The Riemann Hypothesis: Aspiring toward Perfect Simplicity

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ABSTRACT[1]

The oft-ventured yet elusive Riemann Hypothesis allows for some unparalleled, perfecting simplicity which arguably denies any more-economical means. An overlap obtains with prior work from a drastically different angle.

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For Aught Simple[r]

RH would seem to be building on *individual* terms that are [quadratic] *zero*-values (sufficiency) or on *sign-alternation/reversal* (necessity) which will be shown to effectively imply a minor extension of the same. The rationale is presented in (1), whilst a set of implications in (2).

$$\zeta(s) \equiv \sum_{n=1}^{T} n^{-s} = 0 \sim \frac{1}{T} \; \boldsymbol{IF} \; n^{-s} \equiv n^{-(a+it)} = 0^2 \sim T^{-2}, \qquad T \to \infty \quad (1)$$

$$n^{-s} = n^{-a} * [\cos(2i\pi k - tlogn) + isin(2i\pi k - tlogn)] = 0^2, \qquad \forall k, n \in \boldsymbol{N}$$

$$\sqrt{\cos(\pm 2i\pi k + tlogn) - isin(\pm 2i\pi k + tlogn)} = 0, \qquad \forall a$$

$$\cos\left(\pm i\pi k + \frac{t}{2} * logn\right) - isin\left(\pm i\pi k + \frac{t}{2} * logn\right) = 0 \quad (2a)$$

$$cot\left(\frac{t}{2} * logn \pm i\pi k\right) = i \qquad (2b)$$

What (2a) suggests is that, an argument/phase delta of *i*pi*k* accounts for a minus sign under *k odd*, or *k=2l+1*. In effect, *Re(s)=**1/2*** is implied as *-1=exp (log1*(2l+1)/2)*, if only because the *potential Re(s)=a* is rendered irrelevant (*a*-invariance), whilst *Re(s)=**k/2*** collapses to ½ invariably due to *2l/2=1*. Please note that the *log1* power obtains from:

$$n^{-s} = e^{log1 + logn^{-s}} = e^{\log(1 * n^{-s})} \qquad (3)$$

Put differently, ½ prevails as an *implied* real part if only phenomenologically (the underlying structural ontology being well there too).

[1] For those binding the world back to sense, beauty, chastity, and truth…

Now, this was a *sufficient* scenario. One other way a zero zeta could obtain would *necessarily* involve either an *n*-specific extension/transform *phi(n)* (a finite, bounded function or factor or operator) or some kind of sign-variability across the n^{-s} terms, without necessarily building on adjacency. In other words (4):

$$\zeta(s) = 0 \;\; \boldsymbol{ONLY\ IF} \;\; \forall\varphi_n \exists\Delta_n : n^{-s} = \begin{cases} \varphi_n * 0^2 \\ -(n+\Delta_n)^{-s} \equiv -(\varphi_n n)^{-s} \;\; \leftrightarrow n^{-s} = 0 \; OR \; \varphi_n = 1^{\frac{2}{s}} \end{cases} \quad (4)$$

This does appear to reduce the necessary criterion to a minor extension of the sufficient core. For that matter, the overlap with Shevenyonov (2022) looks striking—more so despite the drastically divergent core approaches!

Of interest could be to discern some further, "constructive" implications for ***Im****(s)* based on (2b). By dint of standard identities, one arrives at (5a) through (5c) as equivalents pointing to (6).

$$cot\left(\frac{t}{2} * logn \pm i\pi k\right) \equiv \cot(\cdot) = \sqrt{\frac{cos^2(\cdot)}{1 - cos^2(\cdot)}} = \sqrt{\frac{1}{cos^{-2}(\cdot) - 1}} = i \; IF \; cos^{-2}(\cdot) = 0 \quad (5a)$$

$$cot\left(\frac{t}{2} * logn \pm i\pi k\right) \equiv \cot(\cdot) = \pm\sqrt{\frac{1 + cos2(\cdot)}{1 - cos2(\cdot)}} = i \;\; IF \; cos2(\cdot) = T \to \pm\infty \quad (5b)$$

$$cos^{-2}(x) = 0 \;\; IF \; (\frac{e^{ix} - e^{-ix}}{2})^2 = T^2 \;\; \leftrightarrow \;\; \frac{1}{n^{it} - n^{-it} - 2} \sim 0^2 \quad (5c)$$

$$n^{-it} \sim 0^2 \;\; \leftrightarrow it \sim T^2 \sim (2i\pi k)^{-2} \;\; \leftrightarrow t = \begin{cases} -2T \sim -T \\ -T^2/2\pi \sim -T^2 \end{cases} \quad (6)$$

$$cos2x = cos^2x - sin^2x \to cos^2x \;\; IF \; sin^2x \to 0 \;\; \leftrightarrow \;\; \frac{t}{2} * logn \pm i\pi k,$$
$$t = \mp\frac{2i\pi k}{logn} = \begin{cases} 0, & k = 0 \\ -T, & k \to T \end{cases} \;\; OR \;\; n^{-it} = e^{-2\pi k}, \qquad k \to T \quad (7)$$

One other standard representation would hold as per (8):

$$cos^2x = \frac{1 + cos2x}{2} \to cos2x \;\; \leftrightarrow \;\; cos2x = \begin{cases} \pm T \\ 1 \end{cases} \;\; \leftrightarrow x \equiv t * logn \pm 2i\pi k = \begin{cases} 0 \\ \pm 2i\pi k \end{cases}$$

$$t \sim \frac{2i\pi k}{logn} \;\; \leftrightarrow \;\; n^{-it} = e^{-2\pi k}, \qquad k \to T \quad (8)$$

References

Shevenyonov, Arthur V. (2022). Generalizing [Un]Even Series-Sums Toward an Eventual Demonstration for the Riemann Hypothesis & Implied Extensions. *viXra: 2201.0062*