The expansion of spacetime

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Abstract

This paper is based on the idea that the expansion of the universe is an expansion of spacetime, involving not just an expansion of *space*, as in the *FLRW* metric, but also an expansion of the *time* coordinate. It is shown that the standard cosmological model in which the universe consists of two-thirds dark energy in the form of a cosmological constant, results from misinterpreting observational data, and that dark energy is an artefact that arises due to not including the expansion of time in the usual metric. It is argued that the proposed model is not physically equivalent to the standard *FLRW* model, since it predicts new physics. The acceleration in universal expansion is understandable on the basis of a mass-dominated universe, where the deceleration due to gravitation is more than compensated for by the slowing down of time as the universe expands.

1 Introduction

Observations of redshifts of light from distant galaxies are interpreted to mean that the universe is expanding. By assuming that the cosmological principle applies, i.e. the universe is homogeneous and isotropic on a large enough scale, this expansion is described in the currently accepted standard cosmological model [1] via the Friedmann-Lemaitre-Robertson-Walker (*FLRW*) metric, which may be written:

$$d\tilde{s}^2 = -c^2 dt^2 + a(t)^2 ds^2 \qquad (FLRW metric) \qquad (1)$$

Here, $d\tilde{s}$ is a spacetime increment, c the speed of light, dt a time increment, and ds is a space increment with a time-dependent scale factor

$a(t)$ that describes the expansion. The scale factor a is conventionally set to unity in the present. The time coordinate t is usually referred to as the cosmic time, and the spatial coordinates are comoving coordinates. For each t the spatial slices are maximally symmetric; $a(t)$ is called the scale factor, since it tells us how the distance between two points scale with time. This metric has been discussed in countless papers and textbooks on cosmology, and is now regarded as generic in its description of an expanding universe as well as satisfying Einstein's field equations of general relativity, and is one of the foundational assumptions in modern gravitational physics [2].

Although this is currently regarded as standard theory, several scientists believe it does not describe universal expansion correctly. We know from Einstein's special theory of relativity [3], that *space* and *time* are linked via the speed of light, and in the description of spacetime as a 4D Minkowski space, *time* can be regarded as another orthogonal dimension related to the three spatial dimensions by the conversion factor ic, where i is the imaginary unit. Verkhovski has pointed out [4] that in the original paper of Einstein, a scale factor ϕ was included as a multiplying factor for all 4 coordinates of spacetime in the equations known as the Lorentz transformation. At the time, Einstein argued that this multiplier was equal to unity, which was, of course, before universal expansion was known about. In any case, Einstein's theory did not treat, or attempt to treat, an expanding spacetime. However, in the light of present day knowledge that the universe is expanding, this multiplying factor may be just what is required theoretically to describe the scale of the universe as it expands. This suggests that not just *space* is expanding, but also the *time* axis.

Observational evidence has been mounting that can be interpreted as due to *time* expanding. For example, Senovilla, Mars and Vera proposed such an explanation for the phenomenon that very distant supernovae seem to be moving faster than those nearer the centre of the universe - not as a result of hypothetical dark energy - but caused by time itself slowing down [5]. Johan Masreliez in particular has published several papers and books on what he calls a scale-expanding cosmos [6]. In his theory, space and time are expanding in a scale-invariant way, such that each epoch in the development of the universe is identical to any other epoch.

The purpose of my paper is to present an outline of my own independent view and analysis on this subject of a changing time scale. Some of the ideas will be similar to those of Masreliez, but others are not. I shall use as simple a mathematical analysis as possible, in an attempt to make the paper accessible to a maximum number of people.

2 Expanding time scale model

Before I even begin this paper in ernest, I want to mention a few ideas that sound more like fantasy than real physics. Please bear with me as I consider some thought experiments - the kind of thing that Einstein himself was very fond of. The danger with thought experiments, however, is that you cannot actually carry them out in reality - which then opens up the door to completely untestable conclusions.

Firstly, imagine you could instantaneously "beam" someone back in time with a one-metre rule to when the universe was half its present size, and could compare this rule with a metre rule in the present. Would it be half its current length, or would it have the same length as now? It would be the same length. What is smaller is the spatial distance between material objects, not the size of material objects themselves. Secondly, with regard to time, we can measure the progression of time on a laboratory clock - a cesium atomic clock, for example. Now imagine this same intrepid time traveller has taken an atomic clock back in time with him/her. Would the clock rate have altered? No, atomic vibrational frequencies are the same in every epoch. What is different is that a virtual clock that has been devised to tick according to the scale of the time coordinate will be ticking much more quickly in the past, since the time axis has been compressed. In other words, real clocks and rigid rods do not scale with the universe. It is only spacetime itself that expands, and the factor linking space and time is the invariant speed of light.

According to the *FLRW* spacetime metric (Equation 1), time advances at the same rate, whether in the past, present, or future. However, if the time axis changes in the same way as the three space axes, i.e. by the same scale factor, we could write a new metric that takes this into account, i.e. by using the same multiplier for all 4 coordinates:

$$d\tilde{s}^2 = -c^2 dt'^2 = a^2[-c^2 dt^2 + ds^2] \tag{2}$$

Even though this new metric looks mathematically like a simple coordinate transformation of time intervals in the *FLRW* metric from dt to $a\,dt$, I consider that it contains new physics, and is not physically equivalent to the *FLRW* metric. This will be justified in more detail below.

3 Einstein tensor components

This section contains completely standard procedure to derive geodesic equations from the new metric of Equation 2, and calculate the Christoffel coefficients describing the curvature, before applying Einstein's *GR* theory.

Since NASA's *WMAP* satellite observations indicate that the large-scale universe is approximately flat [8], I shall simplify the following analysis by writing the metric in Equation 2 using polar coordinates as

$$d\tilde{s}^2 = a(t)^2[-c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2] \tag{3}$$

having ignored any possible spatial curvature in ds. The Lagrangian is given by

$$L = a^2\left(-c^2\dot{t}^2 + \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right) \tag{4}$$

From the Euler-Lagrange equation we then obtain the following geodesic equations for t, r, θ and ϕ:

$$\ddot{t} + \frac{\dot{a}}{c^2a}\left(c^2\dot{t}^2 + \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin\theta^2\,\dot{\phi}^2\right) = 0 \tag{5}$$

$$\ddot{r} + \frac{2\dot{a}}{a}\,\dot{r}\dot{t} - r(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2) = 0 \tag{6}$$

$$\ddot{\theta} + \frac{2\dot{a}}{a}\,\dot{\theta}\dot{t} + \frac{2}{r}\,\dot{\theta}\dot{r} - \sin\theta\cos\theta\,\dot{\phi}^2 = 0 \tag{7}$$

$$\ddot{\phi} + \frac{2\dot{a}}{a}\,\dot{\phi}\dot{t} + \frac{2}{r}\,\dot{\phi}\dot{r} + 2\cot\theta\,\dot{\phi}\dot{\theta} = 0 \tag{8}$$

The Christoffel symbols are found from the coefficients in front of each term. (The Christoffel symbol for a mixed term is half the coefficient, since the term is really the sum of two equal terms, i.e., $\Gamma^{\phi}_{\phi t} = \Gamma^{\phi}_{t\phi} = \dot{a}/a$, for example.) The "dots" above t, r, θ and ϕ refer to differentiation with respect to the Lagrangian parameter λ (which is the proper time t' in *GR*), whereas the "dot" over a refers to an "ordinary" or coordinate time derivative. We thus obtain for the set of non-zero connection coefficients:

$$\Gamma^{t}_{tt} = \frac{\dot{a}}{a}; \Gamma^{t}_{rr} = \frac{\dot{a}}{c^2a}; \; \Gamma^{t}_{\theta\theta} = \frac{r^2\dot{a}}{c^2a}; \; \Gamma^{t}_{\phi\phi} = \frac{r^2\dot{a}\sin^2\theta}{c^2a}$$

$$\Gamma^{r}_{rt} = \frac{\dot{a}}{a}; \; \Gamma^{r}_{\theta\theta} = -r; \; \Gamma^{t}_{\phi\phi} = -r\sin^2\theta$$

$$\Gamma^{\theta}_{\theta t} = \frac{\dot{a}}{a}; \; \Gamma^{\theta}_{\theta r} = \frac{1}{r}; \; \Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta$$

$$\Gamma^{\phi}_{\phi t} = \frac{\dot{a}}{a}; \; \Gamma^{\phi}_{\phi r} = \frac{1}{r}; \; \Gamma^{\phi}_{\theta\phi} = \cot\theta \tag{9}$$

Next we need to calculate the components of the Ricci curvature tensor. For example, R_{tt} is obtained from

$$R_{tt} = \frac{\partial \Gamma^{\gamma}_{tt}}{\partial x^{\gamma}} - \frac{\partial \Gamma^{\gamma}_{t\gamma}}{\partial t} + \Gamma^{\gamma}_{tt}\Gamma^{\delta}_{\gamma\delta} - \Gamma^{\gamma}_{t\delta}\Gamma^{\delta}_{t\gamma}$$

where we sum over γ and δ for t, r, θ, ϕ. Then the following components are obtained:

$$R_{tt} = -\frac{3\ddot{a}}{a} + \frac{3\dot{a}^2}{a^2};\ R_{rr} = \frac{\ddot{a}}{c^2 a} + \frac{\dot{a}^2}{c^2 a^2}; R_{\theta\theta} = \frac{r^2\ddot{a}}{c^2 a} + \frac{r^2\dot{a}^2}{c^2 a^2};$$

$$R_{\phi\phi} = \sin^2\theta \left(\frac{r^2\ddot{a}}{c^2 a} + \frac{r^2\dot{a}^2}{c^2 a^2} \right) \tag{10}$$

The Ricci scalar curvature R, which is the trace of the Ricci curvature tensor with respect to the metric, is then calculated using

$$R = R^a_a = g^{ab}R_{ab} = g^{tt}R_{tt} + g^{rr}R_{rr} + g^{\theta\theta}R_{\theta\theta} + g^{\phi\phi}R_{\phi\phi} \tag{11}$$

Since g_{ab} is a diagonal matrix, the components of the inverse matrix are just the inverse of the components of the original matrix, i.e., with $g_{tt} = -a^2c^2$, we have $g^{tt} = -1/(a^2c^2)$. We then find that the scalar curvature (for $k = 0$, a flat space) is given by

$$R = \frac{6\ddot{a}}{c^2 a^3} \tag{12}$$

Using Einstein's equation:

$$G_{ab} = R_{ab} - \frac{1}{2} R\, g_{ab} = \kappa T_{ab} \tag{13}$$

we then obtain all four non-zero diagonal components of the Einstein tensor:

$$G_{tt} = 3H^2 \qquad ; \qquad G_{ii} = \frac{-(2\dot{H} + H^2)}{c^2} \qquad (Owen) \tag{14}$$

where I have written $\dot{a}/a = H$, which is called the Hubble parameter, and I have surreptitiously converted to Cartesian coordinates (x, y, z), instead of (r, θ, ϕ), in order that the three diagonal terms labelled ii, where $i = x, y, z$, are the same. Also, $\dot{H} = dH/dt = (\ddot{a}/a - \dot{a}^2/a^2)$.

Next, exactly the same procedure for the *FLRW* metric (Equation 1) may be carried out, and for that we obtain the following Einstein tensor components for comparison:

$$G_{tt} = 3H^2 \qquad ; \qquad G_{ii} = \frac{-(2\dot{H} + 3H^2)}{c^2} \qquad (FLRW) \tag{15}$$

Equations 14 and 15 differ only in the factor H^2 versus $3H^2$, respectively, in the ii components, but this difference is crucial for a proper understanding.

Now assume that the universe contains a combination of mass-energy (which I shall write as ϱ_M) plus vacuum energy in the form of a cosmological constant (which I shall denote as ϱ_Λ). The cosmological constant is described by diagonal terms in the Einstein tensor proportional to $(\varrho, -\varrho/c^2, -\varrho/c^2, -\varrho/c^2)$, whereas mass-energy gives $(\varrho, 0, 0, 0)$. Thus, we may write

$$G_{tt} \sim \varrho_M + \varrho_\Lambda \quad ; \quad G_{ii} \sim -\frac{\varrho_\Lambda}{c^2} \tag{16}$$

where the symbol "$\sim$" means "is proportional to". For the two metrics we thus have

$$\varrho_M + \varrho_\Lambda \sim 3H^2 \;\; ; \;\; \varrho_\Lambda \sim 2\dot{H} + H^2 \qquad (Owen) \tag{17}$$

$$\varrho_M + \varrho_\Lambda \sim 3H^2 \;\; ; \;\; \varrho_\Lambda \sim 2\dot{H} + 3H^2 \qquad (FLRW) \tag{18}$$

From *ESA's* Surveyor Probe and using the *FLRW* metric as the cosmological model, the universe is thought to contain 68.3% dark energy (almost exactly 2/3), which means - using a reverse physics calculation - the ratio $\varrho_M : \varrho_\Lambda \approx 1 : 2$ must have been obtained. This then implies (using *FLRW*): $\dot{H} = -\frac{1}{2} H^2$, and on integrating, we find: $H = 2/t$.

Substituting this behaviour into my model, we obtain $\varrho_\Lambda = 0$, and the cosmological constant disappears completely. (That is the most important statement in this paper.) This finding is very strongly suggestive that the large value of Λ is an artefact that arose entirely due to using the *FLRW* metric, which does not correctly describe universal expansion, viz. expansion of the time coordinate must also be taken into account, as I have done in my metric.

With $\dot{H} = -\frac{1}{2} H^2$, we have $a \sim t^2$, i.e. an accelerating expansion. This is now understandable, even for an entirely mass-dominated universe, since the gravitational attraction tending to slow down the expansion is more than compensated for by the acceleration due to the time coordinate expanding into the future.

So, repeating what I said, ESA's probe found that the universe is expanding as t^2. They then used *FLRW* to calculate that the universe consists of 2/3 dark energy, whereas if they had used the expanding time scale in my metric they would have concluded there is no need to invent dark energy.

4 Replacement Friedmann equations

The standard cosmological model with the *FLRW* metric leads to a set of equations called the Friedmann equations. The new metric in Equation 2 leads to different equations for describing the same quantities, which I shall now derive.

Firstly, as a reminder, the Friedmann equations derived using the *FLRW* metric and the resulting Einstein tensor components are:

$$H^2 = \frac{8\pi G\varrho}{3} \;\; ; \;\; \dot{H} = -4\pi G\left(\varrho + \frac{P}{c^2}\right) \tag{19}$$

where ϱ is the mass density and P the hydrostatic pressure, which lead to

$$\dot{\varrho} = -3\frac{\dot{a}}{a}\left(\varrho + \frac{P}{c^2}\right) \tag{20}$$

Using the cosmological equation of state

$$P = w\varrho c^2 \tag{21}$$

we then arrive at

$$\varrho \sim a^{-3w-3} \;\; ; \;\; a \sim t^{\frac{2}{3+3w}} \tag{22}$$

For a mass-dominated universe, $w = 0$, this gives $\varrho \sim a^{-3}$ and $a \sim t^{2/3}$; for a radiation-dominated universe, $w = 1/3$, we have $\varrho \sim a^{-4}, a \sim t^{1/2}$, while a cosmological constant, $w = -1$, gives $\varrho = \textit{constant}$, and an exponential expansion.

In my model the equivalent expressions are:

$$H^2 = \frac{8\pi G\varrho}{3} \;\; ; \;\; \dot{H} = -4\pi G\left(\frac{\varrho}{3} + \frac{P}{c^2}\right) \tag{23}$$

This leads to

$$\dot{\varrho} = -3\frac{\dot{a}}{a}\left(\frac{\varrho}{3} + \frac{P}{c^2}\right) \tag{24}$$

from which is obtained

$$\varrho \sim a^{-3w-1} \;\; ; \;\; a \sim t^{\frac{2}{1+3w}} \tag{25}$$

For a mass-dominated universe ($w = 0$) we now have $a \sim t^2$, which implies the universe will accelerate its expansion even though gravity is trying to slow it down. (The Friedmann model gives $a \sim t^{2/3}$, which is a deceleration.) For a radiation-dominated universe ($w = 1/3$) my model gives $a \sim t$ (whereas Friedmann gives $t^{1/2}$), and a cosmological constant ($w = -1$) doesn't give a sensible result.

Another interesting case would be for $w = -1/3$, which can be regarded as a dust model of non-interacting particles. In that case, my model predicts an exponential increase in scale factor (not immediately apparent from the equation), while the Friedmann model gives a corresponding linear increase in scale factor.

5 Discussion

5.1 Galactic red-shift

In standard cosmology the usual relationship between red-shift and the expansion is derived by considering a light wave emitted at the frequency of the source f_s with a wavelength λ_s, related via the speed of light as

$$\lambda_s = \frac{c}{f_s} \tag{26}$$

It is then imagined that during transit the wave is stretched as space expands. On arrival at the observer it will accordingly have a new wavelength λ_o determined by the amount space has expanded during transit. The red-shift is defined as

$$z = \frac{\lambda_o - \lambda_s}{\lambda_s} \tag{27}$$

This is then related to the scale change of space by the expression

$$\frac{\lambda_o}{\lambda_s} = \frac{1}{a} = 1 + z \tag{28}$$

where a is the scale factor when the light was emitted.

The model presented here does not alter this interpretation, since atomic frequencies in the past were the same as today. The abstract spacetime clock that can be imagined to have been ticking more rapidly in the past, has no bearing on atomic frequencies. This would mean that atomic vibrational frequencies emitting light we observe today were not higher in the past, and then the conventional argumentation applies for understanding galactic red-shifts, viz. they are due to the expansion of space during transit of photons from source to observer.

I mention this because Masreliez [9] proposed a different mechanism for the observed red-shifts, which I shall now explain briefly. Consider the equation of motion of a free particle moving on a radial path ($d\theta = 0; d\phi = 0$) relative to an observer in a scale-expanding spacetime. From the radial geodesic obtained earlier we have:

$$\ddot{r} + 2\frac{\dot{a}}{a}\dot{r}\dot{t} = 0 \tag{29}$$

Furthermore, the metric line element in Equation 2 may be rearranged to read:

$$\frac{1}{\dot{t}} = a\sqrt{1 - \frac{v^2}{c^2}} \tag{30}$$

where $v = dr/dt$. Substituting this into the previous equation gives the following differential equation (after several lines of working):

$$\frac{dv}{dt} + Hv\left(1 - \frac{v^2}{c^2}\right) = 0 \tag{31}$$

where dv/dt is the coordinate acceleration.

Masreliez's model of a scale-expanding spacetime represents a special case of my model, in which he sets H as constant. This is equivalent to assuming only a cosmological constant, with no mass. With H constant, he then integrated the same equation as above from an initial velocity v_0, resulting in:

$$v = \frac{v_0 e^{-Ht}}{\sqrt{1 - v_0^2/c^2 + (v_0^2/c^2)e^{-2Ht}}} \tag{32}$$

This result predicts that a freely moving particle will decelerate. If the particle velocity is much smaller than the speed of light, the expression can be approximated as:

$$v = v_0 e^{-Ht} \;\; ; \;\; [v \ll c] \tag{33}$$

He called this effect cosmic drag, since the acceleration is proportional to the velocity and in the opposite sense (like a friction force, or Newtonian viscosity), i.e. H acts like a drag coefficient. By writing the relativistic energy of the particle as

$$E = mc^2 = m_0 c^2/\sqrt{1 - v^2/c^2} \tag{34}$$

(where m = relativistic mass, m_0 = rest mass), and inserting the expression for v given in Equation 32, one obtains the expression:

$$\frac{E}{E_0} = \frac{v_0}{v} e^{-Ht} \tag{35}$$

where E_0 is the initial energy. Masreliez then considered this particle to be a photon of light (which has no rest mass, only relativistic mass), and wrote $v_0 = v(= c)$, and thus obtained an exponential decay in energy:

$$\frac{E}{E_0} = e^{-Ht} \tag{36}$$

Since the energy of a photon is $E = hf$, where $f = c/\lambda$ is the frequency, he obtained

$$\lambda = \lambda_0 e^{Ht} \tag{37}$$

where λ_0 is the emitted wavelength, and claimed that the red-shift of light increases exponentially with the time that elapsed since the light was emitted by the source. This is clearly a different explanation of red-shift of galactic light from that given by the standard cosmological model, which is based on the expansion of space alone. The functional form resembles that of Zwicky's tired light phenomenon [10], however, the perceived energy loss here is not due to dissipative losses (such as collisions, or scattering with other photons), but due to the expansion of time during transit. Unfortunately, I believe Masreliez's mathematical argumentation (jumping from the equation of motion of a particle with mass to a light photon) might possibly be flawed, rendering the analysis partially incorrect.

5.2 Conclusion

The new exponents in my model are of importance for the cosmological model, and if adopted would alter the predictions considerably. However, it is sure to be rejected on the grounds that the metric appears to be physically equivalent to the *FLRW* model, since one metric can be transformed into the other by a simple coordinate transformation. Indeed, a well-known way of rewriting the *FLRW* metric is via the conformal time τ, which is defined as

$$\tau = \int_0^t \frac{dt}{a(t)} \qquad or \qquad a\, d\tau = dt \tag{38}$$

The distance $c\,\tau$ represents the comoving distance travelled by a photon in a time t. This then allows one to write the *FLRW* metric as

$$d\tilde{s}^2 = a(\tau)^2[-c^2 d\tau^2 + ds^2] \tag{39}$$

which appears to be the same as my metric in Equation 2 with my time quantity equal to the conformal time. Clifton et al [2] have indeed calculated the solutions to the standard-model Friedmann equations using both cosmic time and conformal time, and obtained the same exponents as I obtained in a previous section, but without making the same deductions as I have made about the expansion of time.

I maintain that my model introduces new physics, and is not another way of writing the same thing. If the expansion of time is not taken into account (as in *FLRW*), then τ is the conformal time, but if time expansion is taken into account, then what was the conformal time

τ becomes the same quantity as the actual time coordinate t in the new spacetime metric. The proof that this is something different lies in the obvious prediction that there is no need to invent a substance called dark energy to describe the accelerated universal expansion. If an expanding time scale is not considered, as in the standard model, then dark energy in a very unlikely relationship of $2/3$ has to be introduced as an *ad hoc* hypothesis, which must surely be regarded as very unlikely indeed.

References

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