000 (512)	2064.846				
10 (8192) =	160.000 (512)	2077.833				
	161.000 (512)	2090.819				
	162.000 (512)	2103.806				
	162.500 (512)	2110.298	2111	43/25	0.702	arXiv:2012.07360
	163.000 (512)	2116.792				
	163.750 (512)	2126.531	2126.5	16.8/12.4	0.031	arXiv:2001.04131
	164.000 (512)	2129.778				
	165.000 (512)	2142.765				
	166.000 (512)	2155.751				
	664.666 (128)	2157.915	2158	30/33	0.085	arXiv:2112.13219
	167.000 (512)	2168.738				
	167.750 (512)	2178.477	2179	21/3	0.523	arXiv:2009.08099
	168.000 (512)	2181.724				
	169.000 (512)	2194.711				
	170.000 (512)	2207.697	2208	19/24	0.303	arXiv:2202.06447
	171.000 (512)	2220.684				
	684.666 (128)	2222.848	2223	16/11	0.152	arXiv:2112.15076
	172.000 (512)	2233.670				
	173.000 (512)	2246.657				
	174.000 (512)	2259.643				
	175.000 (512)	2272.629				
11 (8192) =	176.000 (512)	2285.616				
	177.000 (512)	2298.602	2298	60/44	0.602	arXiv:2112.13219
	178.000 (512)	2311.589				
	179.000 (512)	2324.575				
	180.000 (512)	2337.562				
	181.000 (512)	2350.548				
	182.000 (512)	2363.535				
	183.000 (512)	2376.521				
	184.000 (512)	2389.508				
	185.000 (512)	2402.494				
	186.000 (512)	2415.480				
	187.000 (512)	2428.467				
	188.000 (512)	2441.453				

‘Reference’ refers to the source of ‘ExpMass’ and ‘Error’ data.

4. Commentaries on Select Mass Spectrums

4.1 K(493) Mass Spectrum Commentary

Of the 15 experimental mass data points reported for K(493) by PDG, only 14 are plotted in this mass spectrum, because two of the 15 data points are the same (493.640 by LUM and 493.640 by CHENG), which leaves only 14 unique data points.

11 of the 14 experimental masses plotted in this mass spectrum factor with a divisor of $3^8(100)$. The other three factor to integers in the numerator of the factoring fraction if a divisor of $3^9(100)$ is used. These three masses can be identified in the mass spectrum by their **n**'s, which end with either .333 or .666.

The experimental masses of K(493) have the smallest errors of any kaon. They range in size from .007 to .095 with an average of .0385 MeV/c². If one assumes that the placements of the experimental masses in this mass spectrum are correct, then by examining the values in the dm/Error column one sees that the experimental errors assigned to the experimental masses are bigger than necessary by anywhere from 16 to 500 times. (Excepting two extreme cases. dm/Error = 0.09% translates to 1111 times too big and dm/Error = 0.06% translates to 1666 times too big.)

The resolution (step size, block size) of this mass spectrum is:

$$(16/(3^8(100))) S6h = 0.0050 \text{ approximately.}$$

This is 1.4 to 19 times smaller than the experimental errors, so one might argue that the assignments of the experimental masses to the theoretical masses may be incorrect. But if the experimental errors are on average 50 times too big, then the adjusted experimental errors (errors divided by 50) are 6.5 times smaller on average than 0.0050. (Using the average of the experimental errors, which is 0.0385.) It might be argued that this is a circular argument, but the fact that there are three occurrences of two experimental masses plotted sequentially in this mass spectrum (only 0.0050 MeVc² apart) that have overlapping error sizes lends credence to the belief that the experimental errors assigned to K(493)'s experimental masses are larger than they should be, i.e. - they do not reflect the true degree of accuracy of the experimenters determinations.

4.2 K(700) Mass Spectrum Commentary

Only 7 of the 24 experimental masses reported by PDG for K(700) are plotted in this mass spectrum to emphasize their positions relative to large factor blocks. Four of the plotted masses fall on large factor blocks and three fall very close to large factor blocks.

4.3 K(892) Mass Spectrum Commentary

Only 35 of the 65 experimental masses reported by PDG for K(892) are plotted in this mass spectrum, because 17 are redundant, 5 are outliers, 2 are almost identical to two others, and 6 factor better with a divisor of $3^5(100)$ rather than the $3^4(100)$ divisor used in this mass spectrum.

The experimental mass data associated with K(892) is the third most accurately determined of all kaon mass data. Experimental mass errors for the 35 experimental masses plotted in K(892)'s mass spectrum vary from 0.1 to 2.6 with an average of 0.87 MeV/c². The base resolution of this mass spectrum is $(8/8100)S6h = 0.2029$ approximately, so since the resolution of this mass spectrum is 4.29 times smaller than the average error, it could be argued that the assignment of an experimental mass to its closest matching theoretical mass in the mass spectrum could be incorrect. This would be true if the true error size was equal to the reported error size, but there are reasons to believe they are larger on average for the same reasons given in the K(493) mass spectrum commentary.

4.10 **K(1770), K(1780)** Mass Spectrum Commentary

The experimental masses of these kaons seem to be symmetrically arranged around the large block 8.5(8192).

4.16 **K(3100)** Mass Spectrum Commentary

Of the 11 experimental masses reported by PDG for K(3100), 2 are redundant. Of the 9 left that are plotted in this mass spectrum 3 factor with a factor divisor of $3^5(100)$ rather than a divisor of $3^4(100)$, which was used to construct this mass spectrum. Those three can be identified by n's that end in .333 or .666.

4.17 **BESIII Kaons** Mass Spectrum Commentary

Six of the eight kaons plotted in this mass spectrum factor to fractions with integer numerators using a divisor of $3^4(100)$. The other two kaons factor to fractions with integer numerators using a divisor of $3^5(100)$. Those two can be identified as the ones with n's ending in .666.

5. Summary

The good agreement between the values of experimental and theoretical kaon masses shown in the various mass spectrums presented in this paper, lends strong support for belief in the idea that kaons are composed of matter that occupies simply defined fractions of 6-sphere surface volumes. Specifically, kaon masses can all be specified by the expression:

$$\frac{n(2^y)}{3^x(100)} S6h$$

Where n, x, and y are integers, $S6 = \pi^3$, and $h = 6.62607015$.