Assessing the black hole solution

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Abstract

In this paper it is shown that the solution enabled by Hilbert that predicts black holes and an event horizon is flawed both mathematically and physically. With the correct physical ansatz linking *GR* theory with the real physical world, spacetime is shown to be completely regular outside a point mass, there is no event horizon, observed velocities do not exceed the speed of light, the effective gravitational mass falls to zero as $r \to 0$, and there is no singularity at the origin.

1 Introduction

Black holes play a major part in the current paradigm in gravitational physics and cosmology, and the vast majority of mathematicians, astrophysicists and cosmologists believe they exist in the universe as a natural phenomenon. For many scientists the rationale has been fully established, and they have moved on in their thinking, with no further consideration of the possibility that they may be a mathematical or physical artefact caused by false interpretation of theory and its link with the real physical world. Indeed, there have recently been claims that black holes have been "observed" by both gravitational wave [1] and VLBI [2] measurements, but it is not my intention here to show that those claims are false. It is indeed true to say that the concept of black holes has captured the imagination of countless scientists and non-scientists alike, and spawned enormous amounts of theoretical and experimental work in astrophysics and astronomy.

Nevertheless, I shall show in this paper that the theoretical foundation for black holes is flawed. Firstly, I shall discuss the steps leading to the currently accepted solution, and then pinpoint why this paradigm is wrong. Einstein's general theory of relativity *per se* is not being questioned here, only the way it has been interpreted.

2 Theory

2.1 Preamble

The starting point in the story of black holes is a solution using Albert Einstein's general theory of relativity (GR) [3] for the gravitational field due to a point mass in a vacuum, first obtained in 1916 by Karl Schwarzschild [4]. A year later Droste [5] and Weyl [6] independently obtained a further variant of the solution. Subsequently, Hilbert [7] extended Droste and Weyl's solution in such a way that the solution showed a discontinuity in spacetime that was later interpreted to be an event horizon obscuring the central mass, which came to be known as a black hole.

2.2 Spacetime metric and solution

Using polar coordinates the metric line element for a spherically symmetric spacetime outside a point at $r = 0$ may be written:

$$d\tilde{s}^2 = c^2 dt'^2 = Ac^2 dt^2 - B\,dr^2 - C\,d\Omega^2 \tag{1}$$

where $d\tilde{s}$ is a spacetime increment, c is the speed of light, dt' an increment of proper time, dt an increment of coordinate time, dr an increment of radial distance and $d\Omega$ an angular increment given by $d\Omega^2 = d\theta^2 + sin^2\theta\,d\phi^2$. A, B and C are radially dependent functions that describe the time, radial and angular spacetime curvature metric coefficients, respectively.

Using Lagrangian formalism the radial equation of motion for a free-falling test object (along a radius with $d\Omega = 0$) may be found from the geodesic equation in r, and is given by

$$\ddot{r} + \frac{A'}{2B}\,c^2\dot{t}^2 + \frac{B'}{2B}\,\dot{r}^2 = 0 \tag{2}$$

where "dot" refers to differentiation with respect to proper time t', and $A' = dA/dr, B' = dB/dr$.

We don't yet know how A, B and C are related, but using Einstein's vacuum field equations in GR, the following relationships are obtained as the solution:

$$A = \frac{1}{B} = \left(1 - \frac{\alpha}{\tilde{r}}\right) \quad ; \quad C = \tilde{r}^2 \tag{3}$$

where $\tilde{r}$ is some radial coordinate, defined by $\tilde{r} = \sqrt{C}$, which is related to the (true) radial distance r in an as-yet unspecified way.

Using $B = 1/A$, it is then straightforward to show that Equation 2 can be "simplified" to read

$$\ddot{r} + \frac{1}{2}c^2 A' = 0 \tag{4}$$

Integration of this expression then gives

$$A = 1 - \frac{\dot{r}^2}{c^2} \quad ; \quad B = 1/A \tag{5}$$

2.3 Weak-field approximation

Newton's law of gravitation for a free-falling test object may be written:

$$a + \frac{GM}{r^2} = 0 \tag{6}$$

where a is the classical acceleration, G is Newton's universal gravitational constant, and M is the gravitational mass at the coordinate origin. This gives the following expression for the free-fall velocity of an object falling from rest at infinity:

$$v = \sqrt{\frac{2GM}{r}} \tag{7}$$

In order to obtain correspondence between GR and classical physics in the weak-field region where Newton's law holds, the conventional procedure is then to equate this proper velocity $\dot{r}$ with v, giving:

$$A = 1 - \frac{2GM}{c^2 r} \qquad [weak\ field\ approx.] \tag{8}$$

Strange though it may seem to any rigorous mathematician or physicist not personally involved with black-hole physics, this expression is then assumed to hold for any value of r, including when it becomes very small and of the order of magnitude of $2GM/c^2$. It is then falsely envisaged by the current scientific community that this function A can become negative as $r \rightarrow 0$, and it is this mistake that is used as the rationale for believing in an event horizon at $r = 2GM/c^2$, with the point mass M obscured behind it as a black hole. This is certainly incorrect, as I shall explain in the next section.

3 The proper velocity

The fallacy arises because the speed of a free-falling object has been equated with the proper velocity $\dot{r} = dr/dt'$, which is defined via distance increments measured in a flat coordinate space (dr), which could be imagined to be a long way from the gravitational field, where space is not curved, or in a hypothetical frame of reference with the mass causing gravitation somehow removed, while the time increments dt' are measured by observing the time on a clock co-moving with the falling object.

Now consider Newton himself sitting underneath an apple tree, musing about gravity, when an apple lands on his head. He is essentially acting as an observer positioned within the gravitational field of the Earth, where - to be accurate about it - space is curved compared to that at infinity. The space is approximately flat, when the field is weak, but to ignore the curvature of space in defining the observed velocity is the crucial mistake that leads to the black-hole prediction.

It is therefore correct to identify the proper velocity according to this observer using a distance increment in terms of the curved radial space, viz. $\sqrt{B}\,dr$ (not simply dr). We then have for the observed proper velocity

$$v_{obs'} = \frac{\sqrt{B}\ dr}{dt'} \tag{9}$$

It is this quantity that should be set equal to the Newtonian expression for the free-fall velocity. We then have:

$$\frac{\sqrt{B}\ dr}{dt'} = \sqrt{\frac{2GM}{r}} \tag{10}$$

which gives

$$\dot{r}^2 = \frac{1}{B} \times \frac{2GM}{r} \tag{11}$$

Using Equation 5 and $B = 1/A$ we then obtain

$$A = \left(1 + \frac{2GM}{c^2 r}\right)^{-1} \tag{12}$$

You will notice that A never becomes negative when $r \to 0$, and it approximates the conventional solution for $r >> 2GM/c^2$. In addition, the GR proper velocity may be expressed in the following form

$$\dot{r}^2 = \frac{2GM}{r}\left(1 + \frac{2GM}{c^2 r}\right)^{-1} \tag{13}$$

4 Showing how this is linked to *SR*

Special relativity (*SR*) published in 1905 was a forerunner to *GR*. Both theories adopt a four-dimensional Minkowski spacetime composed of three spatial coordinates and one time coordinate of opposite signature. *SR* satisfies the principle of relativity of inertial motion, and is called *Lorentz covariant*, which means that there is no preferred reference frame and physical laws remain unchanged under a Lorentz transformation of the space and time coordinates. On the other hand, *GR* is *generally covariant*, such that its laws remain unchanged in form under arbitrary or general transformation of the spacetime coordinates, and the concept of there being no preferred frame is lost. Relating the two theories is, therefore, problematical. Nevertheless, in this section I shall use *SR* to show what it can say about the free-fall of a test object, even though it was not really conceived to do so.

Writing the four-force $\tilde{F}$ as the rate of change of four-momentum in the co-moving (proper) frame, we have:

$$\tilde{F} = \frac{d\tilde{p}}{dt'} = \left(\frac{iP}{c}, \vec{F}\right) \tag{14}$$

where P is power, given by $P = dE/dt'$, and E is the energy. We then write

$$\tilde{F}.d\tilde{s} = \left(\frac{iP}{c}, F\right).(ic\,dt, dr) = -P\,dt + F\,dr = 0 \tag{15}$$

for conservation of energy. Now rearrange this as

$$P\,dt = F\,dr \tag{16}$$

and then we may write

$$\frac{dE}{dt'}\,dt = -\frac{GmM}{r^2}\,dr \tag{17}$$

where the term F on the right is Newton's law for the gravitational force, and m is the rest mass of the free-falling test object. This expression represents the differential gain in kinetic energy balanced against the differential loss in potential energy. From the metric of *SR* we have

$$dt'^2 = dt^2 - \frac{dr^2}{c^2} \tag{18}$$

or

$$\frac{dt'}{dt} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{\gamma} \tag{19}$$

which gives

$$\gamma \, dE = -\frac{GmM}{r^2} \, dr \tag{20}$$

A factor γ has appeared because we transformed from proper to coordinate frame. Next, in *SR*, the gain in kinetic energy may be expressed as

$$E = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 \tag{21}$$

so that

$$dE = mc^2 d\gamma \tag{22}$$

We then have

$$\gamma \, d\gamma = -\frac{GM}{c^2}\frac{dr}{r^2} \tag{23}$$

which on integration from $r = \infty$ to r gives

$$v^2 = \frac{2GM}{r}\left(1 + \frac{2GM}{c^2 r}\right)^{-1} \tag{24}$$

This equation describes how the observed speed v of a free-falling object changes with distance r from the gravitational mass M. When r is large, the classical expression, $v^2 = 2GM/r$, is recovered, but as r decreases, the velocity lags behind the classical result, and never exceeds c.

Note that this is identical to the expression I obtained above for the proper velocity in *GR*.

5 Discussion

I included the previous section in order to demonstrate that the proper velocity I obtained using *GR* (Equation 13) now contains the correct ingredients of relativistic mechanics. The factor before the bracket is the classical expression, while the factor inside the brackets describes how in *SR* the velocity is modified by the increase in relativistic mass or, entirely equivalently, by the curvature of space in *GR*.

Writing $\alpha = 2GM/c^2$, we may express this result as

$$\frac{\dot{r}^2}{c^2} = \frac{\alpha}{r}\left(1 + \frac{\alpha}{r}\right)^{-1} = \frac{\alpha}{(r + \alpha)} \tag{25}$$

We also have

$$A = \left(1 + \frac{\alpha}{r}\right)^{-1} = \frac{r}{(r + \alpha)} \tag{26}$$

Also, from the GR solution we have

$$A = \frac{1}{B} = \left(1 - \frac{\alpha}{\tilde{r}}\right) = \left(1 + \frac{\alpha}{r}\right)^{-1} \tag{27}$$

which gives

$$\tilde{r} = r + \alpha \tag{28}$$

This now shows how the radial coordinate $\tilde{r}$ is related to the radial distance r. Importantly, $\tilde{r}$ is never smaller than α, which means that the function $(1 - \alpha/\tilde{r})$ never becomes negative, so black holes do not occur and there is no event horizon.

Hilbert [7] first extended the solution in Equation 3 to values of $\tilde{r} < \alpha$, and this became the accepted rationale for allowing black holes. This was justified historically on grounds of physical equivalence, in that changing the coordinates in GR should not alter the physics, since the theory is supposed to be generally covariant. However, Stephen Crothers [8] and Leonard Abrams [9] subsequently showed that the coordinate change also changed the limits of the spacetime manifold and therefore that Hilbert's extension to $\tilde{r} < \alpha$ was incorrect from a pure mathematical point of view.

In my analysis above, I have proved from a physical point of view that Hilbert's extension allowing $\tilde{r} \to 0$ is incorrect, since from Equation 28 $\tilde{r}$ can never be less than α, since $r \geq 0$ and α is a positive real quantity.

The solution I have proposed is one of an infinite set of solutions that satisfy Einstein's field equations of GR for the vacuum outside a point mass [8]. It was mentioned in a paper by Brillouin [10], but he did not develop it in the way I have. Schwarzschild himself never predicted black holes, but rather forced the possible discontinuity in the function $(1 - \alpha/\tilde{r})$ to be at the origin of coordinates by judicious choice of the radial coordinate $\tilde{r}$, with $\tilde{r} = (r^3 + \alpha^3)^{1/3}$, which is just one solution belonging to the infinite set of solutions.

Taking the free-fall velocity in Equation 24 and calculating the observed acceleration gives

$$a = v\frac{dv}{dr} = -\frac{1}{2}c^2\frac{\alpha}{(r+\alpha)^2} \tag{29}$$

We see that for small r the free-fall acceleration does not increase (negatively) as $1/r^2$ as $r \to 0$, but is retarded, and reaches a constant value a_0 given by

$$a_0 = -\frac{1}{2}\frac{c^2}{\alpha} \qquad [r \to 0] \tag{30}$$

This means that gravity is modified and does not diverge when $r \to 0$. From these equations we may define an effective mass M_{eff}, and write

$$\frac{M_{eff}}{M} = \left(1 + \frac{\alpha}{r}\right)^{-2} \tag{31}$$

which is unity for large r and goes to zero as $r \to 0$.

6 Conclusion

Hilbert's solution should only be regarded as an approximation valid for $r >> \alpha$, and not used when r is of the order of α or less. Replacing it with a solution first used by Brillouin, which I have developed here, not only satisfies Einstein's vacuum field equations and agrees with all predictions from *GR* - except those relating to black-hole horizon physics - but also enables a description of the behaviour valid at all r, even in the strong-field region - and then there is no horizon in space-time. The solution predicts that the velocity of a free-falling test object does not exceed the speed of light c, but behaves in a way intuitively expected from the kinematics of special relativity in conjunction with the principle of conservation of energy. It has been demonstrated that the proper velocity of *GR* is equivalent to the coordinate velocity of *SR*, which clears up a few issues with understanding the link between *GR* and *SR*. Furthermore, since the effective gravitational mass falls off to zero as $r \to 0$, this removes the singularity that is otherwise deemed to be present at the coordinate origin.

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