

Speculations On Black Hole Information Storage And The Topology Of Planck Mass Black Holes: Implications for Primordial Planck Mass Black Holes In Cosmology, Dark Matter & Dark Energy

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Abstract

The topology of a Planck Mass Black Hole (PMBH) is derived from a consideration of the characteristics of the lattice unit cell which stores 1 bit of information on the surface of a black hole. Information inviolability implies that the PMBH has a toroidal, not spherical, form. The toroidal form lends itself to a plausible field-based information storage mechanism with the potential for storing either "0" or "1". There are consequences for Cosmology, Dark Matter and Dark Energy all of which are now linked through toroidal Primordial PMBHs.

Topology

Planck Mass Black Holes (PMBHs) are theoretical particles whose topology is generally assumed to be spherical. This assumption is carried through into predictions of its properties and behaviour. How PMBHs store their S_{BH} 1 bit of information is generally not addressed and it is simply assumed that information is conserved somehow when, for example, they merge.

It is proven that to store 1 bit of information on the surface of a black hole requires 4 Planck areas^[1]. This is taken literally; 4 Planck area squares are required.

The analysis therefore starts by zooming in to the surface of a Black Hole until the 4 Planck areas unit cell which stores 1 bit of information can be seen. The first task is to identify the characteristics of the lattice forming this 2D unit cell. For convenience of drawing at this stage the 1 bit of information is located on the central node of the unit cell.

The unit cell is shown in Figure 1.

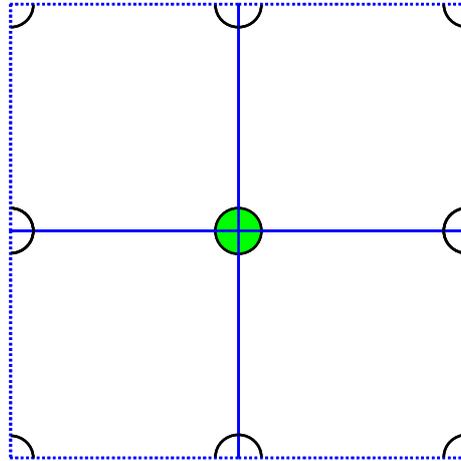


Figure 1

The unit cell has the following characteristics:

- 4 nodes
- 8 (Planck length) connections between the nodes
- 4 connections per node

For convenience this will be described as the 484 characteristic and this characteristic is required to store the 1 bit of information.

The second task, (permitting the 2D unit cell lattice to have unlimited elasticity at this stage), is to produce a fully closed (stand-alone) 3D lattice through a folding and joining procedure, (no breaking of connections within the unit cell is permitted), such that the closed 3D lattice retains the 484 characteristic and thus can still store the 1 bit of information.

Any entity produced by this procedure which satisfies all the conditions will therefore have the geometric characteristics of a PMBH.

The two actions required are shown in Figures 2a and 2b.

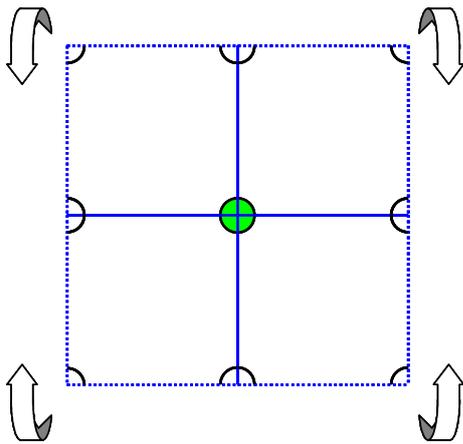


Figure 2a

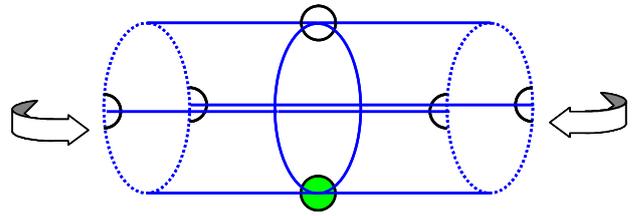


Figure 2b

The first action brings two opposing edges together and produces an (open-ended) cylinder; the second action bends the cylinder round and joins the ends to produce a (single hole) toroid. The 484 characteristic is retained and 1 bit of information is stored. In principle a toroidal black hole can be produced with any desired number of bits simply by choosing a number of (connected) unit cells.

Specifically, it is not possible to produce a (zero hole) sphere which retains the 484 characteristic and it is not possible to produce a 2 hole toroid which retains the 484 characteristic. Given that the 484 characteristic is identified as the means by which the 1 bit of information is stored the inference is clear; if the topology is not toroidal then information will be destroyed and this is currently and widely considered to be forbidden^[2].

An Objection To The Toroidal Concept

There is an instant objection possible to this toroidal form. The PMBH is being considered as an isolated non-rotating (non-charged) Schwarzschild black hole and the geometry of this type of black hole was established by Hawking^[3]. It is widely quoted that he established the geometry of such a black hole as spherical and this is therefore in contradiction to the toroidal geometry proposed here. There was, however, a rider to this spherical geometry; the basis for this rider is considered and summarised in various papers amongst which are these^{[4][5]}.

In simple terms Hawking showed that a black hole of this type could be toroidal but there was a condition which was, in essence, driven by the requirement that information/data could not pass through the toroidal hole. If it did then, essentially, there would be information passing between areas of space-time which should not have such a connection and various conflicts with physical laws would arise; (in extreme cases causality could be lost). Hawking therefore argued that the lifetime of the toroidal black hole could not be greater than the time taken for a photon, with its properties intact, to pass through the toroidal hole. This conclusion has never been challenged and there is no basis for a challenge to be made. It is, however, important to recall that Hawking was dealing with macroscopic black holes where there is a clear delineation between the toroidal body and the hole; a clear "doughnut" exists.

A macroscopic toroidal black hole always has a photon "flightpath" directly along the axis of the toroid and hence a photon can always transit the toroidal hole, thus limiting the lifetime of the toroidal black hole. In practice there is a circular section window of flightpaths around this axis the size of which is determined by the nature of the toroid. At the limit of this window the photon has a "grazing" contact with the toroidal body and outside of the window there is contact.

This macroscopic limitation, where a photon carrying information through the hole in the toroid defines the lifetime of the toroid, needs to be considered in the context of the microscopic PMBH. Moving directly to the extreme end of the range and using the folding/joining procedure above for, say, 3 bits (12 nodes), it is a simple matter to show that the axial flightpath (at least) still exists and it is still possible to discern the axial flightpath for 2 bits (8 nodes). The situation changes, however, for the 1 bit (4 node) PMBH. This will now be discussed.

The Nature Of The PMBH Toroid

The toroidal form was generated by permitting indefinite elasticity and it is now necessary to consider the situation that results when the stretched connections are permitted to "relax" thus permitting all nodes and connections to become equivalent (the 1 bit is no longer localised at one node). It is possible to draw a schematic of the lattice which illustrates the nature of the 484 characteristic required to store the 1 bit of information.

The schematic shown in Figure 3 represents the situation following the "relaxation" of the lattice and emphasis has been given to showing the connectivity of the nodes.

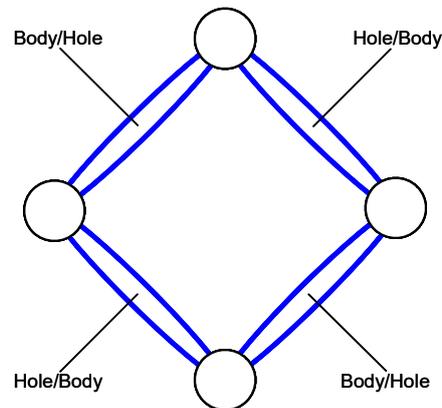


Figure 3

There are some remarkable features of the structure which arise from having only 4 nodes. Firstly, what was previously the 4 Planck areas has now split into two sets of 2 Planck areas each. One set of Planck areas, (part of the "body" of the toroid) and the "hole" of the toroid are actually located *between* the pairs of connections joining the nodes thus ensuring that the two sets of connections are fully independent. Secondly, any differentiation between the "hole" and the relevant parts of the "body" of the toroid has been lost. There is no way of determining which is which and their functions are entirely equivalent and interchangeable. There is also a suggestion in the structure that "hole" and "body" may actually "cross" in some way within the structure. Thirdly, the two "squares" which remain are the remaining set of 2 Planck areas and hence are parts of the toroidal body; specifically there is no "hole" through the squares.

With this understanding it is clear that the photon flightpath along the axis of the toroid has been compromised and there does not seem to be any prospect of a photon passing through the toroidal hole and emerging with its properties intact. If a photon cannot transit the toroidal hole then the PMBH has an indefinite lifetime.

Thus, what should probably be called a Schwarzschild-Hawking toroidal PMBH, is both unique and stable.

Other Forms Of The PMBH

It is instructive to consider whether other plausible 2D unit cells can produce satisfactory results when subject to a procedure like the above.

There is a popular representation of black hole 1 bit information storage based on 4 triangles, (actually 3+1 triangles as not all the triangles are equivalent), representing the 4 Planck areas^[6]. The triangle approach has the benefit of generating a closed spherical structure. It is worth considering whether a 4 triangles unit cell could lead to a satisfactory result under the above general procedure.

Superficially, it appears to be a simple matter. The 4 triangles are folded so as to produce a closed tetrahedral lattice which has no holes and the particle would therefore qualify as a "spherical" PMBH. A more detailed examination reveals that the requirements are not satisfied.

The way in which the 4 triangles unit cell is shared with adjacent unit cells determines that it has only 2 nodes; $3 \times \frac{1}{6} + 3 \times \frac{1}{2} = 2$; (the equivalent calculation for the 4 squares unit cell is $4 \times \frac{1}{4} + 4 \times \frac{1}{2} + 1 \times 1 = 4$). Thus, when folded so as to produce the 3D tetrahedral lattice each corner of the tetrahedron is only half a node and the lattice is not therefore closed and cannot stand-alone. What has been generated is actually a part of a larger closed 3D lattice

where each node is a whole node and not some fraction. The procedure therefore fails; a closed 3D lattice, with the same characteristic as the open 2D 4 triangle lattice which stores the 1 bit of information, cannot be produced. A closed 3D lattice could be produced from 2 unit cells but this would necessarily, somehow, store 2 bits of information and hence it cannot be a PMBH.

No 2D unit cell (1 bit), other than the 4 square Planck area unit cell, has so far been identified which can satisfy all the requirements and the only other commonly illustrated one, the 4 Planck area triangle unit cell, is demonstrated to fail.

Information Storage

Examining the toroidal PMBH it is possible to see what are the critical characteristics in respect of the storage of the bit. Firstly there are 2 Planck areas available as part of the structure and secondly each of these Planck areas has independent connectivity around the nodes. The existence of two independent loops around the four nodes of the (closed) 3D lattice is considered to be a highly significant feature and it offers a realistic physical mechanism for storing the 1 bit of information.

It is postulated that there is a field line running around each of these loops and that these field lines are the mechanism by which information is stored. For convenience these field lines can be described as Information field lines. A new field is therefore postulated as the means by which information is stored.

The independence of the two loops/fields offers a ready mechanism for storing either "1" or "0". For "1" the fields in each of the two loops would be in phase (and reinforce) and for "0" the fields for each of the two loops would be in anti-phase (and cancel).

The requirement to retain the 484 characteristic, so that the 1 bit of information is not destroyed, therefore leads naturally to a plausible field-based physical mechanism by which the 1 bit of information could be stored and equally naturally, whether a "0" or a "1" is stored.

Having established the "2 independent loops" mechanism for information storage it is possible now to return to Figure 1 and to overlay these loops to illustrate how the information is stored on the surface of a standard black hole. Essentially the procedure shown in Figures 2a and 2b is reversed. The result of this is shown in Figures 4a and 4b. The bit remains a property of the lattice and its connections; it is not required to "float" over the Planck area. The colour of the loops, (green/green or green/red) illustrates the phase/anti-phase situation and therefore whether a "1" or a "0" is stored in the 4 Planck area unit cell.

As expected, the field lines (which have been slightly inset from the actual connections for clarity) utilise nodes on adjacent units cells but the field lines do not share a connection between the nodes; they are independent. The absence of a need to share a connection is considered to be an essential component of this, or any other, proposed scheme as it is the absence of the need to share a connection which ensures the independence of the two field lines.

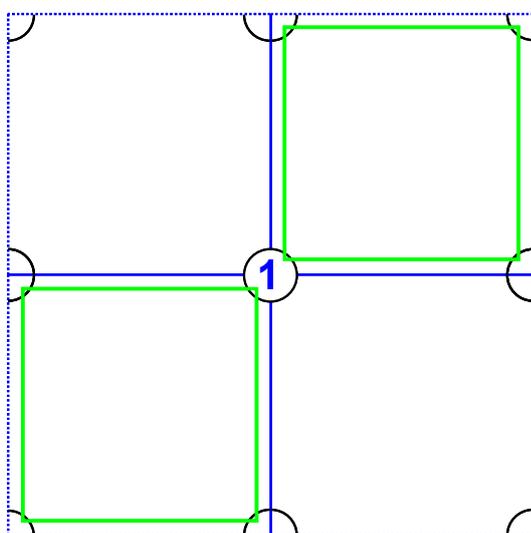


Figure 4a

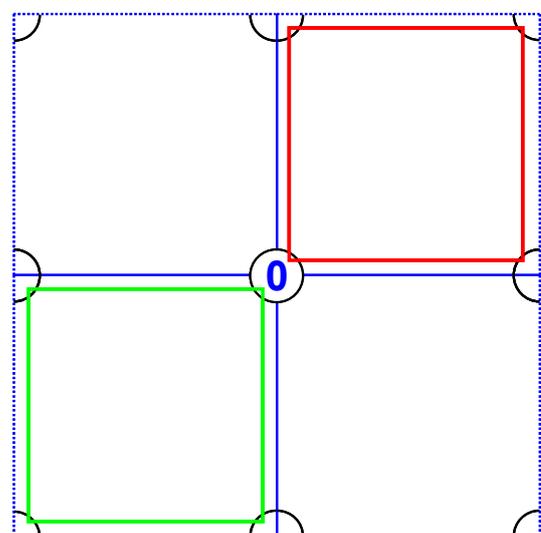


Figure 4b

The unit cells can therefore be combined edge to edge, without interference between them, to produce a surface of any size. This is shown in Figure 5.

It should be noted that if the square, (or more generally, quadrilateral), unit cells are to tessellate to produce a closed surface on a macroscopic gravitational collapse black hole then some mechanism must operate if the structure of such a macroscopic black hole is to be spherical and not toroidal. This is considered later.

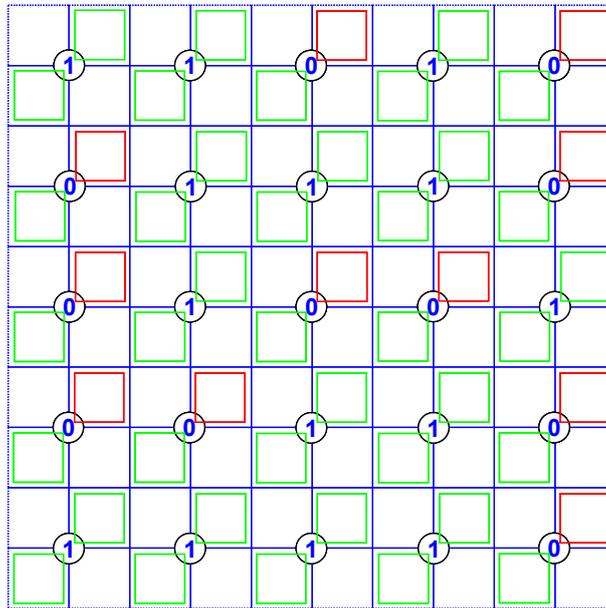


Figure 5

Dark Matter

Opinions differ as to how far down the black hole size range the Hawking radiation mechanism is valid but there is agreement that at the end point of the scale is a PMBH (which stores 1 bit of information) and that this entity is stable against the Hawking radiation mechanism^{[7][8]}. It is also described as a potential "remnant" of the Hawking radiation process^[9] when considering standard black holes^[10].

This has led in the past to it being proposed^{[11][12]}, (by Hawking himself amongst others), as a Dark Matter candidate. It was proposed that it would have been produced in the pre inflation era of the big bang and is therefore described as a Primordial PMBH (PPMBH) and thus theoretically could have been available for galaxy formation and stabilisation when required. Whilst it appeared plausible initially, it has never really been pursued as a candidate for Dark Matter because there are some major problems with it's assumed/predicted properties and behaviour. It is not a true WIMP because such an acronym would normally require the particle to interact with normal matter via the Weak Force and a PMBH does not have this ability. It is a Gravity-only "particle" which if required would lead to another, pleasing, acronym. (It would probably be the only GIMP in existence if it were to be considered to be a particle).

One of the greatest obstacles to accepting a PMBH as a Dark Matter candidate is that when two spherical PMBHs meet, they should simply merge (a process which can be repeated indefinitely). Depending on the view taken as to the point at which the Hawking radiation model becomes valid for microscopic black holes, ever larger (microscopic) black holes are produced until finally the threshold is reached and there is a large release of energy and the microscopic multiple PMBH drops back to a "remnant" PMBH. This behaviour, which can reasonably be described as "cannibalistic", would result in a serious attrition of the PPMBH "stock" over time.

There is no evidence that Dark Matter does this and commonly the contrary view is taken based on observation and modelling, i.e. that Dark Matter is largely unchanging and unreactive. Thus, despite superficially having a combination of properties which make it the ideal Dark Matter candidate, it has been disregarded.

A toroidal PMBH does not suffer from such issues.

When two (assumed) spherical PMBHs meet there is nothing to prevent them merging. There are no topological issues, (zero hole + zero hole = zero hole, "pair of trousers" diagrams are commonly used to show the merger of

spherical black holes) and there are no information loss issues because it is simply assumed that, in the absence of any plausible mechanism, this is taken care of by some means because it can be imagined that the resulting 2 bit black hole has 2 x 4 Planck areas available and hence no information is lost.

As demonstrated, when considering the storage of 1 bit of information, it is the 484 characteristic of 4 Planck area lattice which is critical and not some notional surface area of the Event Horizon of a Planck size particle, although when dealing with macroscopic black holes the surface area approach would obviously remain valid.

When two toroidal PMBHs meet there is a major obstacle to them merging. It is not immediately obvious why they should not merge to form a two hole toroid, (a process which could be repeated indefinitely with at some point breakage of connections and loss of the holes). The barrier to the merging is that the resulting two hole toroid does not comply with the 484 characteristic and hence information is lost. Loss of information is forbidden and this procedure cannot therefore happen.

For two toroidal PMBHs to merge, retaining the 484 characteristic and hence not destroying information, it is necessary that they both are opened up to produce the open cylinders as shown in Figure 2b and that these open cylinders can then join and produce a single hole toroid which satisfies the 484 characteristic. Even if this mechanism were possible (and it is considered not to be viable), then the resulting toroidal black hole would self-destruct due to the "photon transit" issue.

Merging of toroidal PMBHs is therefore prevented firstly by information inviolability and secondly by the (large) activation energy required to open (both) the closed PMBH lattices, (the process is kinetically forbidden) and even if it were possible to open the lattices the resulting 2 bit toroidal black hole would immediately self-destruct by the photon transit mechanism.

A toroidal PMBH is therefore an ideal (WIMP/GIMP) Dark Matter candidate. It is unique and extreme in many ways. It is the only durable Schwarzschild-Hawking toroidal PMBH; it has a mass of 2.2×10^{-8} kg (approx. proton x 10^{19}) and using the Schwarzschild spherical radius as a guide, it has a radius of only 2 Planck lengths (3.2×10^{-35} m, approx. proton x 4×10^{-20}). It has good claims to be the first discrete entity produced in the Big Bang, (pre inflation) and it's stability (inertness) permits it to survive into later periods and to become involved in galaxy formation when required and act as the stabiliser for the galaxy in the long term. It also has a non-Primordial source, as well as a primordial source, as a remnant of black hole evaporation. Given that it has these two sources, the distinction between primordial and non-primordial may no longer be of importance.

It offers gravitational lensing with no occlusion or other optical effects, it has no opportunity to interact with normal matter via the Weak Force and hence has essentially zero chance of being detected in any current terrestrial experiments. It has the highest gravitational field possible at its surface and the highest matter density possible. Its mass also means that a relatively modest number of particles are needed to supply the mass required for Dark Matter. Taking the mass of Dark Matter as 4.8×10^{53} kg the number of PMBHs required to supply this mass is only 2.2×10^{61} , which is a very modest number and hence the density of particles in space is also very modest. Whilst the average density is not a particularly useful value, given that Dark Matter is not evenly distributed, it is interesting to note that, taking the volume of the universe as $3.6 \times 10^{80} \text{m}^3$ as few as 3 particles per 10^{19}m^3 are required.

It also has another unique property in that, given that it is not susceptible to the Hawking radiation mechanism, it will be the only black hole which has a temperature of 0K.

It is useful, at this point, to consider the information storage mechanism and the postulated Information field and what properties this may have, because these properties will be found in the PMBH.

Gravitational Collapse

In order to illustrate the nature of the Information field it is useful to consider the production of standard black holes. The gravitational collapse may be halted due to specific properties of matter^[13]. Electron degeneracy pressure/repulsion can produce a White Dwarf Star; neutron neutron pressure/repulsion may halt the collapse at the Neutron Star stage. There is even a suggestion that a Quark Star may be possible within the shell of a Neutron Star although this is a controversial proposal.

Even though they are now known not to be completely black, the black hole name has "stuck" but they were not always known by this name; previously they were known as "Frozen (in time) Stars". With due consideration of

these various barriers to gravitational collapse and maintaining the "star" theme it is instructive to redescribe them as; "Information Shell Stars". The suggestion therefore is that whilst the mass may have been compressed, (potentially) to a "point" (although there is still much debate as to where a black hole's mass is to be found), the Information bits did not get compressed with it. It may actually be thought of as the mass and information being forcibly separated.

The description is attempting to suggest that the information of a black hole is stored in and by a lattice at the Event Horizon. The analogy with other stages in a gravitational collapse would therefore lead to a description such as; Information inviolability pressure/repulsion resists the gravitational force of the black hole's mass. The lattice resists collapse under the gravitational field which the mass itself could not.

The implication is that the unit cells carrying 1 bit of information have a repulsive force between them and that this repulsive force stabilises the lattice. This same repulsive force will exist between toroidal PMBHs.

Dark Energy

In searching for Dark Energy it is natural to consider it as a property of space; a Cosmological Constant in the style of Einstein's ill-fated attempt to produce a steady state universe. Attempts to find Dark Energy in the zero-point energy of space appear at first sight to be entirely reasonable given that it is considered to be a property of space generally. Unfortunately the zero-point energy of space results in a value which is of the order of 10^{120} times too great compared to what is required of Dark Energy^[14].

Obviously such a result draws an assortment of comments and conclusions but given the scale of the problem it is not going to be resolved by some minor tinkering. When a result of this level of incorrectness is achieved it is not unreasonable to ask; "What message is being sent?". The message may be that Dark Energy is not derived from space, the number of Planck volumes will be of the order of 10^{185} and hence controlling a calculation which is "space-driven" will always be a difficult task. If it is not derived from space then there is only one other option; it must be derived from matter - and there is considerably less matter than space which offers a realistic chance of controlling any calculation.

PMBHs have repulsive forces between them as a result of the Information field repulsion (as well as being gravitationally attractive). If the information field had a different characteristic to the d^{-2} of the gravitational field, say, equivalent to a power dependency between 0 and -2, then a basis exists for changing the attractive/repulsive balance over time as the universe expands after the Big Bang. Purely empirically, it is relatively simple through choice of parameters, to reproduce a curve which is initially dominated by gravitational attraction and subsequently dominated by information field repulsion as is found in the measured rate of expansion of the universe over time^[15]; all driven by the properties of the PMBH.

Big Bang

Toroidal PMBHs would have been produced or, more correctly, their production would have commenced, in the pre inflation era of the Big Bang, shortly after the Planck time when the gravitational force separated out from the unified force. It has been suggested that, because of the Information field, these entities will have a previously unconsidered repulsive force. It is therefore possible that this repulsive force could make a significant contribution to the inflation process generally and potentially, depending on the precise characteristics of the force at very short range, it may even be the principal driver for the inflationary process given the sudden production of a large number of particles filling the pre inflation volume.

Macroscopic (Standard) Black Holes

Logically, given the inviolability of information and the need to tessellate quadrilateral Planck areas, standard black holes should also be toroids but this cannot be and is not, the case and this results in a geometrical problem; quadrilaterals cannot tessellate to a sphere. This problem is generally either dealt with by using triangles, (which it has been shown does not work for PMBHs and hence will not work for black holes generally), or by simply ignoring the issue and allowing an artist's impression to "solve" the problem.

There is a graphic, attributed to T.B. Bakker and Dr. J.P. Van Der Scharr, (Universiteit van Amsterdam; reference to the source document appears to be lost but it may date back to an article/publication in the early 2010s), which is

reproduced on an occasional basis in popular science articles. It is essentially the "square" equivalent of the "triangle" graphic (and similar drawings may be found in other works^[16]). It is reproduced as Figure 6.

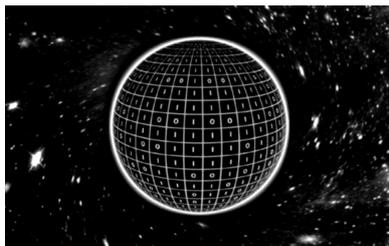


Figure 6

It is assumed that a single square represents a 2x2 Planck length area. The failure of the square cells to tessellate to a sphere and in particular the connection/interface "catastrophe" at the poles, is dealt with initially by distorting the squares, (which is entirely reasonable - there is no suggestion that the Planck area quadrilaterals cannot be distorted), but finally by fading out the cell boundaries completely. Problems such as this are not merely a "graphics" issue; in the context of information storage and information inviolability they are real.

There are many issues which need to be addressed when switching consideration from a PMBH produced directly in the pre-inflation period of the big bang to a (rotating) black hole formed by gravitational collapse but realistically some mechanism must be found for permitting the 4 Planck square unit cell to conform to a spherical geometry without compromising the information storage mechanism.

One possibility is a series of dislocations in the lattice where, essentially, unit cells are removed and the lattice rejoined so as to produce a null element. In this element no information is stored and it exists only to facilitate the tessellation of the quadrilateral unit cells to the curved surface. One example of such a dislocation, (each square should be considered as a single Planck area) adapted from^[17], is shown in Figure 7.

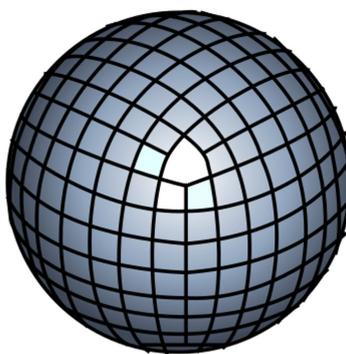


Figure 7

Such a null cell would have no information field pressure/repulsion stabilising it directly. Thus, subject only to the requirements of the field loops and their repulsion in the adjacent squares it could be compressed to a smaller size. A simplistic purely 3D geometric view of the Event Horizon topology as used here to illustrate the nature of information storage and the nature of a PMBH has no prospect of resolving such matters but it seems possible that there are potential mechanisms by which the 4 Planck area scheme could be adapted to generate a spherical geometry for a macro black hole.

Summary

1. (Schwarzschild-Hawking) PMBHs have a toroidal, not spherical, topology and only the PMBH itself is stable.
2. The connectivity of the lattice forming the Event Horizon of the toroidal PMBH offers the potential for Information field lines to store the bit of information and also, due to the two independent loops around the lattice, to store either a "0" or a "1" depending on whether the field lines are in or out of phase.

3. Toroidal PMBHs have large barriers against merging to form "multi" PMBHs and are therefore essentially inert from the point of their formation - this and their other properties make them an ideal Dark Matter candidates.
4. The barrier to their merger is derived from the inviolability of the information bit that they carry and the postulated (pairs of) Information fields forming the unit cell, which stores the 1 bit of information, repel each other thus revealing a potential (particle-based rather than space-based) origin for Dark Energy.
5. Given that the production of PMBHs would have commenced in the pre inflationary phase of the Big Bang and given that they have a repulsive force between them, it is possible that they are a contributor to the energy for the inflation stage.

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