Discussion of cosmological acceleration and dark energy

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Abstract

Following our publications, we argue that the phenomenon of cosmological acceleration has a natural explanation as a consequence of quantum de Sitter symmetry in semiclassical approximation. The explanation does not involve dark energy and other exotic concepts.

Key words: quantum de Sitter symmetry; cosmological acceleration
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1 Introduction

This article is unusual in the following sense. The usual practice is that the authors first announce their results in the form of a letter and then write a more detailed article. However, in my case, the situation is the opposite. The problem of cosmological expansion was considered in my papers published in known journals, and in the book recently published by Springer.

My works are based on large calculations. To understand them, readers must be experts not only in quantum theory, but also in the theory of representations of Lie algebras in Hilbert spaces. Therefore, understanding my results can be a challenge for many physicists. Since the problem of cosmological acceleration and dark energy is very important, I decided to write this short note, which outlines only the ideas of my approach without calculations. I hope that after reading this note, many readers will have an interest in studying my approach which considerably differs from typical approaches in the literature.

2 History of dark energy

The history of this problem is well-known. First Einstein introduced the cosmological constant Λ because he believed that the universe was stationary, and his equations can ensure this only if $\Lambda \neq 0$. But when Friedman found his solutions of equations of General Relativity (GR), and Hubble found that the universe was expanding, Einstein said (according to Gamow's memories) that introducing $\Lambda \neq 0$ was the greatest blunder of his life. After that, the statement that Λ must be zero was advocated even in textbooks, and now it is accepted by most physicists.

The explanation of this statement was as follows. According to the philosophy of GR, matter creates a curvature, so when matter is absent, there should be no curvature. That is why when the data on supernovae showed that $\Lambda \neq 0$, the impression was that it was a shock of something fundamental. However, the explanation has been found: the term with Λ in the Einstein equations was moved from the left part to the right one, and it was declared that in fact $\Lambda = 0$, but the impression that $\Lambda \neq 0$ was the manifestation of dark energy. And physicists were not confused that, with the found value of Λ, the energy of dark energy approximately equals 70% of the energy of the universe.

Let us note that currently there is no physical theory which works under all conditions. For example, it is not correct to extrapolate nonrelativistic theory to the cases when speeds are comparable to c, and it is not correct to extrapolate classical physics for describing energy levels of the hydrogen atom. GR is a successful classical (i.e. non-quantum) theory for describing macroscopic phenomena where large masses are present, but extrapolation of GR to the case when matter disappears is not physical. One of the principles of physics is that a definition of a physical quantity is a description how this quantity should be measured. The concepts of space and curvature are pure mathematical. Their aim is to describe the motion of real bodies. But the concepts of empty space and its curvature should not be used in physics because nothing can be measured in a space which exists only in our imagination. Indeed, in the limit of GR when matter disappears, space remains and has a curvature (zero curvature when $\Lambda = 0$, positive curvature when $\Lambda > 0$ and negative curvature when $\Lambda < 0$) while, since space is only a mathematical concept for describing matter, a reasonable approach should be such that in this limit space should disappear too.

Let us also note the following. A common principle of physics is that when a new phenomenon is discovered, physicists should try to first explain it proceeding from the existing science. Only if all such efforts fail, something exotic can be involved. But in the case of cosmological acceleration, an opposite approach was adopted: an exotic explanation with dark energy was accepted without serious efforts to explain the data in the framework of existing science.

3 Elementary particles in relativistic and de Sitter-invariant theories

Standard particle theory is based on Poincare symmetry where elementary particles are described by irreducible representations (IRs) of the Poincare group or its Lie algebra. This description has been first given in the famous Wigner's paper [1] and then discussed in many papers and textbooks.

The representation generators of the Poincare group Lie algebra commute according to the commutation relations

$$[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho),$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$, P^μ are the operators of the four-momentum and $M^{\mu\nu}$ are the operators of Lorentz angular momenta.

Although the Poincare group is the group of motions of Minkowski space, the description in terms of expressions (1) does not involve Minkowski space at all, and *those expressions can be treated as a definition of relativistic invariance on quantum level* (see a discussion in Refs. [2, 3]). In classical field theories, a background space (e.g., Minkowski space) is an auxiliary mathematical concept for describing real fields and bodies. In quantum theory, any physical quantity should be described by an operator, but there is no operator corresponding to the coordinate x of the background space. In quantum field theory, Minkowski space also is an auxiliary mathematical concept for describing interacting fields. Here a local Lagrangian $L(x)$ is used, and x is only an integration parameter. The goal of the theory is to construct the S-matrix in momentum space, and, when this construction has been accomplished, one can forget about space-time background. This is in the spirit of the Heisenberg S-matrix program according to which in quantum theory one can describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$.

The fact that the S-matrix is the operator in momentum space does not exclude a possibility that, in semiclassical approximation, it is possible to have a space-time description with some accuracy but not with absolute accuracy (see e.g., Ref. [3] for a detailed discussion). For example, if $\mathbf{p}$ is the momentum operator of a particle then, in the nonrelativistic approximation, the position operator of this particle in momentum representation *can be defined* as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$. In this case, $\mathbf{r}$ is a physical quantity characterizing a given particle, and is different for different particles.

In relativistic quantum mechanics, for considering a system of noninteracting particles, there is no need to involve Minkowski space. A description of a single particle is fully defined by its IR by the operators commuting according to Eq. (1) while the representation describing several particles is the tensor product of the corresponding single-particle IRs. This implies that the four-momentum and Lorenz angular momenta operators for a system are sums of the corresponding single-particle operators. In the general case, representations describing systems with interaction are not tensor products of single-particle IRs, but there is no law that the construction of such representations should necessarily involve a background space-time.

In his famous paper "Missed Opportunities" [4] Dyson notes that de Sitter (dS) and anti-de Sitter (AdS) theories are more general (fundamental) than Poincare one even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one. The transition from the former to the latter is described by a procedure called contraction when a parameter R (see below) goes to infinity. At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

The paper [4] appeared in 1972 and, in view of Dyson's results, a question arises why general theories of elementary particles (QED, electroweak theory and QCD) are still based on Poincare symmetry and not dS or AdS ones. Probably, physicists believe that, since the parameter R is much greater than even sizes of stars, dS and AdS symmetries can play an important role only in cosmology and there is no need to use them for describing elementary particles. We believe that this argument is not consistent because usually more general theories shed a new light on standard concepts. The discussion in our publications and, in particular, in this paper is a good illustration of this point.

By analogy with relativistic quantum theory, the definition of quantum dS symmetry should not involve the fact that the dS group is the group of motions of dS space. If M^{ab} ($a, b = 0, 1, 2, 3, 4,\ M^{ab} = -M^{ba}$) are the operators describing the system under consideration, then, *by definition of dS symmetry*, they should satisfy the commutation relations *of the dS Lie algebra* so(1,4), *i.e.*,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \quad (2)$$

where η^{ab} is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

The procedure of contraction from dS and AdS symmetries to Poincare one is defined as follows. If we *define* the operators P^μ as $P^\mu = M^{4\mu}/R$ where R is a parameter with the dimension *length* then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (2) become Eqs. (1). This procedure is the same for the dS and AdS symmetries.

In Refs. [3, 5, 6] it has been proposed the following

Definition: *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken, one cannot return to theory A, and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.*

As argued in Refs. [3, 5, 6], in contrast to Dyson's approach based on Lie groups, the approach to symmetry on quantum level should be based on Lie algebras. Then it has been proved that, on quantum level, dS and AdS symmetries are more general (fundamental) than Poincare symmetry, *and this fact nothing to do with the comparison of dS and AdS spaces with Minkowski space*. It has been also proved that classical theory is a special degenerate case of quantum one in the formal limit $\hbar \to 0$, and nonrelativistic theory is a special degenerate case of relativistic one in the formal limit $c \to \infty$. In the literature the above facts are explained from physical considerations but, as shown in Refs. [3, 5, 6], they can be proved mathematically by using properties of Lie algebras.

Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8$m/s and $\hbar \approx 1.054 \cdot 10^{-34}$kg·m^2/s. At the same time, they usually believe that physics should derive the value of the cosmological constant Λ, and that the solution of the dark energy problem depends on this value. However, background space in General Relativity (GR) is only a classical concept, while on quantum level symmetry is defined by a Lie algebra of basic operators.

As follows from the above remarks, the parameters $(c, \hbar, R)$ are on equal footing because each of them is the parameter of contraction from a more general Lie algebra to a less general one, and therefore those parameters must be finite. In particular, the formal case $c = \infty$ corresponds to the situation when the Poincare algebra does not exist because it becomes the Galilei algebra, and the formal case $R = \infty$ corresponds to the situation when the de Sitter algebras do not exist because they become the Poincare algebra.

Quantum de Sitter theories do not need the dimensionful parameters $(c, \hbar, R)$ at all. They arise in less general theories, and the question why they are as are does not arise because the answer is: $\hbar$ is as is because people want to measure angular momenta in kg·m^2/s, c is as is because people want to measure velocities in m/s, and R is as is because people want to measure distances in meters. The values of the parameters $(c, \hbar, R)$ in (kg, m, s) have arisen from people's macroscopic experience, and there is no guaranty that those values will be the same during the whole history of the universe (see e.g., Ref. [3] for a more detailed discussion). The fact that particle theories do not need the quantities $(c, \hbar)$ is often explained such that the system of units $c = \hbar = 1$ is used. However, the concept of systems of units is purely classical and is not needed in quantum theory.

It is difficult to imagine standard Poincare invariant particle theory without IRs of the Poincare algebra. Therefore, when Poincare symmetry is replaced by a more general dS one, dS particle theory should be based on IRs of the dS algebra. However, as a rule, physicists are not familiar with such IRs. The mathematical literature on such IRs is wide but there are only a few papers where such IRs are described for physicists. For example, an excellent Mensky's book [7] exists only in Russian.

4 Explanation of cosmological acceleration

In this section, following our publications, we argue that, in classical theory, the explanation of the fact that $\Lambda \neq 0$ is natural. Let us note first that, although the phenomenon of the cosmological acceleration is macroscopic, this does not mean that quantum theory should not be involved for the explanation of this phenomenon. Moreover, ideally every result of classical physics should be a consequence of quantum theory in semiclassical approximation, and, with the above approach, the explanation of the cosmological acceleration is natural.

Consider a system of free macroscopic bodies, i.e., we do not consider gravitational, electromagnetic and other interactions between the bodies. Suppose that the distances between the bodies are much greater than their sizes. Then the motion of each body as a whole can be formally described in the same way as the motion of an elementary particle with the same mass. In semiclassical approximation, the spin effects can be neglected, and we can consider our system in the framework of dS quantum mechanics of free particles.

The explicit expressions for the operators M^{ab} in dS IRs have been derived in Ref. [8] (see also Refs. [3, 6, 9, 10]). The representation describing a free N-body system is a tensor product of the corresponding single-particle IRs. This means that every N-body operator M^{ab} is a sum of the corresponding single-particle operators. Then, as shown in Refs. [3, 6, 8, 9, 10], in any two-body system, the relative acceleration in semiclassical approximation equals

$$\mathbf{a} = \mathbf{r}c^2/R^2 \tag{3}$$

where $\mathbf{a}$ and $\mathbf{r}$ are the relative acceleration and the relative radius vectors of the bodies, respectively.

The fact that the relative acceleration of noninteracting bodies is not zero does not contradict the law of inertia, because this law is valid only in the case of Galilei and Poincare symmetries. At the same time, in the case of dS symmetry, noninteracting bodies necessarily repulse each other (see e.g., the discussion in Ref. [11]). In the formal limit $R \to \infty$, the acceleration becomes zero as it should be.

The result (3) coincides with that in GR if the curvature of dS space equals $\Lambda = 3/R^2$, where R is the radius of this space. Therefore the cosmological constant has a physical meaning only on classical level, and, in semiclassical approximation, the parameter of contraction from dS symmetry to Poincare one coincides with the radius of dS space. Every dimensionful parameter cannot have the same numerical values during the whole history of the universe. For example, at early stages of the universe such parameters do not have a physical meaning because semiclassical approximation does not work at those stages. In particular, the terms "cosmological constant" and "gravitational constant" can be misleading. General Relativity successfully describes many data in the approximation when Λ and G are constants but this does not mean that those quantities have the same numerical values during the whole history of the universe.

5 Conclusion

The result (3) has been derived without using dS space and its geometry (metric and connection). It is a simple consequence of dS quantum mechanics in semiclassical approximation. We believe that this result is more important than the result of GR because any classical result should be a consequence of quantum theory in semiclassical approximation.

Therefore, the phenomenon of the cosmological acceleration has nothing to do with dark energy or other artificial reasons. This phenomenon is a purely kinematical consequence of dS quantum mechanics in semiclassical approximation.

References

[1] E.P. Wigner, *On unitary representations of the inhomogeneous Lorentz group.* Annals of Math. **40**, 149–204 (1939).

[2] F.M. Lev, *de Sitter Symmetry and Quantum Theory.* Phys. Rev. **D85**, 065003 (2012).

[3] F.M. Lev, *Finite Mathematics as the Foundation of Classical Mathematics and Quantum Theory. With Application to Gravity and Particle theory.* ISBN 978-3-030-61101-9. Springer, https://www.springer.com/us/book/9783030611002 (2020).

[4] F. G. Dyson, *Missed Opportunities.* Bull. Amer. Math. Soc. **78**, 635–652 (1972).

[5] F.M. Lev, *Symmetries in Foundation of Quantum Theory and Mathematics.* Symmetry **12**(3), 409 (2020).

[6] F.M. Lev, *Cosmological Acceleration as a Consequence of Quantum de Sitter Symmetry.* Physics of Particles and Nuclei Letters **17**(2), 126-135 (2020).

[7] M.B. Mensky, *Method of Induced Representations. Space-time and Concept of Particles.* Nauka: Moscow (1976).

[8] F.M. Lev, *Finiteness of Physics and its Possible Consequences.* J. Math. Phys. **34**, 490-527 (1993).

[9] F.M. Lev, *Could Only Fermions Be Elementary?* J. Phys. **A37**, 3287-3304 (2004).

[10] F.M. Lev, *Positive Cosmological Constant and Quantum Theory.* Symmetry **2**, 1945-1980 (2010).

[11] F.M. Lev, *The Problem of Interactions in de Sitter Invariant Theories.* J. Phys. **A32**, 1225-1239 (1999).