Approaching the Collatz Conjecture over A Re-Mystifying Demonstration

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ABSTRACT[1]

The Collatz conjecture, elusive as it may prove, has shown to be collated to other arcane results, notably ABC, Goldbach's, and their generalized ilk. The present paper demonstrates that, whilst a straightforward proof scheme could (ironically) be a near trivial enterprise, the philosophical implications would posit more of a far-reaching agenda.

Collatz Conjecture: The Maths

One may feel amazed and stunned at just how any starting number, or initial value tends to converge to unity when embarking on a trivial affinity transform, or transition algorithm. While one could sense some *a*symmetry to it, in that the *converse* might not hold functionally (unity giving rise to the whole of the natural set!), still the awe this inspires only gets further fueled by the prior simplicity somewhat at odds with the posterior [distribution of] distributions. Indeed a sheer instance of what I have long referred to as *"azimuthality"* (Shevenyonov, 2016) bordering on perceived (*phenomeno*logical) complexity or chaos that may have little to do with structural fine-tuning or knife-edge divergence around small neighborhoods or quadratic recursion-generated, fixed point based 'multiverse' patterns (underlying *onto*logies that are a bit more involved than what CC posits yet still fairly simple).

That said, we shall attempt a very straightforward approach to CC. Firstly and informally, suffice it to observe that a *match* between the starting value and one of the *success hits* (within the series: …13, 40, 20, 10, 5, 16, 8, 4, 2, 1) would amount to a *sufficient* criterion, whereas a

[1] *In memoriam* Evgenia Brik & Zinaida Kirienko, deceased 40 versus 88 of age about simultaneously days back, all of a sudden despite all too divergent lifespans, or paths—quantitatively though not quality-wise so…

good chance of arriving at such a series from an arbitrary initial value would complement it as a *necessary* condition.

We will for now assume this dimension away and focus on the more 'down-to-earth' routine. Namely, the entire algorithm could be reduced to a *functional equation* by observing how it collapses to alternating states or scenarios with alpha-weights (in essence Boolean corners) readily applying thereto.

Not only can the initial (or *locally* initial, meaning the last observed) value allow for divisibility by *higher* powers of 2, even if it does not at the ongoing stage, it will assuredly qualify next by dint or design of the very *X to 3X+1* transform. Better yet, although the latter may account for the bulk of interim divergence and slowing-down, it likewise tends to build in the potential of garnering high-powered 2^k-reducibility at any subsequent trial. All of this somehow resembles the RAD operator as part of the *ABC conjecture* for large enough constituencies which boast the potential of proving high powers of some underlying radix—or, equivalently, a low effective radix.

$$x_k = \begin{cases} x_{k-1}/2 \\ (3x_{k-1}+1)/2 \end{cases} \equiv \frac{1}{2} * [\alpha_k * x_{k-1} + (1-\alpha_k) * (3x_{k-1}+1)]$$

$$= 1/2^k * [x_0 * \prod_{i=0}^{k} \widehat{A_\iota} + \sum_{i=1}^{k} \widehat{B_\iota} * \prod_{s=1}^{i} \widehat{A_s}] \qquad (1)$$

$$A = 3 - 2\alpha = \begin{cases} 1 \\ 3 \end{cases}, \quad B = 1 - \alpha = \begin{cases} 0 \\ 1 \end{cases} \qquad (2)$$

Whilst a zero (minimum) value *B* takes on (at the same time a minimal *A* obtains) need not nullify the entire sum residuale, still from appreciating the power-of-2 inverse factor in (1), one should not be stunned to see how the *end* value, whatever it is, is bound to converge to a low one. Overall, based on the effective domain(s) for *A* and *B* as in (2), the solution reduces to (3) by replacing *x(k)* with *any* "success hit" value, or to (3a) when a *unity* asymptote is adopted, if only for simplicity's sake.

$$x_k = \frac{1}{2^k} * \left[x_0 * 3^{m_k} + \sum_{t=1}^{k} \widehat{B_t} * 3^{n_t} \right] \qquad (3)$$

$$2^k = \left[x_0 * 3^{m_k} + \sum_{t=1}^{n \le k} 3^{n_t} \right] \qquad (3a)$$

Again, X_0 is the equivalent of an arbitrary starting value. In the event of divisibility by 2, the ongoing alpha weight is unity, which persists at subsequent stages thus nullifying the B's implied. At the power-of-2 extreme, all of the B's will make 0 thus waiving the sum-residuale while keeping the *m(k)* identically zero too, such that $X_0=2^k$. Indeed, this does stand the scrutiny as a definition: A straight-linear path obtains whenever the initial value is just a power of 2. One other special case of importance could be inspected, e.g. *m(k)=1*, which automatically collapses the sum residual to a unity, so that the candidate *(k, X_0)* solutions could be (2, 1) or (4, 5). In any event, the solution remains *Diophantine* in nature even in this, simplest possible instance tantamount to the very transform *3X+1* applied in a *reversed* manner.

Now, what of other, more general solutions, in particular posed as *k=k(X_0)* as either a closed form or an implicit function? One could benefit from resorting to the well-established result, the *Chen's theorem* (Chen, 1966-73) whereby, if somewhat stylized (C-CC), any even number can be represented as a sum of either two primes or one prime and a 'semiprime' (product).

$$\exists N\ \forall x = 2m > N:\ \ x = \begin{cases} p_1 + p_2 \\ p_1 + p_2 * p_3 \end{cases} \qquad (C - CC)$$

Obviously, *2^k* qualifies as a *2m* "large enough," and so does the simplest possible case of bare-bones affinity wherein *3^m(k)=1* and sum collapsed to either unity or a product of primes as long as X_0 is a prime. Alternatively, the very sum could, for a particular X_0 and/or *k* value and per m_k*=1*, be aligned to a prime—indeed suggesting an *alternate prime-generating structure* (4), fully in line with those proposed in Shevenyonov (2022) making use of 9-factor residuales!

$$p(n) = \sum_{t=1}^{k} \widehat{B_t} * 3^{n_t}\ , \qquad B = \begin{cases} 0 \\ 1 \end{cases} \quad (4)$$

Notably, B does *not* follow a naïve *sign-reversal* path, or at any rate, all that can be stated for certain without studying the particular initial or interim value is what was posited from the outset: B=1 is necessarily followed by B=0, but not mandatorily the other way around unless *X_0=2^k* or *l*2^k*.

Not least, one alternative way of depicting the interlinkage could be to decompose 2^k *combinatorily*, with the combinations somehow juxtaposed against the in-sum B-times-product-A terms as of (3a).

Metaphysics & Dialectics Converging: The Aftermaths

The philosophical, whether moral or epistemic, implications are even more profound than the core result aims at. For one thing, this could be a productive as well as efficient or 'parsimonious' [meta-]metaphor of what anyone's path of choice-making amounts to or how it can be put on wheels or assessed in the interim. Many of us may have presumed a kind of straight-line trend, an asymptote, a "ladder of ascension," or otherwise a [weak] propensity to be rational and benevolent beyond pragmatic, oftentimes manipulative concerns. The scrupulous follower of his or her own path (inner self, tao or Heavenly Kingdom within, which also may echo *'hyh [']sh[r] 'hyh*= "Am even as/whatever/howsoever Am" or *lech lecha*= "heed/follow thyself") may apply their best effort, say in abiding by the single yet bifocal law (roughly and mnemonically codified by the CC algorithm), yet the path ends up excessively winding, at times more divergent per lower initial values such as 27 than it is for X_0=88 or 40 (both complying with X_0=n*2^3), which appears counterintuitive and may invoke the "*judge not*" imperative alongside the "*repent*" one.

This holds irrespective of whether the initial/interim value $(X_0, X_{s<k})$ pertains to a mass of sin/karma accumulated, cost incurred, risks taken, or talents granted yet to be invested or augmented—or indeed the age as a shadow metric of path that has been trodden amid the *k* residuale implied and being so hard or uncertain to solve for.

References

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